GBP-Based Detection and Symmetric Information Rate for Granular Tiling Model in TDMR

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***Abstract*—Two dimensional magnetic recording (TDMR) is a new paradigm in data storage which envisions densities up to $10\ \mathrm{Tb/in^2}$ as a result of drastically reducing bit to grain ratio. In order to reach this goal aggressive write (shingled writing) and read process are used in TDMR. Kavcic *et al.* proposed a simple magnetic grain model called the granular tiling model which captures the essence of read/write process in TDMR. Capacity bounds for this model indicate that 0.6 user bit per grain densities are possible, however, previous attempt to reach capacities are not close to the channel capacity. In this paper, we provide a truly two-dimensional detection scheme for the granular tiling model based on generalized belief propagation (GBP). Factor graph interpretation of the detection problem is provided and formulated in this paper. Then, GBP is employed to compute marginal a posteriori probabilities for the constructed factor graph. Simulation results show huge improvements in detection. A lower bound on the symmetric information rate (SIR) is also derived for this model based on GBP detector.**

I. Introduction

In the pioneer work of Wood *et al.* [1], TDMR was first introduced as a technology for approaching magnetic recording density of $10\ \mathrm{Tb/in^2}$ resulting 0.5 user bits per grain. The key component of TDMR is the aggressive write process called shingled writing [1], that is, to achieve narrow track width comparable to the grain diameter, each sweep of the write head partially overlaps and overwrites previously written adjacent track. Evidently, this creates challenges including severe 2-D inter-symbol interference (ISI) and modeling of a nonlinear channel.

Several TDMR channel models have been studied in [2], [3]. In [4], Kavcic *et al.* introduced a relatively simple 2-D magnetic grain model of TDMR channel. The granular tiling model assumes that the medium consists of tiles where each tile represents a grain and is chosen from a predefined set of tile shapes (prototiles). In [5], bounds on capacity of the 1-D version of the granular tiling model are derived. The upper and lower bounds for the capacity of the 2-D granular tiling model is also provided in [4] showing the feasibility of 0.6 user bits per grain, which translates to $12\ \mathrm{Tb/in^2}$ at typical $20\ \mathrm{Teragrains/in^2}$ media grain density. This paper considers a novel detection scheme for the granular tiling model.

In [6] and [7], BCJR [8] based detection algorithms are provided for the granular tiling model. In [6], Pan *et al.* provided a BCJR detection algorithm operating on one track at a time. The BCJR detector is then followed by a rate 0.25 serially concatenated convolutional code (SCCC) and puncturing is used in order to achieve the highest possible rate at 10^{-5} bit error rate (BER). In [7], Carasino *et al.* generalized the detection algorithm to two rows. Their detector considers output from two rows at a time over two adjacent columns. Soft decision feedback is also employed to further improve the estimation of the grain states. Considering more grain and data states, the two row BCJR outperforms the one row BCJR in [6]. Moreover, the iterative exchange of soft information is employed to further improve the performance.

These detection techniques are multi-track rather than 2-D, and consequently are not able to fully exploit the 2-D nature of the channel. As opposed to conventional systems where signal processing algorithms operate on a single track of data, large performance improvements are achieved by processing read-back data as 2-D array. This paper presents a novel 2-D detection technique using generalized belief propagation (GBP) detector [9], [10]. The application of GBP algorithm in the context of 2-D detection has been first considered by Shental *et al.* [11]. GBP operates on undirected graphical models e.g. factor graph, to infer a posteriori probabilities using a message passing algorithm which takes into account the existence of cycles in a graph. In [11], it was shown that the performance of GBP is almost the same as maximum a posteriori (MAP) detector. In this paper, the factor graph [12] interpretation of the 2-D detection problem is provided for granular tiling model. The variables and factor nodes are specified and formulated for the 2-D detection. Then, we applied GBP to operate on the constructed factor graph. Moreover, based on the GBP detector performance, a numerical lower bound on the symmetric information rate (SIR) of the granular tiling model is provided.

This paper is organized as follows: In Section II, we review the granular tiling model and the read/write process is introduced. Section III outlines the detection problem and provides necessary formulation for factor graph representation of the detection problem. In Section III-B, GBP is explained as a tool to compute the marginalized probabilities of the grain states of the factor graph. An estimation of SIR for the channel model based on the numerical results of detection is provided in Section IV. Finally, Section V concludes the paper.

II. Channel model

In this paper, we adapt a simple granular tiling model for the channel model introduced in [4], [6], [7]. In this model, four possible types of rectangular tiles are considered to cover the whole medium where each tile represents a grain. The grain sizes are 1×1, 2×1, 1×2 and 2×2 relative to one channel bit to be of 1×1 grain size. Fig. 1 shows the four grain types. The 1×1 subgrains are labeled from A-I called grain states [7] which are used in the state definition in detection. The arrangement of prototiles on a medium is unknown to the reader, thus the reader naively assumes that the medium is composed of 1x1 prototiles only. The channel bits are assumed to be written in a raster-scan fashion from left to right and top to bottom. Since the track-width (channel bit length) can be smaller than the grain size, it is possible for a grain to be overwritten several times before the final polarization remains. In this model, the channel bits written on the grain states A, C, E and I dominate the final polarization of their grains. We denote $\mathrm{S}_D = \{A, C, E, I\}$ as the set of "dominant states" and $\mathrm{S}_E = \{B, D, F, G, H\}$ as the set of "erased states" since the values written on these channel bits are erased and overwritten. To put the write process in a rigorous form, we define the write function, ϕ, as

$$y_{i,j} = \phi(\mathbf{X}, s_{i,j}) = \begin{cases} x_{i,j}, & s_{i,j} \in \mathrm{S}_D \\ x_{i+1,j}, & s_{i,j} \in \{B, H\} \\ x_{i,j+1}, & s_{i,j} \in \{D, G\} \\ x_{i+1,j+1}, & s_{i,j} = F. \end{cases}$$

where $\mathbf{X}$ is the input data array, $y_{i,j}$ and $s_{i,j}$ are the readback and grain state of the channel bit (i, j), respectively.

We consider a random realization of a medium with area $N \times M$ channel bits, where M and N are large. Let us suppose that within this realization of the medium, there are N_1, N_2, N_3 and N_4 tiles of sizes 1×1, 2×1, 1×2 and 2×2, respectively. For large N and M, we have

$$NM = N_1 + 2N_2 + 2N_3 + 4N_4. \tag{1}$$

The probabilities of occurrence of 1×1, 2×1, 1×2 and 2×2 tiles are denoted by p_1, p_2, p_3 and p_4, respectively and are defined as

$$p_i = \frac{N_i}{N_1 + N_2 + N_3 + N_4} \tag{2}$$

where $i \in \{1, 2, 3, 4\}$. The density in channel bits per grain is defined as

$$D = p_1 + 2p_2 + 2p_3 + 4p_4. \tag{3}$$

In this paper, we assume $D = 2$, similarly to [4], [6], [7]. The probability of occurrence of 2×1 and 1×2 tiles is assumed to be equal (symmetry condition $p_2 = p_3$). By choosing $p_2 \in [0, 0.5]$, it can be easily seen that the occurrence probabilities, $p_i, i \in \{1, 2, 3, 4\}$, are obtained from (3) and the fact that $\sum_{i=1}^{4} p_i = 1$. Assuming large M and N, $N_i, i \in \{1, 2, 3, 4\}$ can be approximated by solving the system of linear equations of (1) and (2).

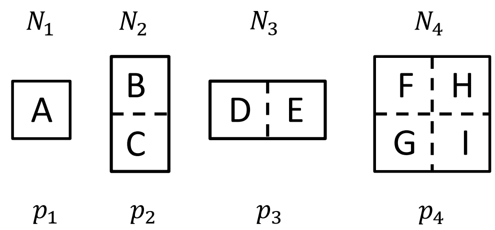


Fig. 1. The discrete grain model with 4 sizes of 1×1, 2×1, 1×2 and 2×2, the probability of occurrence of the tiles and the total number of tiles of each size [4], [7].

Fig. 1 demonstrates that any channel bit (1×1 subgrain) takes on one of the nine states A-I. We now define the grain state probability for a single channel bit based on occurrence probability of the tiles which helps us compute the state probabilities for 2×2 channel bits. The grain states probabilities are necessary in detection to compute the regional probability in GBP described in Section III-B. Let S denote the grain state of a single channel bit and $P(S = s) = p_s$ as the grain state probability which is calculated as

$$p_s = \frac{\mathcal{N}(S = s)}{N \times M} \tag{4}$$

where $s \in \{A, B, .., I\}$ and $\mathcal{N}(S = s)$ denotes the number of channel bits with state s in the medium. Clearly,

$$\begin{aligned} &\mathcal{N}(S = A) = N_1, \\ &\mathcal{N}(S = B) = \mathcal{N}(S = C) = N_2, \\ &\mathcal{N}(S = D) = \mathcal{N}(S = E) = N_3, \\ &\mathcal{N}(S = F) = \mathcal{N}(S = G) = \mathcal{N}(S = H) = \mathcal{N}(S = I) = N_4, \end{aligned}$$

and the state probabilities can be calculated using (4).

III. Detection

Let $\mathbf{X} = [x_{i,j}]$ and $\mathbf{Y} = [y_{i,j}]$ with $x_{i,j}, y_{i,j} \in \{0, 1\}$ be the input and the readback channel bits, respectively in an $N \times M$ channel bit medium. The input, $x_{i,j}$, is assumed to be independent and identically distributed (i.i.d.), equiprobable binary random variable. The optimal maximum a posteriori (MAP) detector calculates the joint conditional a posteriori probabilities $\mathrm{P}(\mathbf{X}|\mathbf{Y})$. Since the input bits are independent, we have

$$\mathrm{P}(\mathbf{X}|\mathbf{Y}) = \prod_{i=1}^{N} \prod_{j=1}^{M} \mathrm{P}(x_{i,j}|\mathbf{Y}) \tag{5}$$

$$\mathrm{P}(x_{i,j}|\mathbf{Y}) = \sum_{s} \mathrm{P}(x_{i,j}|\mathbf{Y}, S_{i,j} = s)\mathrm{P}(S_{i,j} = s|\mathbf{Y}) \tag{6}$$

where $s \in \{A, B, .., I\}$ and $S_{i,j}$ denotes the grain state of channel bit (i, j). Moreover, $\mathrm{P}(x_{i,j}|\mathbf{Y}, S_{i,j} = s)$ only depends on $y_{i,j}$, which is

$$\begin{aligned} \mathrm{P}(x_{i,j}|\mathbf{Y}, S_{i,j} = s) &= \mathrm{P}(x_{i,j}|y_{i,j}, S_{i,j} = s) \\ &= \begin{cases} \frac{1}{2} & s \in \mathrm{S}_E \\ 1 & x_{i,j} = y_{i,j}, s \in \mathrm{S}_D \\ 0 & x_{i,j} \neq y_{i,j}, s \in \mathrm{S}_D. \end{cases} \end{aligned}$$

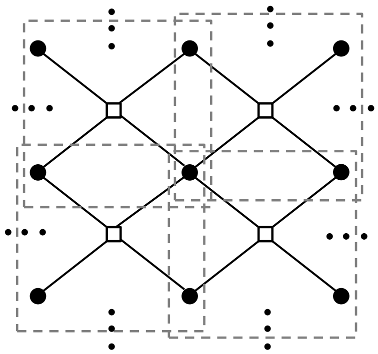


Fig. 2. The factor graph representation of the channel model used in detection and corresponding regions in GBP.

Now, (6) can be rewritten as follows

$$
\begin{aligned}
&\mathrm{P}(x_{i,j}=y_{i,j}|\mathbf{Y})=\frac{1}{2}\times\mathrm{P}(S_{i,j}\in\mathrm{S}_E|\mathbf{Y})+1\times\mathrm{P}(S_{i,j}\in\mathrm{S}_D|\mathbf{Y}),\\
&\mathrm{P}(x_{i,j}\neq y_{i,j}|\mathbf{Y})=\frac{1}{2}\times\mathrm{P}(S_{i,j}\in\mathrm{S}_E|\mathbf{Y}).
\end{aligned}
$$

This shows that in order to calculate $\mathrm{P}(x_{i,j}|\mathbf{Y})$, it is sufficient to obtain $\mathrm{P}(S_{i,j}\in\mathrm{S}_E|\mathbf{Y})=1-\mathrm{P}(S_{i,j}\in\mathrm{S}_D|\mathbf{Y})$. Therefore, the detection problem is reduced to finding the erasure probabilities for all the bits.

A. Factor Graph Interpretation of Detection

Finding MAP solution for $\mathrm{P}(S_{i,j}|\mathbf{Y})$ is intractable for the large N and M, therefore approximate inference methods such as message passing algorithms are employed. As an intermediate tool for message passing algorithms, the factor graph [12] representation of the channel model used in detection is shown in Fig. 2. The variable nodes are shown as circles referring to grain state of the channel bit and the factor nodes are depicted as squares expressing the local dependencies of the grain states. A factor node represents a function of the variable nodes which are connected to it. Fig. 2 demonstrates the fact that the grain state of a channel bit directly depends on its neighboring grain states since the largest grain size is 2×2. Every factor node is connected to a 2×2 channel bit region. Each variable node can accept 9 grain states (A-I), however, in a 2×2 channel bit region, there are only 393 possible grain states out of the total 9^4 grain states. Let region R represent a 2×2 channel bit region. We denote $\mathbf{Y}_R$ as the read-back bits and S_R as the grain state of region R. For the factor graph representation of the detection problem, $\mathrm{P}(S_R|\mathbf{Y}_R)$ is the factor node function connected to the variable nodes of region R.

Using the Bayes' rule,

$$
\mathrm{P}(S_R|\mathbf{Y}_R)=\frac{\mathrm{P}(\mathbf{Y}_R|S_R)\mathrm{P}(S_R)}{\mathrm{P}(\mathbf{Y}_R)}\propto\mathrm{P}(\mathbf{Y}_R|S_R)\mathrm{P}(S_R) \quad (7)
$$

Since $\mathbf{Y}_R$ is given, $\mathrm{P}(\mathbf{Y}_R)$ can just be considered as the normalization factor.

1) Calculating Grain State Probabilities $P(S_R)$: As it is mentioned, there are 393 possible grain states in a 2×2 region. Here, we compute the probability of the grain states based on p_s, $s\in\{A,B,..,I\}$ given in Section II. For simplicity, let S_R be $[S_{1,1},S_{1,2},S_{2,1},S_{2,2}]$ and let $\mathbf{s}=[s_{1,1},s_{1,2},s_{2,1},s_{2,2}]$ be one of the possible grain states in a 2×2 region. Using the chain rule, we have

$$
\begin{aligned}
\mathrm{P}(S_R=\mathbf{s})=&\,\mathrm{P}(S_{1,1}=s_{1,1})\mathrm{P}(S_{1,2}=s_{1,2}|S_{1,1}=s_{1,1})\\
&.\mathrm{P}(S_{2,1}=s_{2,1}|S_{1,2}=s_{1,2},S_{1,1}=s_{1,1}) \qquad (8)\\
&.\mathrm{P}(S_{2,2}=s_{2,2}|S_{2,1}=s_{2,1},S_{1,2}=s_{1,2},S_{1,1}=s_{1,1})
\end{aligned}
$$

To make the problem clear, the probability calculation is explained through an example. Based on (8), the probability of grain state $\mathrm{P}(S_R=[A,D,F,H])$ is calculated as follows:

$$
\begin{aligned}
\mathrm{P}(S_R=[A,D,F,H])=&\;\mathrm{P}(S_{1,1}=A)\mathrm{P}(S_{1,2}=D|S_{1,1}=A)\\
&.\mathrm{P}(S_{2,1}=F|S_{1,1}=A,S_{1,2}=D)\\
&.\mathrm{P}(S_{2,2}=H|S_{1,1}=A,S_{1,2}=D,S_{2,1}=F)
\end{aligned}
$$

where $\mathrm{P}(S_{1,1}=A)=p_A$ which is calculated in (4). Then,

$$
\begin{aligned}
\mathrm{P}(S_{1,2}=D|S_{1,1}=A)&=\mathrm{P}(S_{1,2}=D|S_{1,2}\notin\{E,H,I\})\\
&=\frac{\mathrm{P}(S_{1,2}=D,S_{1,2}\notin\{E,H,I\})}{\mathrm{P}(S_{1,2}\notin\{E,H,I\})}\\
&=\frac{\mathrm{P}(S_{1,2}=D)}{\mathrm{P}(S_{1,2}\notin\{E,H,I\})}=\frac{p_D}{1-p_E-p_H-p_I}
\end{aligned}
$$

Similarly, we have

$$
\begin{aligned}
&\mathrm{P}(S_{2,1}=F|S_{1,1}=A,S_{1,2}=D)\\
&\quad=\mathrm{P}(S_{2,1}=F|S_{2,1}\notin\{C,G,I\})=\frac{\mathrm{P}(S_{2,1}=F)}{\mathrm{P}(S_{1,2}\notin\{C,G,I\})}\\
&\quad=\frac{p_F}{1-p_C-p_G-p_I},
\end{aligned}
$$

and

$$
\begin{aligned}
&\mathrm{P}(S_{2,2}=H|S_{1,1}=A,S_{1,2}=D,S_{2,1}=F)\\
&\qquad\qquad=\mathrm{P}(S_{2,2}=H|S_{2,1}=F)=1.
\end{aligned}
$$

The probabilities of all the 393 possible grain states, $\mathrm{P}(S_R)$, are obtained in a similar manner and stored.

2) Calculating $P(\mathbf{Y}_R|S_R)$: The calculation of $\mathrm{P}(\mathbf{Y}_R|S_R)$ is demonstrated through three examples.

1)
$$
\mathrm{P}(\mathbf{Y}_R=[1,1,0,1]|S_R=[A,D,F,H])=0
$$
since it violates the invariant nature of a grain polarization which means that the readback value of the channel bits of one grain must be the same.

2)
$$
\begin{aligned}
\mathrm{P}(\mathbf{Y}_R=&[1,0,1,1]|S_R=[A,D,F,H])\\
=&\mathrm{P}(Y_{1,1}=1|S_{1,1}=A)\mathrm{P}(Y_{1,2}=0|S_{1,2}=D)\\
&.\mathrm{P}(Y_{2,1}=1,Y_{2,2}=1|S_{2,1}=F,S_{2,2}=H)\\
=&\mathrm{P}(X_{1,1}=1)\mathrm{P}(X_{1,3}=0)\mathrm{P}(X_{3,2}=1)\\
=&\frac{1}{2}.\frac{1}{2}.\frac{1}{2}=\frac{1}{8}
\end{aligned}
$$

3)

$$\mathrm{P}(\mathbf{Y}_R = [1,1,1,1] | S_R = [F,H,G,I]) = \mathrm{P}(X_{2,2}=1) = \frac{1}{2}$$

The conditional probabilities, $\mathrm{P}(\mathbf{Y}_R|S_R)$, are calculated and stored in a 16×393 array since there are 16 data states in Y_R and 393 grain states in S_R.

B. GBP-based Detection

One of the important aspects of factor graphs is the application of message passing algorithms, which efficiently computes all the marginal probabilities of the individual variables of the factor nodes. In message passing algorithms such as belief propagation (BP) and GBP messages are sent from node to node in order to compute marginal a posteriori probabilities of each node. Belief propagation is derived for tree like factor graphs and results in exact inference in these graphs. As it is shown in Fig. 2, there exists many small cycles in the detection factor graph. Since the tree-like assumption used in BP does not hold, the BP approximation is poor. To resolve this issue, GBP algorithm is used. In this algorithm, messages are sent from regions of nodes to other regions of nodes. GBP uses region graph method to specify regions and messages. As the first step in region graph method, the parent regions are defined. All the variable nodes connected to the factor node included in the region, are included in that region. Then, we construct a set with all parent regions, intersection of parent regions, intersection of intersection and so on. A graphical model is then constructed using this set. The explanation on how to choose the appropriate regions was given by Yedidia *et al.* in [13]. Overlapping regions of 2×2 channel bits are chosen as the parent regions in our detection problem since the number of states in this region is small enough to do exact inference and large enough to encompass small cycles in the factor graph. The dashed squares in Fig. 2 represent the parent regions. The parent to child algorithm is used for the formulation of GBP. In this message passing algorithm, messages are only sent from parent regions to child regions. For a more detailed explanation of GBP, the reader is referred to [13].

IV. Numerical Results

In this section, we provide numerical results of the GBP-based detector. In order to minimize the effect of the boundary conditions, the bits near the borders are not considered and just the middle bits in the medium are counted for the simulations.

A. Probability of Correct Decision

As described in the previous sections, the GBP detector provides the erasure probability of the channel bits. If this probability is greater than $\frac{1}{2}$, the bit is considered as an erased bit; otherwise, it is considered as a correctly transmitted bit (dominant state). We simulated 16000 realizations of the medium for $p_2 \in [0, 0.45]$ with 0.05 step size. In each simulation, a medium of 14×14 channel bits is generated. Uniform random binary input is written on the generated medium. The readback signal is provided to the GBP detector which produces erasure probabilities for all the channel bits of the medium. Hard decision is applied to the erasure probabilities with threshold $\frac{1}{2}$. Then, a region of 2×2 in the middle of the 14×14 is selected for performance evaluation of the GBP detection in order to avoid any boundary effect. In the selected 4 bits, the GBP decision and the state of the channel bits (erased or dominant) is compared. We define the GBP "probability of correct decision" as the number of the GBP correct decisions divided by the total number of compared bits. Fig. 3 shows the GBP performance over $p_2 \in [0, 0.45]$.

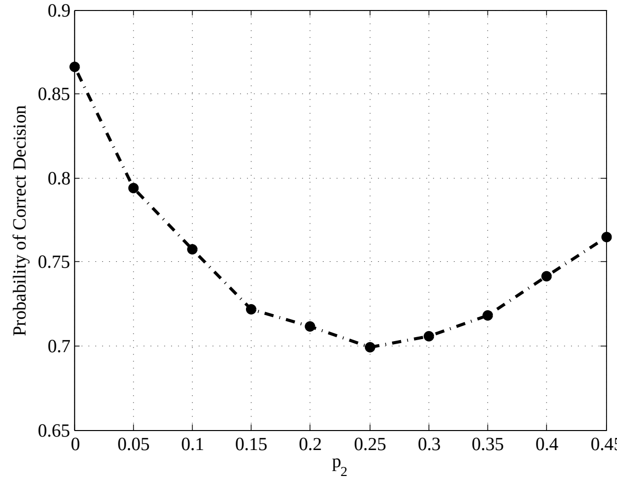
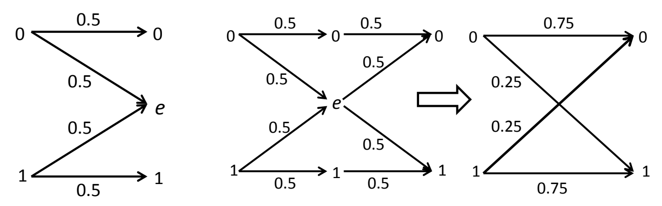


Fig. 3. GBP detector performance for $p_2 \in [0, 0.45]$.

B. GBP-based SIR Estimation

In this section, the SIR of the input data and the GBP detector output is estimated. As Data Processing Inequality states, the post processing can not increase the information. Therefore, the SIR estimation of the input and the GBP detector output is a lower bound on the SIR of granular tiling model. There are two possible cases for each bit after the detection process: The GBP either correctly detects the bit as erased or dominant, or GBP fails and does not detect if a bit has been erased or not. In the first case, the detector correctly detects if the bit is erased or not, thus the channel is modeled as a binary erasure channel (BEC) C_1 which is shown in Fig. 4(a). Since the number of channel bits per grain is 2, each bit is erased with the probability $\frac{1}{2}$. In the second case, since the detector does not know if the bit is erased, the erased bit either remains the same or gets flipped from the original channel bit value with probability $\frac{1}{2}$. Recall that the input data is a binary i.i.d. equiprobable random variable. The channel model corresponding to the second case is considered as a concatenation of a binary erasure channel and a ternary-input, binary-output channel C_2 as depicted in Fig. 4(b). Thus, if the GBP detector does not detect if a bit has been erased or not, the erased bit is detected to 0 or 1 with probability $\frac{1}{2}$. It is easy to see that this concatenated channel can be considered as a binary symmetric channel (BSC) with the crossover probability $\frac{1}{4}$ in Fig. 4(b).



(a) (b)

Fig. 4. The channel model for the GBP detector. (a) the channel corresponding to C_1 (correct detection) (b) the channel corresponding to C_2 and its equivalent binary symmetric channel (incorrect detection).

Having modeled the channel, we are now able to estimate the SIR of the channel. Let $X = \{0, 1\}$ be the input and $Y = \{0, \epsilon, 1\}$ be the output the channel. Thus,

$$I(X;Y) = H(Y) - H(Y|X) \tag{9}$$

where $H(Y)$ and $H(Y|X)$ are the entropy and conditional entropy, respectively. To find $H(Y)$ and $H(Y|X)$, we need to calculate $\mathrm{P}(y)$ and the conditional probabilities $\mathrm{P}(y|x)$ for all $x \in \{0, 1\}$ and $y \in \{0, \epsilon, 1\}$. Let D be the event that the GBP detects the grain state of the bit correctly and let p be the probability of occurrence of D. That is $\mathrm{P}(D) = p$ and consequently, $\mathrm{P}(\bar{D}) = 1 - p$ where $\bar{D}$ is the event of incorrect decision. Probability of correct detection $\mathrm{P}(D)$ for different values of p_2 is calculated in Fig. 3. Then,

$$\begin{aligned} \mathrm{P}(0|0) &= \mathrm{P}(0|0, D)\mathrm{P}(D) + \mathrm{P}(0|0, \bar{D})\mathrm{P}(\bar{D}) \\ &= \frac{1}{2}p + \frac{3}{4}(1 - p). \end{aligned} \tag{10}$$

Similarly,

$$\begin{aligned} \mathrm{P}(1|1) = \mathrm{P}(0|0) &= \frac{1}{2}p + \frac{3}{4}(1 - p), \\ \mathrm{P}(0|1) = \mathrm{P}(1|0) &= \frac{1}{4}p, \\ \mathrm{P}(\epsilon|0) = \mathrm{P}(\epsilon|1) &= \frac{1}{2}p. \end{aligned}$$

By total probability formula, $\mathrm{P}(y)$ for all $y \in Y$ can be obtained as follows:

$$\mathrm{P}(0) = \mathrm{P}(1) = \frac{1}{2}(1 - \frac{1}{2}p), \quad \mathrm{P}(\epsilon) = \frac{1}{2}p.$$

Replacing the above equations in $H(Y)$ and $H(Y|X)$ gives an estimation for the SIR. Fig. 5 shows the SIR estimation of the GBP for this channel. The lower and upper bounds on capacity [4] is also shown in Fig. 5.

V. Conclusion

In this paper, we show that the data state detection can be reduced to grain state detection. The factor graph representation for detection and its necessary formulation are provided. Then, GBP is used to compute the marginalized probabilities of the grain states in the factor graph. Simulation results show huge improvement in detection of the grain states which in turn result in the derivation of a lower bound on the SIR of the granular tiling model in TDMR.

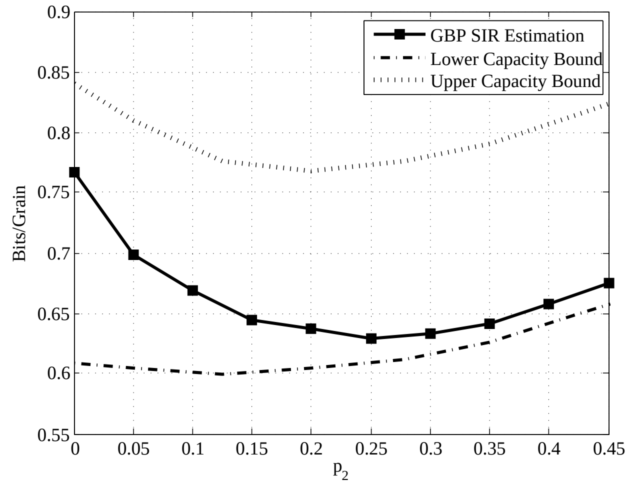


Fig. 5. GBP-based SIR estimation of the granular tiling model for $p_2 \in [0, 0.45]$. The lower and upper bounds on capacity [4] are provided.

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