

# A Summary of Recent Developments on Metamaterial-based and Metamaterial-inspired Efficient Electrically Small Antennas\*

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## Abstract

*This paper summarizes our recent research efforts to realize efficient electrically small antenna (EESA) systems based on ideal analytical and numerical metamaterial-based antenna systems, and physically realized metamaterial-inspired antenna designs. Our theoretical and numerical studies of the radiation and resonance behaviors of the proposed metamaterial-based EESA systems, as well as our efforts to conceptualize structures which might be used to build them, have led to the discovery of several realizable metamaterial-inspired EESA systems. The measurement results confirm the numerical performance predictions.*

## 1. Introduction

Antennas that are electrically small, efficient and have significant bandwidth would fulfill many of today's emerging wireless technology requirements, especially in the areas of communications and sensor networks [1–9]. We immediately remind the reader of several quantities of interest in describing any antenna using the IEEE standard definitions of terms for antennas, including an efficient electrically small antenna (EESA) [10]. The accepted power (AP) is the power delivered to the antenna terminals from the source. It contains information about any mismatch between the source, the feedline, and the antenna. Let  $P_{\text{input}}$  be the input power of the source. Let  $Z_0$  be the characteristic impedance of both the source and the feedline, i.e., assume that the feedline is matched to the source. Let  $Z_{\text{input}}$  be the input impedance of the antenna. The reflection coefficient at the antenna is  $\Gamma = (Z_{\text{input}} - Z_0)/(Z_{\text{input}} + Z_0)$  and the accepted power by the antenna is then  $AP = (1 - |\Gamma|^2) P_{\text{input}}$ , where the mismatch or accepted power efficiency  $AE = AP/P_{\text{input}} = 1 - |\Gamma|^2$ .

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The radiation efficiency is the ratio of the total power radiated to the accepted power, i.e.,  $RE = P_{rad} / AP$ . We also introduce another term that takes into account all of the possible losses in a given antenna system, i.e., the overall efficiency. The overall efficiency of the antenna system is then the ratio of the total power radiated to the input power, i.e.,  $OE = P_{rad} / P_{input}$ . For a 1 W source, it describes what portion of that watt is radiated into the far field of the antenna system. If the directivity of an antenna system is  $D$ , its realized gain is  $G_R = OE \times D$ . In agreement with the definition given by Best in [11], an electrically small antenna in free space is defined by the constraint that  $kr_e \leq 1.0$  ( $kr_e \leq 0.5$ ), where  $r_e$  is the radius of the smallest sphere (hemisphere) enclosing the entire system and  $k = 2\pi/\lambda_0$  is the wave vector corresponding to the free space wavelength  $\lambda_0 = c/f_0$ ,  $f_0$  being the frequency of operation and  $c$  being the speed of light. For example, the effective radius of the system must be smaller than the value  $r_e = 1.0/(2\pi) = 159.16 \text{ mm}$  ( $r_e = 79.58 \text{ mm}$ ) at  $f_0 = \omega_0/2\pi = 300 \text{ MHz}$  to meet this electrically small criterion. The half-power matched VSWR fractional bandwidth, as discussed in [7], was used to compute the  $Q$  value for each system at the resonance frequency  $f_0 = \omega_0/2\pi$ : i.e.,  $Q_{VSWR}(\omega_0) = 2/FBW_{VSWR}(\omega_0)$ . The ratio,  $Q_{ratio}$ , of this  $Q_{VSWR}$  value and the Chu limit value  $Q_{Chu} = 1/ka + 1/(ka)^3$  was obtained using

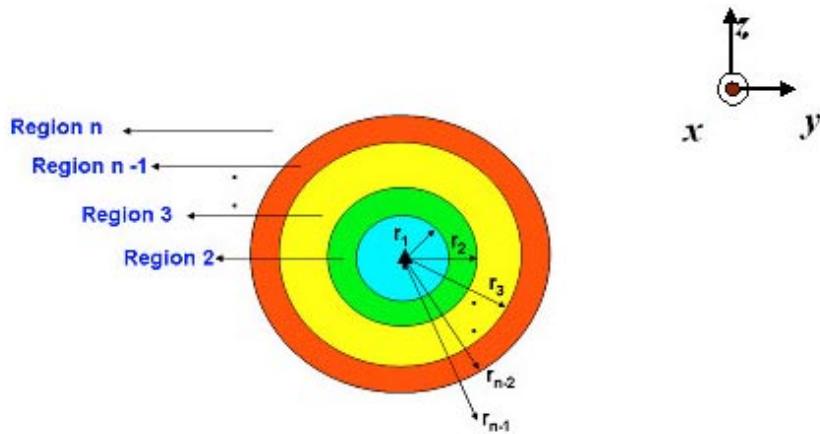
$$Q_{ratio}(\omega_0) = 2/[FBW_{VSWR}(\omega_0) \times Q_{Chu}(\omega_0) \times RE] \quad (1)$$

It is well known that an electrically small dipole antenna by itself is an inefficient radiator, i.e., because it has a very small radiation resistance while simultaneously having a very large capacitance reactance, a large impedance mismatch to any realistic power source exists. The design of reactance and resistive matching networks is a challenging task that often introduces additional constraints on the overall performance of the resulting system. There have been a wide variety of recent approaches to achieve this goal including clever packing of resonant antenna elements into this small volume using natural geometrical configurations [12], fractal curve antennas [13], space-filling curve antennas [14] and structures generated with optimization approaches have also been considered successfully [15]. In addition, numerous matching network approaches have been considered for ESAs (see, for instance, Chapter 9 of [9]). Because they rely on various combinations of (circuit and radiating) components, they all have benefits and drawbacks.

The introduction of the so-called metamaterials (MTMs), artificial materials which have engineered electromagnetic responses, and their exotic properties has provided an alternate design approach that has led to improved performance characteristics of several radiating and scattering systems [16]. The initial analytical research into the metamaterial-based EESA systems given in [17, 18] revealed that it is possible to design an EESA system formed by an electrically small electric dipole antenna radiating in the presence of either an idealized homogeneous and isotropic double-negative (DNG) spherical shell. This paper summarizes our recent research efforts to realize electrically small metamaterial-based antenna systems [19–23], and how our theoretical and numerical studies of the radiation and resonance behaviors of these metamaterial-based EESA systems, as well as our efforts to conceptualize structures which might be used to build them, have led to the discovery of several realizable metamaterial-inspired EESA systems [24, 25]. Section 2 presents an analytical model for an infinitesimal electric dipole antenna-multilayered metamaterial-based spherical shell system, which is developed and combined with a hybrid optimization method to maximize the far-field performance. The analytical model allows a complete understanding of both the near- and far-field behaviors of this antenna system. These idealized metamaterial-based results led to the development of a variety of realizable metamaterial-inspired antennas. The design details, radiation characteristics and measurement results of the electric-based planar versions of these EZ antenna systems at several different frequencies are presented in Section 3. A summary of the research findings is then given in Section 4.

## 2. Infinitesimal Dipole Antenna Surrounded by an ENG Shell: Analytical Results

The geometry of an electrically small electric dipole antenna in the presence of an ideal multilayered metamaterial spherical shell media is shown in Figure 1. The electrically small dipole antenna is assumed to be oriented along the z-axis and is positioned at the center of a set of  $N$  concentric metamaterial spherical shells, each spherical shell being an idealized homogeneous, isotropic, lossy, dispersive material. The dipole produces the fundamental,  $\text{TM}_{10}$ , radial transverse-magnetic mode. The first interior region where the dipole is located and the region exterior to the ideal metamaterial shells are assumed to be free space. The radius of the first spherical region is always greater than the half-length of the electrically small dipole, thus entirely covering the antenna with a free space medium. The multilayered spherical ideal metamaterial shells are modeled as  $N$  consecutive spheres with monotonically increasing radius values.



**Figure 1.** Geometry of the electric dipole-multilayered metamaterial shells system centered at the origin.

The far-field radiation characteristics of this electrically small antenna system are optimized based on a GA-MATLAB<sup>®</sup> based hybrid optimization model given in [22]. The relative electric permittivity  $\varepsilon_r$  and the relative magnetic permeability  $\mu_r$  of each spherical shell layer and the radius of each shell are modeled as optimization parameters. These choices allowed the analysis of all four possible material classes, i.e., double-positive (DPS), epsilon-negative (ENG), mu-negative (MNG), and double-negative (DNG) media, for each of the metamaterial layers and all spherical shell sizes. A fitness function that enabled us to quantify the metamaterial-based antenna-shell system's far-field radiated power characteristics for different metamaterial shell sizes and medium parameter values is given below, i.e., the radiated power ratio (RPR) introduced in [19]:

$$RPR = 10 \log_{10} [P_{\text{with shell}} (1 \text{ A input current}) / P_{\text{without shell}} (1 \text{ A input current})] \text{ (in dB)} \quad (2)$$

i.e., it is the dB value of the ratio of the total power radiated by the infinitesimal dipole antenna driven with a 1 A current in the presence of the spherical shell system to the total power radiated by the infinitesimal dipole antenna driven with a 1 A current in free space.

The RPR value is a metric that is used for the analytical solution to quantify the effect on the radiated power of the metamaterial shell at the far-field region when the metamaterial-based antenna is compared with the antenna itself in free space. The RPR thus, as defined by (2), gives a one-to-one comparison between

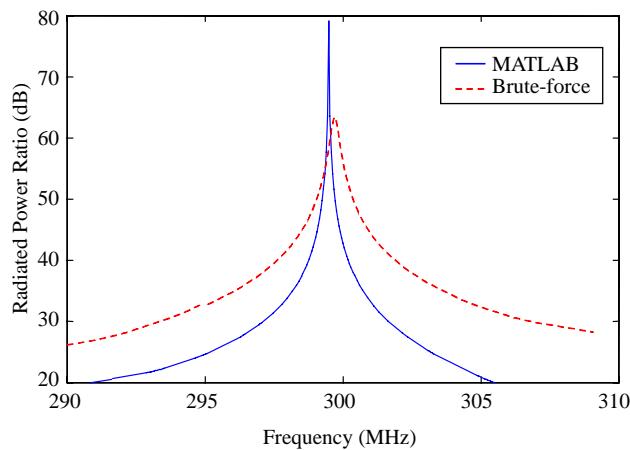
the metamaterial-based and the bare antenna systems. It is noted that when additional comparisons are made, such as the quality factor of the electrically small antenna system, the maximum radius of the antenna system is used. This eliminates any advantages of the metamaterial-based antenna system that could arise from its size. Note that in all of the cases discussed in this paper, the same electrical-sized system with the metamaterial shell replaced by a DPS medium was also tested; and no resonance effects were found.

Consider a non-dispersive, homogenous and isotropic metamaterial shell system consisting of three concentric spherical shells for which the first and third regions were taken to be free space, i.e., the overall system was effectively a single spherical ideal metamaterial shell. The second region was assigned as an ideal ENG medium with a relative permittivity that was optimized to obtain the maximum total radiated power for the given shell dimensions. The inner and outer radius values of the system were set to 10 mm and 18.79 mm, respectively. The relative magnetic permeability of each region was assumed to be that of free space, i.e., in every region  $\mu_r = 1$ . The driving frequency of the antenna was taken to be  $f_0 = 300$  MHz. The total length of the dipole was assumed to be  $l = 10.0\text{ mm} = \lambda_0/100$ . Thus the ENG region was first treated as a non-dispersive, homogeneous and isotropic metamaterial layer.

The proposed metamaterial-based electrically small antenna design produced a resonant system for the optimized relative permittivity that was equal to  $\varepsilon_r = -0.3389713$ . The calculated RPR value was 77.17 dB for the assumed ideal conditions. With the optimized relative permittivity value determined at the target frequency, the well known Drude dispersion model was incorporated into the analysis routine to obtain the corresponding RPR values at different frequencies. A lossy Drude model is given by the expression:

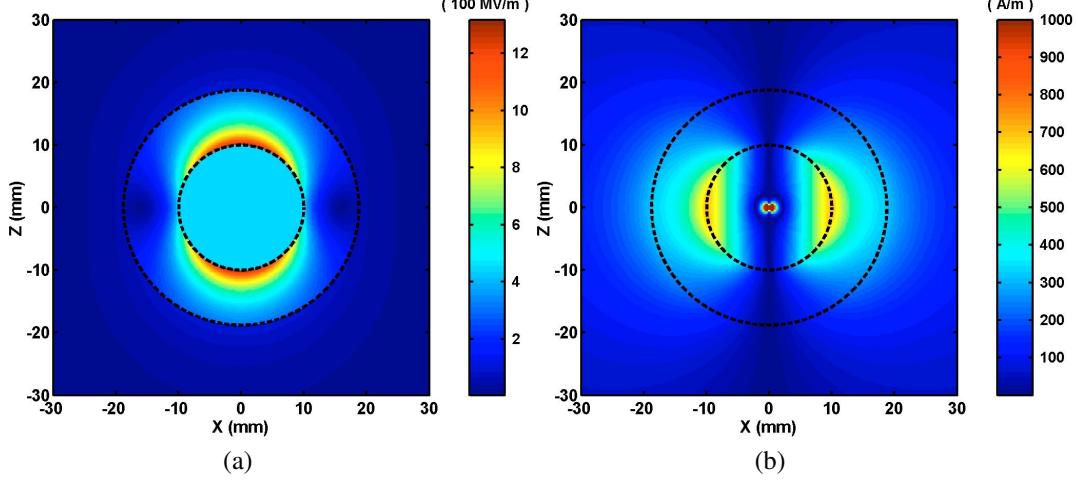
$$\varepsilon(\omega) = \varepsilon_0 \left[ 1 - \frac{\omega_p^2}{\omega(\omega - j\Gamma)} \right], \quad (3)$$

where  $\omega_p$  is the plasma frequency and  $\Gamma$  is the collision frequency. When the Drude medium is lossless,  $\Gamma = 0$ , the permittivity crosses zero at the angular frequency  $\omega = \omega_p$ . The target frequency of the antenna-metamaterial-based shell system was 300 MHz, giving  $\omega_0 = 2\pi \times 300$  MHz. Using a lossless Drude ENG shell with  $r_1 = 10\text{ mm}$  and  $r_2 = 18.79\text{ mm}$  and with  $\omega_p = 1.3389713\omega_0$  to give  $\text{Re}[\varepsilon_r(\omega_0)] = -0.3389713$ , the RPR values were computed with the hybrid optimization method. These RPR values are shown and compared with the Brute-force generated values [19] in Figure 2. The optimized RPR values were found to be larger and more narrowband than the Brute-force values. The E- and H-field distributions for the optimized antenna system are given in Figure 3.



**Figure 2.** The RPR values as functions of the source frequency obtained using the MATLAB® and the Brute-force optimization models for a  $l = 10\text{ mm}$  infinitesimal electric dipole in a resonant, lossless dispersive ENG shell with

$r_1 = 10 \text{ mm}$  and  $r_2 = 18.79 \text{ mm}$ .



**Figure 3.** (a) Electric and (b) magnetic field distributions generated at 300 MHz with the hybrid optimization method for a  $l = 10 \text{ mm}$  infinitesimal electric dipole in the lossless dispersive ENG shell with  $r_1 = 10 \text{ mm}$  and  $r_2 = 18.79 \text{ mm}$ .

Because of their small size, the inner sphere and the ENG shell act as electrically small dipole radiators. Thus, the dipole and shell act as capacitive elements. However, because the material in it is an ENG medium, the shell actually acts as an inductive element. This behavior is depicted in Figure 4. The reactance of the ENG shell at the driving frequency is given approximately by the expression

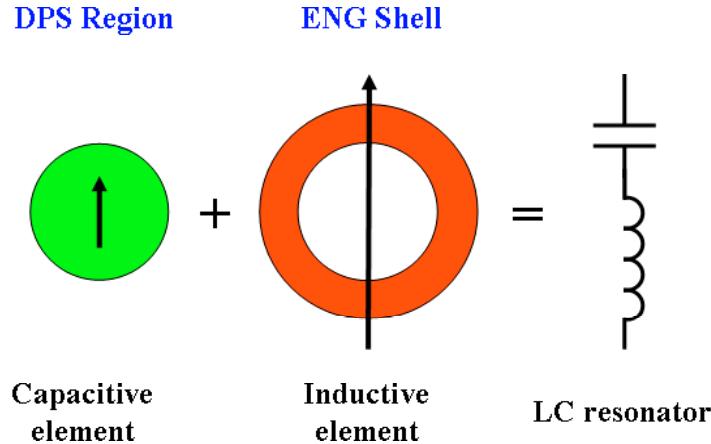
$$X_{shell} = \frac{1}{j\omega_0 C_{shell}} \approx j\omega_0 \frac{1}{\omega_0^2 |\varepsilon| \Delta r} = j\omega_0 L_{eff} \quad (4)$$

where  $\Delta r$  is the thickness of the ENG shell. Thus, the resonant frequency is

$$\omega_{res} = \frac{1}{\sqrt{L_{eff} C_{eff}}} \approx \omega_0 \sqrt{\frac{|\varepsilon| \Delta r}{C_{eff}}} \quad (5)$$

where  $C_{eff}$  is taken to be the effective capacitance of the dipole antenna and the inner shell region. For a fixed inner radius, the variation in the outer shell radius,  $r_2$ , then tunes this LC system through the resonance. A more negative relative permittivity requires a corresponding decrease in the thickness of the shell to maintain the resonance frequency at a particular value.

While theoretically interesting because of their hypothetical performance properties, the metamaterial spherical shells required for these metamaterial-based antenna systems are themselves very electrically small, and they have not been realized physically as yet. Nonetheless, progress towards possible realizations of these spherical shells has been made. For example, as recently reported in [23], a slab of DNG metamaterial constructed from a low loss unit cell whose largest dimension is  $\lambda_0/75$  has been reported at 400 MHz.



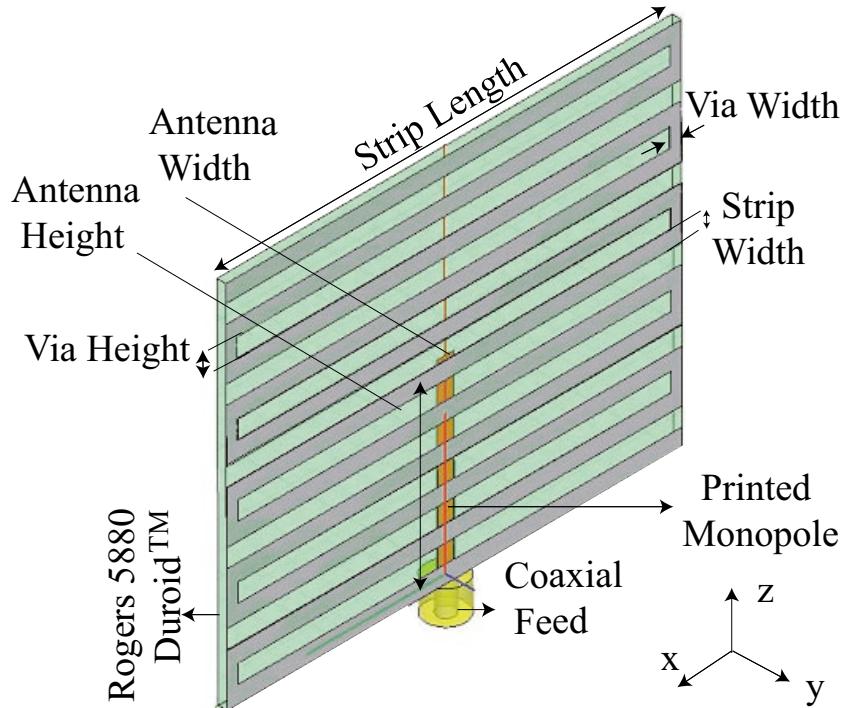
**Figure 4.** The antenna and the surrounding DPS (free space) region act as an electrically small dipole; hence, it acts as a capacitive element. The surrounding ENG shell, once excited by the dipole antenna, also acts as an electrically small dipole. However, this capacitive element, because of the presence of the negative permittivity, acts as an inductive element. The combined system thus forms an LC resonator.

### 3. Metamaterial-inspired Electric-based EZ Antenna Systems

By metamaterial-inspired we mean that resistive and reactance matching is achieved not with an effective metamaterial (spherical shell) medium but rather with an element such as an inclusion that has or could be used in a metamaterial unit cell design to realize an ENG, MNG, or DNG medium. In fact, if one of the elements introduced below is placed in a slab unit cell scattering geometry and a material property extraction code is applied to the resulting S-parameters, the metamaterial-inspired element will exhibit the ENG, MNG, or DNG properties required for the corresponding metamaterial-based antenna system, i.e., an ENG (MNG) metamaterial element must be used with an electric (magnetic) dipole radiator. These metamaterial-inspired EESA systems are easy to design; are easy and inexpensive to build; are easy to test; and, hence, are called EZ antenna systems. The EZ antenna designs are shown to be self-resonant, naturally matched to a  $50 \Omega$  source and shown to have high overall efficiencies.

A planar metamaterial-inspired electric-based EZ antenna design was proposed by integrating an electrically small printed monopole antenna with a 2D meander-line structure over a finite PEC ground plane. The bottom of the 2D meander-line is connected directly to the finite ground plane. The design specifications of the proposed 2D electric-based EZ antenna achieved with a planar 2D meander-line matching element are illustrated in Figure 5. ANSOFT HFSS (High Frequency Structure Simulator) models were used to predict the performances of the physical designs. Independent calculations of the meander-line element alone as a unit cell inclusion show that it acts like a ENG medium; this behavior agrees with the predictions in Section 2 that an ENG metamaterial is required to provide the necessary inductance to achieve a resonant system and to enable the impedance matching of the capacitive monopole antenna to the source.

The extended 2D copper surface of the meander-line serves as a current path for the induced current generated by the electric-field distribution of the electrically-small printed monopole antenna fed through a finite PEC ground plane. The 2D meander-line is positioned in very close proximity to the monopole antenna, approximately  $\lambda_0/275$ . It provides a large inductance, which again allows the combined system to form an RLC resonator. Another valid explanation for the properties of the inductance can be made



**Figure 5.** Geometry and detailed specifications of each design variable for the 2D electric-based EZ antenna.

if one visualizes each electrically-small copper strip as a transmission line terminated in a short circuit. The complex impedance of such a transmission line is inductive. The entire meander-line can then be thought of as a series of inductors that are driven by the electrically-small printed monopole antenna and, consequently, it provides enough inductance to achieve the desired matching system. Increasing the antenna height enhances the resonant coupling of the driving printed monopole antenna to the 2D meander-line; and thus, it enhances the resulting resonant response of the antenna system. A thinner substrate thickness would also enhance the resonant coupling between the antenna and the 2D meander-line. The antenna width affects the reactance part of this antenna system, where a smaller printed dipole width requires a larger inductive value to maintain the resonance effect. The mutual capacitance between the copper strips strictly depends on the distance that separates them. Increasing the distance between two copper strips in the meander-line reduces the mutual coupling; and, therefore, the resonance behavior shifts towards lower frequencies. The resonant effect, however, is reduced due to the lower copper strip density, i.e., the strip density determines the amount of current induced by the electric field distribution. Increasing the strip length and decreasing the strip width both provide larger inductance to the resonant system, but with different magnitudes. Again, this resonance phenomenon can be explained using transmission line theory. The proposed antenna design is electrically small and, thus, each strip length should be much smaller than  $\lambda_0/4$ . Consequently, the characteristic impedance of the given transmission line, i.e., the inductive value of the proposed design, should increase as a tangent function when a longer strip length is used providing that the overall length still remains smaller than  $\lambda_0/4$ . Reducing the strip width, on the other hand, provides a logarithmic increase of the inductance dictated by the strip width and the substrate thickness ratio. The electric-based 2D EZ antennas reported here were designed using Rogers 5880 Duroid<sup>TM</sup> with a 31 mils (0.787 mm) thick substrate and a 0.5 oz. (17  $\mu m$ ) electrodeposited copper.

Tables 1 and 2 give the variable specifications of three different 2D electric-based EZ antenna designs at two different frequencies: at 430 MHz and 1373 MHz. The HFSS predicted radiation characteristics of these 2D antenna systems are given in Table 3. The overall radiation efficiencies of the 2D electric-based antenna systems depend on the electrical size of the antenna system agreeing with all of our findings given above. The predicted complex impedance values and the far-field patterns for Design 3 are shown, respectively, in Figures 6a and 6b. The expected resonant nature of the input impedance is apparent; the E-field pattern is clearly a maximum along ground plane as is expected for this monopole configuration. Design 3 was fabricated and measured.

**Table 1.** 2D electric-based EZ antenna resonant frequency specifications, printed monopole antenna and ground plane dimensions.

	Design Frequency (MHz)	Monopole Antenna Height along z-axis (mm)	Monopole Antenna Width along x-axis (mm)	Ground Plane (x × y) (mm <sup>2</sup> )
Design 1	430	17.0	1.5	442 × 442
Design 2	430	28.0	1.5	500 × 500
Design 3	1373	8.3	1.5	156 × 156

**Table 2.** 2D matching meander-line element dimensions at 430 MHz and 1373 MHz.

	Total Height (mm)	Copper Strip Length (mm)	Copper Strip Width (mm)	Number of Copper Strips	Via Height (mm)	Via Width (mm)
Design 1	33.274	41	1.524	11	1.651	1.016
Design 2	46.15	49.5	4.75	5	5.6	3.1
Design 3	14.732	18	1.524	5	1.778	1.016

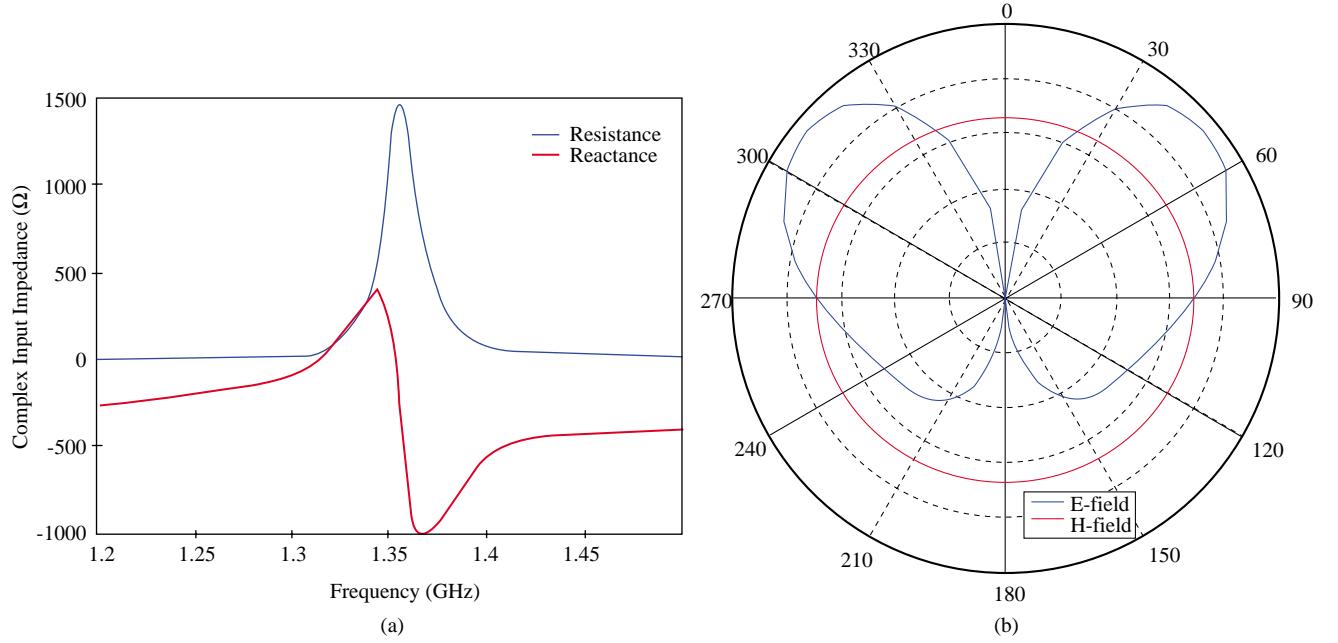
**Table 3.** Summary of the 2D electric-based EZ antenna radiation characteristics at 430 MHz and 1373 MHz.

	$F_{resonant}$ (MHz)	ka	FBW <sub>VSWR</sub> (%)	$Q/Q_{Chu}$	AP (W)	RE (%)	OE (%)	D
Design 1	429.9	0.352	2.32	5.77	0.999	57.86	57.80	1.18
Design 2	430.4	0.494	3.60	5.9	0.996	93.10	92.75	1.46
Design 3	1373	0.497	4.079	5.35	0.985	89.34	88.00	1.44

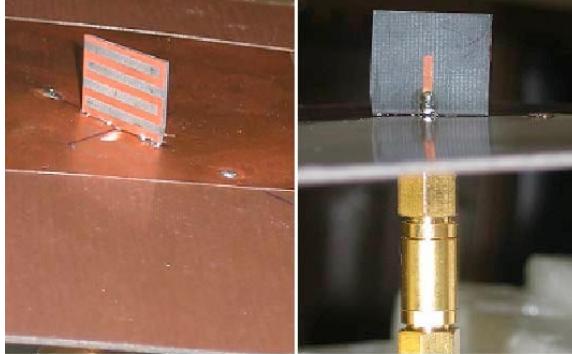
The 2D electric-based EZ antenna, Design 3, was fabricated using a photolithography technique, and was mounted on a 0.8 mm thick, 156 mm × 156 mm large copper ground plane, as shown in Figure 7. It was decided to use the reverberation chamber at NIST-Boulder for the power efficiency measurements at the recommendation of Dr. Chris Holloway [26]. A summary of the literature for all these types of reverberation chamber measurements can be found in [27].

In the NIST reverberation chamber, a dual-ridged horn antenna was used for the reference antenna for all of the measurement results discussed below. The overall efficiency of the horn antenna was determined previously to be 94% [28]. The ratio of the total power received from the AUT to that from the reference antenna gives a measure of the relative total radiated power and, hence, the *relative* overall efficiency (i.e., overall efficiency of the AUT relative to that of the reference antenna). The total power radiated by Design 3 relative to the reference horn antenna was measured. In addition, a reference monopole antenna, i.e., the same monopole antenna used in Design 3 without the matching 2D meander-line, was also measured to obtain a baseline relative overall efficiency result. The intent of these measurements was to confirm the

well-known fact that an electrically-small monopole by itself is a poor radiator and to demonstrate, by direct comparison, that in the presence of the metamaterial-inspired element it becomes, as predicted, a highly efficient radiating system.



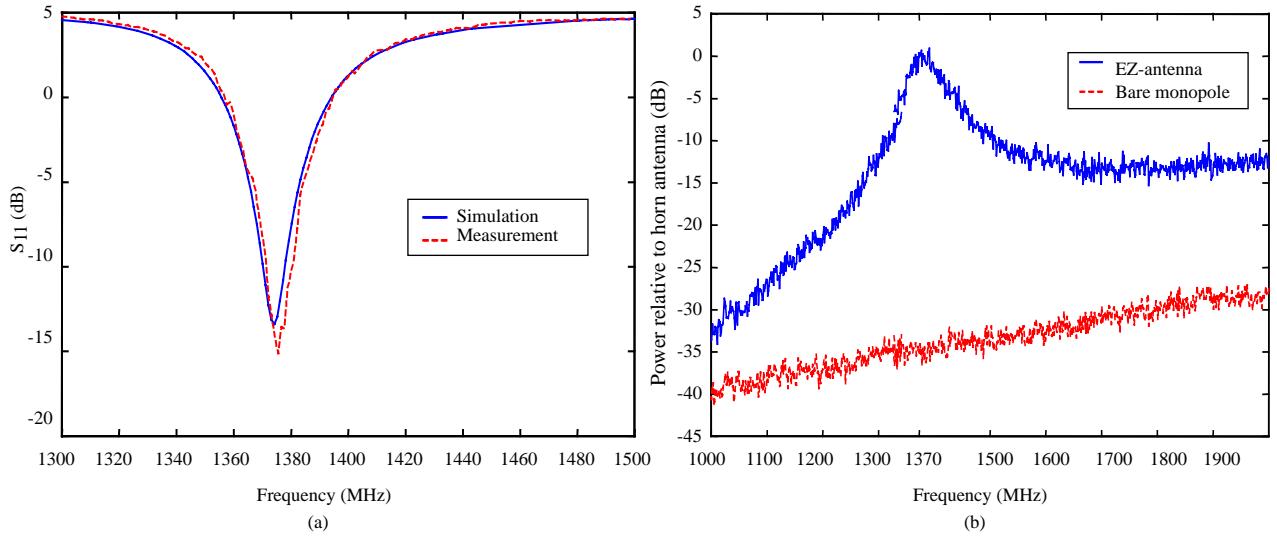
**Figure 6.** (a) Complex input impedance values, and (b) far-field E-field (blue) and H-field (red) patterns for Design 3.



**Figure 7.** The fabricated 2D electric-based EZ antenna, Design 3, which was designed to operate at 1373 MHz.

The predicted and measured  $S_{11}$  values and the measured relative total radiated power results for the 2D electric-based EZ antenna, Design 3, are given in Figures 8a and 8b, respectively. The measured relative overall efficiency is approximately equal to or slightly greater than that of the reference horn antenna at the design frequency of 1373 MHz. Since the efficiency of the reference horn antenna is 94%, these results prove that the proposed metamaterial-inspired 2D electric-based EZ antenna system has a very high overall efficiency. As expected, the reference monopole antenna, on the other hand, did not show any resonance behavior in the neighborhood of the design frequency. The measured relative total radiated power of the 2D electric-based EZ antenna at the design frequency was approximately 35 dB larger than the reference

monopole antenna. These measured results confirm that the design methodologies introduced in Section 2. are valid, and that the theoretical metamaterial-based and physical metamaterial-inspired antenna systems provide an attractive alternative to existing electrically-small antenna designs. The measured  $FBW_{VSWR}$  was 4.1%; and, because the radiation efficiency can be obtained from the  $S_{11}$  and overall efficiency values at 1376.16 MHz, the corresponding  $Q$  and  $Q/Q_{Chu}$  values at the resonance frequency were 49.15 and 4.91, respectively, in very good agreement with the predicted values.



**Figure 8.** (a) HFSS predicted and measured  $S_{11}$  values, and (b) measured relative total radiated power results obtained for the fabricated 2D electric-based EZ antenna, Design 3.

#### 4. Conclusions

The research work presented in this paper summarizes our recent developments of metamaterial-based and metamaterial-inspired EESAs. An electrically small electric dipole antenna in the presence of a multilayered metamaterial shell system was developed analytically and the total radiated power of this system was optimized using a hybrid GA-MATLAB<sup>®</sup> optimization approach. Electrically small antenna systems formed by combining an infinitesimal electric dipole with an ENG spherical shell were considered analytically. It was demonstrated that such systems can be made to be resonant with a resulting very large enhancement of the radiated power in comparison to the antenna alone in free space. The inductive nature of the ENG shell was used to compensate for the capacitive nature of the electrically small dipole antenna to form this resonant system. Dispersion characteristics of the materials were included in these analytical models.

An EESA design methodology was developed in which a resonant LC structure is achieved by introducing an appropriately designed electrically-small metamaterial-inspired ENG (MNG) element into the extreme near field of a driven electrically-small electric (magnetic) antenna. The total far-field radiated power, hence overall efficiency, and the input reflection coefficients of the 1373 MHz version of the 2D electric-based EZ antenna designs were measured in the reverberation chamber facility at NIST Boulder. These experimental results confirmed the numerical predictions of their performance. The MTM-inspired EESA design methodology introduced in this paper and the initial proof-of-concept EESAs realized with this approach may provide attractive alternatives to existing electrically-small antennas.

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