Free-Space Optical Communication using Orthogonal Optical Angular Momentum Modes

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***Abstract*—A Multi-channel free-space optical (FSO) communications system based on orbital angular momentum (OAM)-carrying beams is studied. We numerically analyze the effects of atmospheric turbulence on the system and find that turbulence induces attenuation and crosstalk among channels. Based on a model in which the constituent channels are binary symmetric and crosstalk is a Gaussian noise source, we find optimal sets of OAM states at each turbulence condition studied, and determine the aggregate capacity of the multi-channel system at those conditions. OAM-multiplexed FSO systems operating in the weak turbulence regime are found to offer good performance. We verify that the aggregate capacity decreases as the turbulence increases. A per-channel bit-error rate evaluation is presented to show the uneven effects of crosstalk on the constituent channels.**

***Index Terms*—Free-space optical communication; Orbital angular momentum (OAM); Atmospheric turbulence.**

I. NOVEL MULTIPLEXING TECHNIQUES USING ORBITAL ANGULAR MOMENTUM

ORBITAL Angular Momentum (OAM) is a property of light associated with the helicity of a photon's wavefront. Optical beams carrying OAM are usually called optical vortices, because they feature a phase discontinuity at their center. A vector normal to a vortex wavefront follows a spiral trajectory around the optical propagation axis. The momentum of a vortex field is proportional to the number of turns that this vector completes around the beam's axis after propagating a distance equal to one wavelength. This number is equal to the OAM state.

Unlike spin angular momentum, for which only two states are possible, the OAM state of a photon can take any integer value. This infinite set of OAM states forms an orthonormal basis [1,2]. This property may be exploited in the context of optical communications [2,3]. The orthogonality among beams with different OAM states allows the simultaneous transmission of information from different users, each on a separate OAM channel.

A feasible transmitter architecture for the OAM-multiplexed FSO communication system comprises the following. As shown in the diagram of Fig. 1, a number of data-carrying TEM00 modes (each independently modulated by a data stream) are shone onto a series of volume holograms, each programmed with a Laguerre-Gauss (LG) field with a distinct OAM state. The diffraction angles of the holograms are designed to produce coaxial propagation of the outgoing OAM-carrying beams. After

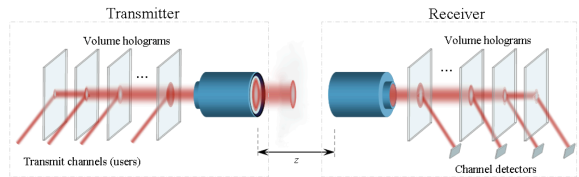

Fig. 1. Diagram of FSO multi-channel OAM.

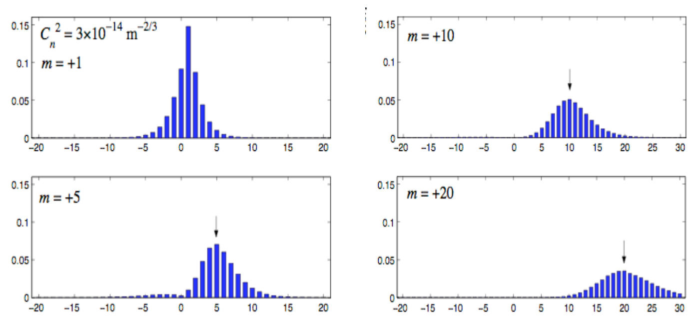

Fig. 2. Average OAM crosstalk observed on receive channels $n = [-20, 20]$ for transmit channels $m = 1, 5, 10$, and 20 [5]. The turbulence strength is $C_n^2 = 3 \times 10^{-14}$ m$^{-2/3}$, which correspond to medium-to-strong turbulence conditions. The arrows indicate the transmit channel. The vertical axis indicates normalized optical power.

the set of independently modulated OAM beams have been made collinear (i.e., optically multiplexed), the resulting superposition field is expanded by a telescope, as shown in Fig. 1. The design of the links receiver uses the principle of the orthogonality among LG beams with different azimuth mode numbers. Although the superposition and simultaneous propagation of a set of OAM beams with different vorticity may produce an unrecognizable intensity pattern, the constituent fields after propagating through vacuum are separable by optical means. For terrestrial FSO applications however, it is important to note that orthogonality is no longer maintained in the presence of atmospheric turbulence [4,5]. In such case, part of the energy launched into a single OAM state will be redistributed into other OAM states after turbulent propagation. Consequently, atmospheric turbulence induces a time-varying crosstalk among OAM channels.

II. SIGNAL DETECTION AND BER CALCULATION

In the turbulent channel most of the transmit power remains in the transmit OAM state and the rest is redistributed on immediately adjacent channels, that is, on OAM channels with similar state number. If turbulence strength grows, the power is further spread onto a larger

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number of channels. We characterize the crosstalk of the turbulent OAM channel by means of numerical propagation simulations as a function of OAM state number and turbulence strength. Figure 2 shows a few examples of the average OAM crosstalk measured on the range of receive state numbers [-20, 20]. The vertical axis indicates the fraction of power observed on each receive channel (normalized to the total transmit power) for transmit channels with OAM state number $m = 1, 5, 10$, and 20. This crosstalk induces unwanted interference on other communication channels, with a consequent reduction in information capacity. The channel fluctuates over time in a random way that is independent of the transmitted signal. That is, the free-space optical channel is not frequency selective. Despite the limitations imposed by turbulence, OAM multi-channel systems can be an excellent solution for wireless communications if proper selection of OAM state is performed.

The channel gains determined by numerical simulations of the channel serve to characterize the OAM-multiplexed optical channel. Each channel matrix H fully determines the communication limits for a fixed receiver noise power. Determining the maximum information rate for such a multiple-input multiple-output channel is a complicated mathematical task. We instead take a simpler approach assuming that the system comprises a set of non-collaborative channels (for instance, the channels are used by independent users without knowledge of other channel's statistics) for which we maximize the information rate, considering a uniform distribution of input symbols in each constituent channel and a uniform power policy among them. Furthermore, it is assumed that crosstalk from each channel is an independent Gaussian noise source that adds to the receiver noise. This intuitive approach gives a lower bound on channel capacity, and is explained in detail below.

Let us denote a set of M OAM channels by the calligraphic letter $\mathcal{O}$, where $\mathcal{O} \subset \mathcal{S}$. Each constituent channel $m \in \mathcal{O}$ is modeled as a binary-symmetric channel (BSC). To determine the flip probabilities for each BSC we want to define a SNR per bit that accounts for channel losses and crosstalk. We denote this SNR by γ, defined in the electrical domain as the ratio between the transmit power P_{Tx} times the squared channel efficiency η_{mm}, and the total noise power. Because each contributing crosstalk signal is modeled as an independent Gaussian source, the total noise power on a given channel is determined by the sum of all contributing crosstalk power and the receiver noise power N_0, also modeled as additive white Gaussian. Thus, γ may be written as

$$\gamma \triangleq \frac{\eta_{mm}^2}{\sum_{n \in \mathcal{O}}^{n \neq m} \eta_{nm}^2 + N_0/P_{Tx}} \,. \tag{1}$$

For a given matrix $\mathbf{H}$, we seek to find an optimum set of OAM channels in the sense of maximizing the system's aggregated information rate. The optimum set, which we denote by $\hat{\mathcal{O}}$, will depend on the number of channels M, and is found by performing the maximization over all possible subsets of $\mathcal{S}$

$$\hat{\mathcal{O}} = \arg\max_{\mathcal{O} \subset \mathcal{S}} \sum_{m \in \mathcal{O}} C(p_m) \,, \tag{2}$$

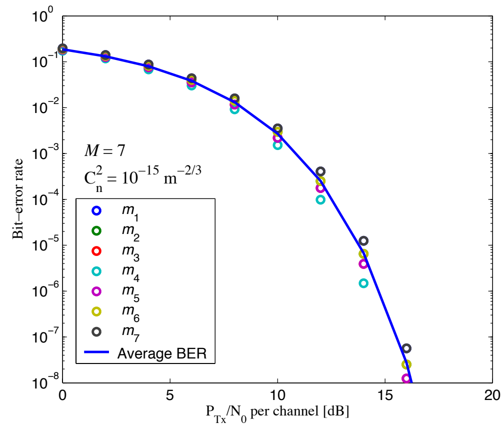

Fig. 3. Bit-error rate versus P_{Tx}/N_0 in dB, with $M = 7$ at (a) $C_n^2 = 10^{-15}$ m$^{-2/3}$.

with

$$C(p_m) = 1 + p_m \log_2 p_m + (1 - p_m) \log_2 (1 - p_m) \tag{3}$$

$$p_m = \frac{1}{2} \text{erfc}(\sqrt{\gamma/2}) \,, \tag{4}$$

where $C(p_m)$ is the capacity of a BSC with flip probability p_m and erfc$(\cdot)$ is the complementary error function. The expression in (4) assumes orthogonal signal modulation, such as binary PPM.

At $P_{Tx}/N_0 = 32$ dB the optimal set is $\hat{\mathcal{O}} = \{-16, -9, -5, -2, 0, +2, +5, +9, +16\}$. We repeat the exercise with 7 channels for $C_n^2 = 3 \times 10^{-14}$ m$^{-2/3}$ and find that at $P_{Tx}/N_0 = 32$ dB the optimal set is $\hat{\mathcal{O}} = \{-16, -8, -3, 0, +3, +8, +16\}$. For $C_n^2 = 3 \times 10^{-14}$ m$^{-2/3}$, $P_{Tx}/N_0 = 32$ dB is no longer near the saturation point of capacity. Therefore, optimal sets require a larger range of state numbers as the transmit power is further increased.

A. *Bit-error rate*

Knowledge of the optimum number of OAM channels and the aggregate rate at each turbulence condition provides a good measure of the system performance. However, it does not describe the quality of each constituent channel. We plot the BER curves for one cases to illustrate the results. As seen in the previous graphs, systems operating at $C_n^2 \leq 10^{-15}$ m$^{-2/3}$ will have very good performance with low P_{Tx} requirements. In the plot, every circle corresponds to the BER of a different constituent channel. At each P_{Tx}/N_0 there are 7 circles (as before, BER is determined using the optimal channel sets). The continuous curves indicate the average BER of these systems. It is evident that BER is not constant within a set (or equivalently, at a given value of P_{Tx}/N_0)).

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