Investigation into Harmful Patterns over Multi-Track Shingled Magnetic Detection Using the Voronoi Model

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In this paper, we demonstrate that how avoiding harmful patterns during the coding process leads to have a better detection performance in Two Dimensional Magnetic Recording (TDMR) systems. We look into the generic Voronoi model and present our analysis on multi-track detection with constrained coded data. We show that two dimensional (2D) constraints imposed on input patterns result in an order of magnitude improvement in the bit error rate for TDMR systems. The use of constrained codes can reduce the complexity of 2D intersymbol interference (ISI) signal detection since lesser 2D ISI span can be accommodated at the cost of a nominal code rate loss. Furthermore, we explain the main idea of our method for generating 2D constrained sequences for TDMR systems using the Generalized Belief Propagation (GBP). Applied to a wide family of constraints, this method produces a convenient method for investigating the benefits of implementing 2D constrained waveforms in data storage systems.

***Index Terms*—TDMR systems, 2D no isolated bit constraint, multi-track detection, bit error rate, GBP algorithm.**

I. Introduction

Many novel approaches have been recently proposed to increase the areal densities for magnetic recording systems beyond $1\,\mathrm{Tb/in}^2$. These technologies include heat assisted magnetic recording (HAMR) [1], bit patterned media (BPM) [2] and two dimensional magnetic recording (TDMR) [3]. TDMR is a purely systems driven approach centered around sophisticated signal processing and coding algorithms [4], [5] to achieve high areal densities; and can provide additive gains over HAMR and BPM technologies. In TDMR, the bits are densely packed leading to 2D inter-symbol interference (ISI) and media noise that need to be mitigated via 2D signal processing algorithms. Shingled magnetic recording (SMR) is a first step towards TDMR, where, the existing wide read/write heads are used to write tracks in an overlapping/shingled fashion. Since the TDMR technology is still emerging, several models for the TDMR channels at various interfaces are being proposed to facilitate the design of a viable read-channel architecture.

TDMR channel models for the media can be classified into a) discrete grain models, b) Voronoi media models and c) micro magnetic media models. Discrete grain models consider the recording medium as a tiling of grains of various known shapes on a 2D plane. Voronoi models treat the distribution of grain centers as a point process. Micro magnetic models consider the sizes, shapes and distribution of the grains closely resembling the actual magnetic recording medium [6]. Recently, a communication theoretic framework [5] was proposed to model TDMR channels by considering 2D ISI from physical characteristics along with noise effects from the media and read electronics. Though the jitter noise in [5] is modeled using a first order approximation and Gaussian statistics, the framework can be used to include the second order noise statistics empirically computed from the Voronoi model.

Efficient coding and signal processing algorithms are central for realizing areal density gains within TDMR systems. Several 2D signal detection algorithms have been proposed over the last few years with an eye towards getting close to the MAP/ML performance [1]. Sullivan *et al.* [7] have proposed an iterative detection algorithm for 2D ISI using 1D row-column detectors that iteratively exchange information to make soft-decision on the bit. A low complexity version of the algorithm optimized for separable 2D ISI is proposed in [8]. Chen and Srinivasa [9] have proposed a 2D joint equalization and detection (JTED) algorithm that combines a self iterating 2D equalizer with multi-row-column detectors over the full signal span to iteratively achieve near MAP performance with tractable complexity. Generalized belief propagation (GBP) algorithm is a different class of signal detection algorithms that uses message passing between regions instead of the message passing between nodes as seen in the traditional belief propagation algorithm. The performance of the GBP algorithm in relation to the MAP/ML algorithm is not known and requires a rigorous theoretical framework to study this. The GBP algorithm was studied by Khatami and Vasić [10] for different TDMR channel models.

Matcha et al. [11] have recently proposed a 2D partial response maximum likelikhood for 2D ISI channels using a 2D SOVA equivalent algorithm. The proposed method is within 1.5 dB of the full JTED performance with noise prediction [5]. While we have advanced methods for signal processing towards a full blown TDMR system, it is of practical interest to study shingled magnetic recording (SMR) systems using multitrack detection to assess areal density gains for read channels of immediate timely interest.

[1]2D MAP detection is NP hard.

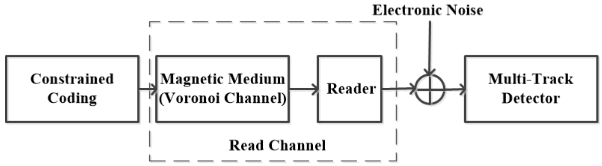

Fig. 1. The block diagram of TDMR system includes constrained coding, read channel and multi-track decoder. Prior to being written to the channel, user data is first encoded by a constrained code in which occurrence of harmful patterns is forbidden or suppressed (constrained coding).

In this paper, we investigate into the Voronoi based media model to study the harmful patterns over multitrack shingled recording systems. Through realistic quasi micromagnetic simulations studies [12], we identify 2D data patterns that contribute to high media noise. By avoiding such patterns at the source, we evaluate the performance of a multitrack detector and assess areal density gains over various TDMR system parameters. This paper is organized as follows. In Section II, we describe a system model that includes the read, write procedure along with the detection scheme. In Section III, we describe the noise characteristics based on empirical results from the Voronoi media model and quantify the signal-to-noise ratio (SNR) using the peak power constraints. In Section IV, we describe a procedure for creating 2D constrained patterns satisfying the *no isolated bit* (n.i.b) constraint. We evaluate the performance of various SMR systems through simulations, followed by conclusions in Section V. Finally, we explain the main idea of our method for generating 2D constrained sequences achieving the 2D noiseless channel capacity for a wide family of constraints based on the GBP algorithm in Appendix.

II. System Model

The study of the effects of jitter noise on the signal processing algorithms in TDMR systems requires sophisticated channel models that include the random grain distribution on the recording medium. Fig. 1 provides a block diagram of the TDMR system utilized in this paper. We model the TDMR channel using a Voronoi model [6] where each grain is specified by a Voronoi region. 2D constrained sequences from the input alphabet $\mathcal{X} = \{-1, +1\}$ are written on the magnetic medium. Without loss of generality -1 and $+1$ denote the bits 0 and 1, respectively. A magnetic reader is utilized to read data written on the Voronoi channel, and produces symbols from the alphabet $\mathcal{Y} = \mathbb{R}$. The electronic noise is modeled by an Additive White Gaussian Noise (AWGN) with variance σ_e^2. The noisy output is equalized and detected using a multi-track detector in order to retrieve the symbols written on the Voronoi channel. In this section, we introduce the details of the model used in this paper.

TDMR channel models typically involve three components: a) media model: models the distribution of grains on the medium b) write-head procedure: models the magnetization process of grains while writing data on to the Voronoi channel and c) read-head procedure: models the readback signal. For the sake of completeness we give these models as described in [13].

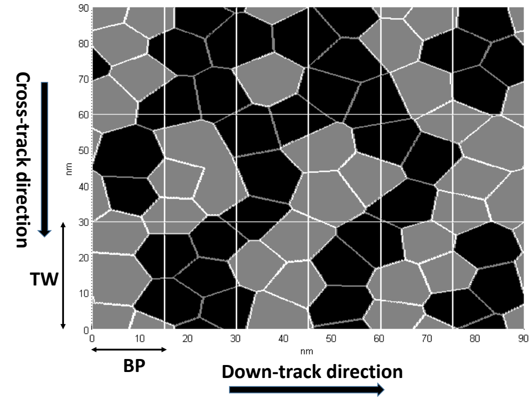

Fig. 2. An example of the Voronoi channel model. The grains on the medium are modeled as the Voronoi regions formed from the random grain centers generated using Poisson disk process. The centers are separated by at least CTC=10nm. The rectangular cells indicate the channel bits. All grains whose centers are within a bit region are polarized according to the bit value. The bit size is TW×BP=30 nm×15 nm.

A. Magnetic Medium

In TDMR systems, a grain is the smallest region that is uniformly magnetized. A Voronoi model is utilized to simulate the non-ideal features of the medium [6]. In this model, the medium is visualized as a random tiling of Voronoi regions where each Voronoi region represents a grain on the medium. A Voronoi region $\mathcal{S}$ with a center c is the collection of points on a 2D (Euclidean) plane that are closer to the center c than to any other grain center. The points on the boundary of a Voronoi region are equidistant from their two closest centers. There is more than one way to to generate a random Voronoi tiling of a plane. In this paper, the grain centers are generated according to the Poisson-disk distribution with boundary sampling introduced in [14]. In the following, the Voronoi model parameters are introduced. The Poisson-disk distribution is characterized by the center-to-center (CTC) distance, the minimum permissible distance between any two grain centers.

A rectangular grid is defined on the medium, where each rectangular cell corresponds to a channel bit and is characterized by

- Bit Period (BP): the length of each bit in the down-track direction.
- Track-Width (TW): the length of each bit in the cross-track direction.

An example of the TDMR channel generated based on the Voronoi model is given in Fig. 2.

B. Write Procedure

Constrained sequences are written on the Voronoi channel at this step. The channel input signal $x(t)$ is defined by

$$x(t_1, t_2) = \sum_i \sum_j x_{i,j} \Pi_{TW}(t_1 - iTW)\Pi_{BP}(t_2 - jBP),$$

where $x_{i,j} \in \mathcal{X}$ is the symbol which will be written on the $(i, j)^{\text{th}}$ bit area and

$$\Pi_T(t) = \begin{cases} 1, & 0 \le t < T, \\ 0, & \text{otherwise.} \end{cases}$$

In TDMR systems, the write head procedure does not have any a *priori* information of the grain shapes, sizes and positions on the magnetic medium. Therefore, the bit areas are considered to be in the form of rectangles. The write head induces a magnetization pattern on the track directly below its head at the center of each rectangular cell such that all grains whose centers are within the bit area are appropriately polarized according to the value of $x_{i,j}$.

C. Read Procedure

We model the read-head response to be a 2-D Gaussian pulse with a span of three bit-areas in both directions. The 2-D Gaussian pulse is characterized by the pulse widths PW_{50} and TW_{50} at half-amplitude in the down-track and cross-track directions, respectively. The read-back signal is the total of contributions of all grains on the medium. Let us suppose that the read-head picks up magnetization only from $m \times n$ cells. The read-back signal samples are obtained by convolving the magnetization of the recording medium with the 2D read-head response and then sampling the resulting signal at the centers of bit areas. As a result, the read-head output samples $y_{i,j}$ at the center of the $(i,j)^{\text{th}}$ cell depending only on the polarity of the grains in the $m \times n$ neighborhood around the $(i,j)^{\text{th}}$ cell, denoted as $C_{i,j}$. Any change in the read-back signal due to the shift in the grain-boundaries is considered as media noise. This depends not only on the regions of the grains in $C_{i,j}$, but also on their polarities. Therefore, this noise is correlated in both down-track and cross-track directions and is data-dependent. The read-head parameters are chosen such that the ISI span does not exceed 3×3 bit areas throughout the simulations i.e., $m = n = 3$. In the sequel, we model the noise of Voronoi channel as an additive noise.

Let $s_{i,j} \in \mathbb{R}$ be the read-back signal samples of the ideal magnetic medium, where the bit areas considered to be rectangular, and $y_{i,j} \in \mathbb{R}$ be the read-back signal samples of the non-ideal medium for the bit cell (i,j). The ideal read-back signal, $s_{i,j}$, is obtained by convolving the magnetization pattern of ideal medium with the read-head impulse response $h(t_1, t_2)$ and sampling at each center of bit area in the down-track direction. We consider that the read-head impulse response of $m \times n = 3 \times 3$ span. Therefore, the ideal read-back signal can be written as:

$$s_{i,j} = \sum_{k_1=-1}^{+1} \sum_{k_2=-1}^{+1} x_{i-k_1,j-k_2} h_{k_1,k_2}, \tag{1}$$

where h_{k_1,k_2} is the sampled output of impulse response of read-head,

$$h_{k_1,k_2} = \iint_{\mathcal{A}_{k_1,k_2}} h(t_1,t_2)\, dt_1\, dt_2, \tag{2}$$

that $\mathcal{A}_{k_1,k_2}$ is the rectangular area of bit (k_1,k_2).

D. Detection Scheme

The read-back signal is detected using a multi-track MAP detector based on the Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm. The BCJR algorithm provides the *a-posteriori* probability (APP) for each symbol given the detector input samples. The BCJR algorithm operates on the trellis representing the noiseless channel output sequences. It recursively computes the forward state metrics and the backward state metrics, which are combined with the branch metrics to produce the APP of each symbol. A detailed description of the BCJR algorithm can be found in [15].

In this study, we extend the BCJR algorithm to operate on the symbols denoted by $x_{C_{i,j}} = \{x_{k,l} | (k,l) \in C_{i,j}\}$ instead of operating on the bit $x_{i,j}$. $x_{C_{i,j}}$ denotes the information bits contributing to the readback sample $y_{i,j}$, i.e., the bits at $C_{i,j}$ where $C_{i,j}$ denotes the 3×3 region with (i,j) as its center. In order to compute the bit error rate (BER) by using the BCJR algorithm, each trellis branch b at time k is assigned the metric

$$\mu(b_k) = p(y_{i,j}|b_k) p(b_k|x_{C_{i,j}}), \tag{3}$$

where $x_{C_{i,j}}$ is the starting (left-hand) 3×3 input state of b_k and $y_{i,j}$ is the output of the Voronoi channel corresponding to the input state $x_{C_{i,j}}$. In fact, $p(y_{i,j}|b_k)$ indicates the noise distribution of Voronoi channel.

In order to model the effect of irregular boundaries on the read-back signal of ideal magnetic medium, we define the media noise $n_{i,j}$ as an additive noise which is dependent on each 3×3 span of input data, the coded signal which is written on the Voronoi channel. Thus, we incorporate the effect of media noise to the the ideal read-back signal, $s_{i,j}$, in the following form

$$y_{i,j} = s_{i,j} + n_{i,j}, \tag{4}$$

where $y_{i,j}$ is the noisy read-back signal sample for the bit (i,j). As we assume the read-head response to be a 2D truncated Gaussian pulse which spans 3×3 bit areas, the media noise is only dependent on a 3×3 span of the input data. Based on extensive simulations, the media noise distribution is shown to be close to Gaussian distribution for most cases of the input states of a 3×3 bit region [6]. Thus, we approximated the media noise distribution of each state of input $x_{C_{i,j}}$, i.e. each 3×3 bit region, with the Gaussian distribution with mean and variance dependent on input information. Therefore, we have

$$p(y_{i,j}|b_k) = \frac{1}{\sqrt{2\pi\sigma^2_{x_{C_{i,j}}}}} \exp\left(\frac{-(y_{i,j} - s_{i,j} - m_{x_{C_{i,j}}})^2}{2\sigma^2_{x_{C_{i,j}}}}\right), \tag{5}$$

where $m_{x_{C_{i,j}}}$ and $\sigma^2_{x_{C_{i,j}}}$ are the mean and variance of the media noise for the case of 3×3 input state $x_{C_{i,j}}$. For the case of ideal medium where the bit areas are in the form of rectangles, the discrete read-head output or "ideal values", $s_{i,j}$, is obtained by convolving the magnetization pattern of the ideal recording medium with the read-head impulse response and sampling at the center of bit area in the down-track direction. The second term $p(b_k|x_{C_{i,j}})$ of the branch metric denotes the *a-priori* probability by which constrained sequences are generated. The *a-priori* probabilities for all the forbidden input patterns by the constraint are set to zero. The BER is obtained by applying the BCJR to the given trellis.

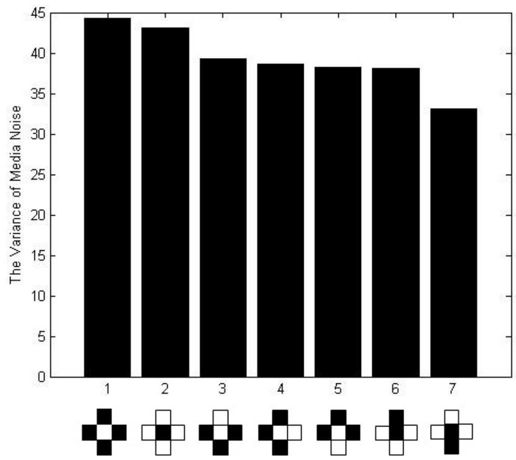

Fig. 3. Observation of media noise variance for the Voronoi channel with the parameters CTC = 7nm, BP = 7.5nm and TW = 16nm. In the 3×3 input patterns 0 and 1 are represented by white and black, respectively. It is shown that the harmful patterns for the Voronoi channel with 2D ISI are ones eliminated by the no isolated bit constraint.

III. Noise Characteristics of the TDMR Systems

In TDMR systems, the primary source of noise comes from irregular boundaries of grains and the random distribution of grain centers [6]. In addition, the noise distribution in TDMR is dependent on input information bits written on the Voronoi channel as the polarity of grains effects on the read-back signals. The Voronoi model with 2D ISI [13] is considered as the magnetic recording channel. As explained in the Subsection II-C, we first consider an ideal magnetic medium, where bit areas assumed to be rectangular, and then we apply the effect of irregular boundaries as the "media noise" [6] by an additive noise which is added to the read-back signal of ideal magnetic medium. We have analyzed the media noise characteristics for a read-head response of 2D truncated Gaussian pulse with 3×3 span. Fig. 3 shows the media noise variance for different 3×3 input patterns. The media noise variance is greater for the input patterns with more transitions in cross-track and down-track direction. The most harmful input patterns are the ones with consecutive transitions in both cross-track and down-track directions. We have also observed the same characteristics of media noise studied with a more realistic channel model in [12]. In the sequel, we introduce three definitions of SNR corresponding to overall noise, media noise and the electronic noise in TDMR systems.

Let $h_{i,j}(p,q)$ be the discrete-time response of $(i,j)^{\text{th}}$ bit. These response coefficients are random and dependent on the position and shape of grains within the bit area. The average bit-response is obtained by taking the expectation on these random response coefficients

$$h(p,q) = \mathbb{E}_{PQ}\big\{h_{i,j}(p,q)\big\}, \tag{6}$$

where P and Q are random variables indicating the distribution of the grain positions and sizes. Therefore, the above averaging is taking into account all possible realizations of grain positions and sizes. The read-back signal samples without considering the electronic noise are

$$y_{i,j} = \sum_p \sum_q x_{i-p,j-q} h_{i-p,j-q}(p,q), \tag{7}$$

where $x_{i,j}$ is the symbol written on the $(i,j)^{\text{th}}$ bit-cell. Furthermore, the ideal read-head output, $s_{i,j}$, is obtained by considering the average discrete-time output of $(i,j)^{\text{th}}$ bit area such that

$$s_{i,j} = \sum_p \sum_q x_{i-p,j-q} h(p,q). \tag{8}$$

The peak value of read-back signal, V_p, is defined by

$$V_p^2 = \sum_p \sum_q |h(p,q)|^2. \tag{9}$$

The media noise comes from the random perturbations of $h_{i,j}(p,q)$ around the average response $h(p,q)$. Therefore, the variance, or, equivalently the energy of media noise σ_m^2 is obtained by

$$\sigma_m^2 = \mathbb{E}_{P,Q}\big\{\sum_p \sum_q |h_{i,j}(p,q) - h(p,q)|^2\big\}. \tag{10}$$

Then, we can define three SNRs for a TDMR system according to the above definitions such that

$$\begin{aligned} \text{SNR} &= 10\log_{10}\big(\frac{V_p^2}{\sigma_m^2+\sigma_e^2}\big), \\ \text{SNR}_{\text{Media}} &= 10\log_{10}\big(\frac{V_p^2}{\sigma_m^2}\big), \\ \text{SNR}_{\text{Elec}} &= 10\log_{10}\big(\frac{V_p^2}{\sigma_e^2}\big), \end{aligned} \tag{11}$$

where SNR is the overall SNR, and $\text{SNR}_{\text{Media}}$ and SNR_{Elec} are the SNRs corresponding to the media and electronic noise, respectively. A detailed description of these SNRs can be found in [5].

IV. Evaluation of utilizing constrained coded data in TDMR systems

In this section, we investigate the performance gain due to using the constrained input waveforms in TDMR systems based on the BER criterion. Let $R_{\mathcal{C}}$ denote the rate of the code with a given constraint $\mathcal{C}$. To achieve the same storage density for a constrained coded system and an uncoded system, the rate loss due to the constrained input sequence is compensated by scaling the bit size of the coded system by a factor of $R_{\mathcal{C}}$. This reduction in bit size is justifiable only if the gain in performance due to constrained coding is high enough to compensate the effect of increased ISI. Therefore, the choice of the constrained code is dependent to the parameters of the TDMR system as well as the detector.

A. Constrained Codes

The harmful data patterns contributing to high media noise are avoided using constrained codes. In our method, constraints are imposed locally and are given by a set of admissible input data patterns. Not all sequences of symbols from the input alphabet may be stored. Let $\mathcal{S}_X \subset \{-1,+1\}^{N\times N}$ be a set of admissible $N\times N$ patterns for the constraint $\mathcal{C}$. An indicator function is defined as

$$f(\mathbf{x}) = \begin{cases} 1, & \mathbf{x}\in\mathcal{S}_X \\ 0, & \text{other} \end{cases} \tag{12}$$

where $\mathbf{x}$ is a random pattern. Consider a set of bit cells a in the neighborhood of the cell (i,j) on the medium. Let x_a be a 2D input pattern indexed by the elements of a, and $f_a(x_a)$ be the indicator function of x_a. f_a is referred to as a local constraint. As an example, the elements of a may correspond to the set of 3×3 bit cells with the center bit (i,j). The indicator function of the $N\times N$ pattern is the product of all local constraints

$$f(\mathbf{x}) = \prod_a f_a(x_a). \tag{13}$$

Here, we introduce the *2D no isolated bit* constraint which is utilized in the simulations of this paper.

2D No Isolated Bit (n.i.b) Constraint: The input patterns which is a 1 surrounded by -1's and a -1 surrounded by 1's are forbidden. This constraint is known as the no isolated bit constraint. The local constraint for the $(i,j)^{\text{th}}$ cell of the code is given as

$$f_a(x_{i,j-1},x_{i-1,j},x_{i,j},x_{i+1,j},x_{i,j+1}) =$$
$$\begin{cases} 0, & x_{i-1,j}=x_{i+1,j}=x_{i,j-1}=x_{i,j+1}\neq x_{i,j}, \\ 1, & \text{other}, \end{cases} \tag{14}$$

where $x_{i,j}$ is the symbol written on $(i,j)^{\text{th}}$ bit area of the magnetic medium. The forbidden patterns for the local kernel of this constraint are

$$\begin{matrix} & \mathbf{-1} & \\ \mathbf{-1} & \mathbf{1} & \mathbf{-1} \\ & \mathbf{-1} & \end{matrix} \quad \text{and} \quad \begin{matrix} & \mathbf{1} & \\ \mathbf{1} & \mathbf{-1} & \mathbf{1} \\ & \mathbf{1} & \end{matrix}$$

in each 3×3 span of input pattern. We explain our method for generating 2D constrained sequences achieving the 2D noiseless capacity in Appendix.

B. Evaluation of performance of the Constrained Codes

We have simulated the TDMR system at different combinations of parameters denoted by TDMR(i), $1\le i\le 4$ as given in the Table I. The parameters chosen are realistic physical values and the parameter combinations TDMR(i) differ only in the size of each bit.

Fig. 4 compares the performances of the TDMR(1) and TDMR(2) configurations as a function of track-width in the absence of the electronic noise. In this comparison, the TDMR(1) configuration is used with unconstrained input while the TDMR(2) configuration is used with the n.i.b constraint on the input sequences. To compensate for the rate loss due to the constrained coding, the BP in TDMR(2) in relation to the BP in TDMR(1) is chosen to match the rate of the n.i.b constrained code 0.9238, i.e.,

$$\frac{\text{BP}_{\text{TDMR(2)}}}{\text{BP}_{\text{TDMR(1)}}} \simeq 0.9238.$$

As it is shown in Fig. 4, using 2D constrained sequences in TDMR systems improves the performance by about an order of magnitude.

In TDMR systems, decreasing the bit size to the limits comparable to the grain size leads to a reduction in the signal to noise ratio (SNR) due to augmentation of the media noise. Since the media noise is caused by the polarity change in magnetization of neighboring grains due to consecutive transitions in the input data, low-pass constraints that restrict the consecutive transitions can be deployed to increase the SNR. Therefore, the constrained sequences can be deployed to reduce the harmful effects of the media noise. In addition to this, constrained coding reduces the state space of the detector and hence reduces the computational complexity of the detector.

We add the electronic noise to the readhead's output of both TDMR(3) and TDMR(4) systems in order to find the SNR trade-off point where the 2D n.i.b constraint can compensate the effects of both media and electronic noises. The TDMR(3) is a constraint free system, but the TDMR(4)'s input sequences

TABLE I
RS_{CT} (RS_{DT}) DENOTES THE READER RESPONSE SPAN IN CROSS-TRACK (DOWN-TRACK) DIMENSION. CTC IS ASSUMED TO BE 7 NANOMETERS. ALL THE PARAMETERS IN THE TABLE ARE SPECIFIED IN NANOMETERS. $\star$ INDICATES THAT THE PARAMETER IS VARIED IN THE SIMULATIONS.

	TW	BP	RS_{CT}	RS_{DT}	TW_{50}	PW_{50}
TDMR(1)	$\star$	7.5	30	21	20	14
TDMR(2)	$\star$	7	30	21	20	14
TDMR(3)	16	7.5	30	21	20	14
TDMR(4)	16	7	30	21	20	14

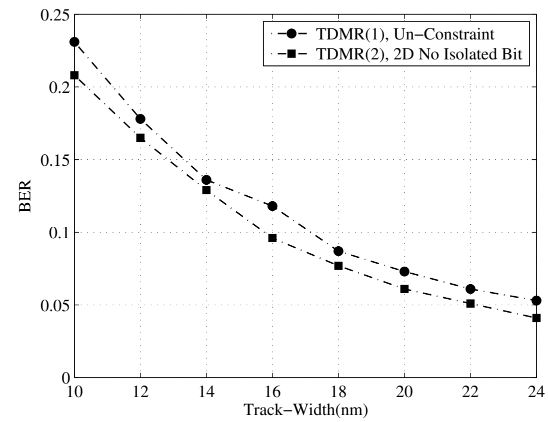

Fig. 4. BER comparison of un-coded (TDMR(1)) and coded (TDMR(2)) systems with different bit areas and the same storage density in absence of electronic noise. Constrained coding improves the performance by avoiding the data patterns that result in high media noise

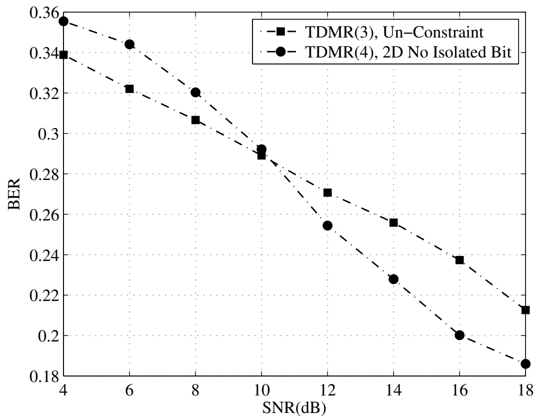

Fig. 5. BER comparison of un-coded (TDMR(3)) and coded (TDMR(4)) systems with different bit areas and the same storage density in the presence of electronic noise. The impact of constrained coding is higher at high SNRs as the media noise dominates the electronic noise in thie region. SNR=10 dB is a trade-off point where the performance gain due to constrained coding compensates the effects of both media and electronic noise.

obey the n.i.b constraint. Similar to the previous experiment, the BP of TDMR(4) is altered based on the rate of n.i.b constraint. Let σ_e^2 denotes the variance of electronic noise which is assumed to be Gaussian $\mathcal{N}(0, \sigma_e^2)$ and statistically independent of the media noise components in two-dimensions. The signal to noise ration corresponding to the electronic noise was defined in (11), where V_p^2 is the peak value of read-back signal. It can be seen from Fig. 5 that the $\mathrm{SNR}_{\mathrm{Elec}}$ =10db is the trade-off point between the performance gains of n.i.b constrained coding and the effects of electronic and media noise. Constrained coding is targeted to handle the media noise, and hence is suitable to use at high SNRs where the media noise dominates the electronic noise. Therefore, at high SNRs, higher gains in BER performance is observed with the n.i.b constraint giving an overall improvement over the TDMR(3) system.

V. Conclusion

We have studied the pattern dependent characteristics of media noise in TDMR using a Voronoi media model. We have identified the no isolated bit constraint that reduces the impact of media noise. We have studied the performance of the constrained coding using a BCJR based multi-track detector. When the media noise is high compared to the electronic noise, the rate loss due to constrained coding is compensated by the performance gains when compared against uncoded systems with the same storage density. Finally, we have introduced the main idea of our method for generating 2D constrained sequences based on the GBP algorithm.

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References

[1] R. E. Rottmayer, "Heat-Assisted Magnetic Recording," *IEEE Transactions on Magnetics*, vol. 42, no. 10, pp. 2417–2421, Oct. 2006.

[2] B. Terris, T. Thomson, and G. Hu, "Patterned Media for Future Magnetic Data Storage," *Microsyst. Technol.*, vol. 13, no. 2, pp. 189–196, Nov. 2006.

[3] R. Wood, M. Williams, A. Kavcic, and J. Miles, "The Feasibility of Magnetic Recording at 10 Terabits per Square inch on Conventional Media," *IEEE Transactions on Magnetics*, vol. 45, no. 2, pt.2, pp. 917–923, Feb. 2009.

[4] Y. Chen and S. G. Srinivasa, "Joint self-iterating equalization and detection for Two-Dimensional Intersymbol-Interference," *IEEE Transactions on Communications*, vol. 61, no. 8, pp. 3219–3230, Aug. 2013.

[5] S. G. Srinivasa, Y. Chen, and S. Dahandeh, "A communication-theoretic framework for 2-DMR channel modeling: Performance evaluation of coding and signal processing methods," *IEEE Transactions on Magnetics*, vol. 50, no. 3, pp. 6–12, Mar. 2014.

[6] A. Krishnan, R. Radhakrishnan, B. Vasić, A. Kavcic, W. Ryan, and F. Erden, "2-D magnetic recording: Read channel modeling and detection," *IEEE Trans. Magn.*, vol. 45, no. 10, pp. 3830 –3836, Oct. 2009.

[7] N. Singla, J. O'Sullivan, R. Indeck, and Y. Wu, "Iterative decoding and equalization for 2-d recording channels," *Magnetics, IEEE Transactions on*, vol. 38, no. 5, pp. 2328–2330, Sep 2002.

[8] Y. Wu, J. O'Sullivan, N. Singla, and R. Indeck, "Iterative detection and decoding for separable two-dimensional intersymbol interference," *Magnetics, IEEE Transactions on*, vol. 39, no. 4, pp. 2115–2120, July 2003.

[9] Y. Chen and S. Srinivasa, "Joint self-iterating equalization and detection for two-dimensional intersymbol-interference channels," *Communications, IEEE Transactions on*, vol. 61, no. 8, pp. 3219–3230, Aug. 2013.

[10] M. Khatami and B. Vasić, "Generalized belief propagation detector for TDMR microcell model," *IEEE Trans. Magn.*, vol. 49, no. 7, pp. 3699–3702, Jul. 2013.

[11] C. K. Matcha, S. G. Srinivasa, S. Khatami, and B. Vasić., "Two-dimensional noise-predictive maximum likelihood method for magnetic recording channels," *IEEE International Symposium on Information Theory and its Applications*, Oct. 2014.

[12] B. Vasic, S. M. Khatami, Y. Okamoto, Y. Nakamura, Y. Kanai, J. R. Barry, S. W. McLaughlin, and E. B. Sadeghian, "A study of TDMR signal-processing oportunities based on quasi-micromagnetic simulations (invited talk)," in *Proc. The Magnetic Recording Conference (TMRC)*, Berkeley, CA, USA, August 11–13 2014, pp. 1–5.

[13] M. Khatami and B. Vasic, "Constrained coding and detection for tdmr using generalized belief propagation," in *IEEE Int. Conf Commun. (ICC 2014)*, June 2014, pp. 3889–3895.

[14] D. Dunbar and G. Humphreys, "A spatial data structure for fast poisson-disk sample generation," *ACM Trans. Graph.*, vol. 25, no. 3, pp. 503–508, Jul. 2006. [Online]. Available: http://doi.acm.org/10.1145/1141911.1141915

[15] L. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate (corresp.)," *IEEE Trans. Inf. Theory*, vol. 20, no. 2, pp. 284 – 287, Mar. 1974.

[16] G. Sabato and M. Molkaraie, "Generalized belief propagation for the noiseless capacity and information rates of run-length limited constraints," *IEEE Trans. Commun.*, vol. 60, no. 3, pp. 669–675, Mar. 2012.

[17] J. Yedidia, W. Freeman, and Y. Weiss, "Constructing free-energy approximations and generalized belief propagation algorithms," *IEEE Trans. Inf. Theory*, vol. 51, no. 7, pp. 2282 – 2312, Jul. 2005.

Appendix

2D GBP-Based Constrained Sequence Generator

In this section, we explain the main idea of using the GBP algorithm for generating 2D constrained sequences achieving the maximum entropy of constraints. The Generalized Belief Propagation (GBP) algorithm was utilized to estimate the 2D noiseless capacity for a wide family of constraints in [13] and [16]. In order to obtain the capacity achieving distribution over the set of admissible patterns, the GBP as a capacity estimation algorithm is utilized. Then, we generate 2D constrained sequences to write on a storage medium according to the obtained distribution of GBP. In order to utilize the GBP algorithm for generating 2D constrained sequences, we need to introduce some preliminary definitions. We start introducing a graphical representation for the procedure as the GBP is a message passing algorithm.

- *The factor graph* corresponding to a local constraint is a bipartite graph consisting of a set of variable nodes $V_{i,j}$ (information bits) and a set of factor nodes $f_{C_{i,j}}$ (local constraints) in which a variable node $V_{i,j}$ is connected to a factor node $f_{C_{i,j}}$ if and only if $V_{i,j}$ is an argument of $f_{C_{i,j}}$. Fig. 6 shows an example of a factor graph for a 4×4 bit grid
- *The region graph* of the given graphical model is generated according to the cluster variation method [17]. In order to obtain the region graph, each parent region is specified by a set of variable nodes which are connected to the same factor node, i.e. for the set $C_{i,j}$ the parent region $\mathcal{R}_i$ is equal to $\{V_{C_{i,j}}, f_{C_{i,j}}\}$, where $V_{C_{i,j}} = \{V_{i,j} | (i,j) \in C_{i,j}\}$. The other sub-regions are established by taking the intersection, the intersections of the intersection, and so on of the parent regions. The region graph of the 4×4 cell square of variable nodes with 3×3 spans of the local constraints is established in Fig. 7.
- *Beliefs of each region* $\mathcal{R}_i$ is the product of all the local factors in that region multiplied by all messages coming into region $\mathcal{R}_i$ from outside region [17]. For each basic region $\mathcal{R}_i$, we have $2^{|\mathcal{R}_i|}$ beliefs of all possible cases for $|\ \mathcal{R}_i\ |$ variable nodes, in the binary domain, participated in the parent region which is denoted by $b_{\mathcal{R}_i}(x_{\mathcal{R}_i})$ where $x_{\mathcal{R}_i} \in \{-1, +1\}^{|\mathcal{R}_i|}$. The belief function is a good approximation of the marginal probability distribution of variables in a region.

The 2D-noiseless channel capacity of a $N \times N$ array of 2D constrained sequence is defined by

$$\mathcal{C}_{2D} = \lim_{N \to \infty} \frac{\log_2(Z(N,N))}{N^2}, \tag{15}$$

where $Z(N,N)$, the 2D partition function, specifies the number of legitimate patterns of the size $N \times N$ which satisfy the constraint. We can obtain the 2D partition function by applying the GBP to the factor graph of a $N \times N$ variable nodes with local constraints. Since the Helmholtz free energy is $F_H = -\ln Z$, computing Z can be done by obtaining the region-based free energy estimate. If the GBP algorithm is used to estimate beliefs of each region $b(x_{\mathcal{R}_i})$ (or the marginal probability of each region), region-based free energy $\hat{F}_H$ can be written as

$$\hat{F}_H = \sum_{R_i \in \mathcal{R}} c_{R_i} \sum_{x_{R_i}} b_R(x_{R_i}) \bigg(\ln b_{R_i}(x_{R_i}) - \ln \prod_{a \in A_R} f_a(x_a) \bigg), \tag{16}$$

where $\mathcal{R}$ is the set of all regions, c_{R_i} is the counting number defined as

$$c_{R_i} = 1 - \sum_{S \in \mathcal{S}_{R_i}} c_S \tag{17}$$

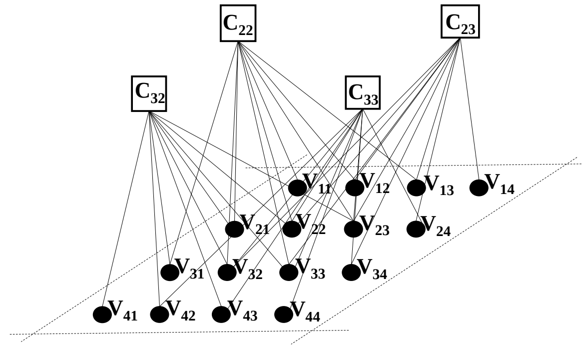

Fig. 6. Factor graph of a 4×4 variable nodes with local constraints.

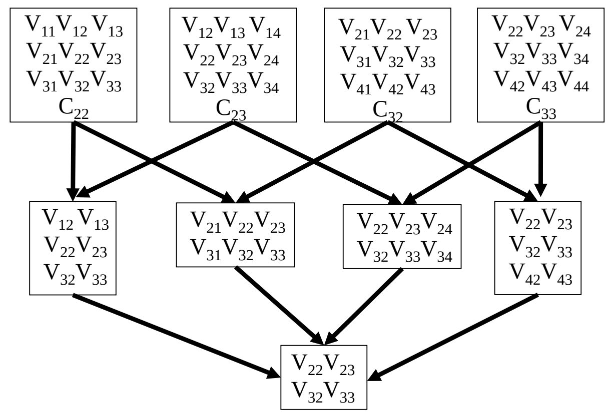

Fig. 7. A region graph of a 4×4 variable nodes generated utilizing the parent to child scheme [17]

where $\mathcal{S}_{R_i}$ is the set of regions which are super-regions of R_i, x_{R_i} is the set of variables in R_i, and finally A_{R_i} is the set of local kernels in region R_i.

The main point is that the GBP as a capacity estimation algorithm provides the distribution over the admissible input patterns $\mathcal{S}_X$ which achieves the 2D noiseless channel capacity of constraint. The 2D noiseless capacity or maximum entropy of a source $\mathbf{X}$ is achieved with uniform distribution of the source outputs. Therefore, the probability distribution achieving the 2D noiseless channel capacity with constraint coding is

$$p(\mathbf{x}) = \begin{cases} \frac{1}{|\mathcal{S}_X|}, & \mathbf{x} \in \mathcal{S}_X \\ 0, & \text{other} \end{cases} \tag{18}$$

Therefore, if we want to generate 2D constrained sequences with maximum entropy, we need to obtain beliefs of regions (or the marginal probability of each region) which establishes a uniform distribution over the set of admissible patterns $\mathcal{S}_X$. In order to achieve this goal, we obtain beliefs of regions for the case that the probability of occurrence of each legitimate pattern is equi-probable by using the GBP algorithm. Notice that the beliefs of forbidden patterns become 0, or, equivalently, the probability of occurrence of such patterns are 0. Then according to the obtained belief distribution achieving the 2D noiseless channel capacity, 2D constrained sequences are generated to write on a storage medium.