Diamond Channel with Partially Separated Relays

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***Abstract*—We consider diamond channels with a general broadcast channel $p(y,z|x)$, with outputs Z and Y at relays 1 and 2, respectively, and where the relays 1 and 2 have noiseless links of capacities R_z and R_y, respectively, to the decoder. For the case when Y and Z are deterministic functions of X, we establish the capacity. We next give an upper bound for the capacity of the class of diamond channels with a physically degraded broadcast channel, i.e., when $X \rightarrow Y \rightarrow Z$ forms a Markov chain. We show that this upper bound is tight, if in addition to $X \rightarrow Y \rightarrow Z$, the output of relay 2, i.e., Y, is a deterministic function of X. We finally consider the diamond channel with partially separated relays, i.e., when the output of relay 2 is available at relay 1. We establish the capacity for this model in two cases, a) when the broadcast channel is physically degraded, i.e., when $X \rightarrow Y \rightarrow Z$ forms a Markov chain, and b) when the broadcast channel is semi-deterministic, i.e, when $Y = f(X)$. For both of these cases, we show that the capacity is equal to the cut-set bound. This final result shows that even partial feedback from the decoder to relays strictly increases the capacity of the diamond channel.**

I. Introduction

The parallel relay network or the diamond channel consists of a transmitter connected to two relays through a broadcast channel $p(y,z|x)$, where Z is the output of relay 1 and Y is the output of relay 2. The relays are connected to the receiver through a multiple access channel. The diamond channel differs from the classical relay channel [1] in the sense that there is no direct link between the transmitter and the receiver. The diamond channel was introduced by Schein and Gallager in [2], where several cases of the diamond channel were studied.

In [3], a special class of diamond channel was considered where relay 2 receives the input X and relay 1 receives Z through a noisy channel $p(z|x)$. Moreover, the relays are connected to the receiver through an orthogonal multiple access channel. In other words, relays 1 and 2 have finite capacity, orthogonal links of capacities R_z and R_y, respectively, to the receiver. The capacity of this class of diamond channels was characterized in [3] and was shown to be strictly less than the cut-set upper bound [4].

In this paper, we consider the diamond channel with a general broadcast channel and an orthogonal multiple access channel as in [3] (see Figure 1). For this class of diamond channels, we establish the capacity when the broadcast channel is deterministic, i.e., when Y and Z are deterministic functions of X. We show that the capacity is given by the cut-set bound and is achieved by Gelfand-Pinsker-Marton coding to the relays. We next consider the case when the broadcast channel is physically degraded, i.e., when $X \rightarrow Y \rightarrow Z$ forms a Markov chain. We provide an upper bound on the capacity of this class of diamond channels and show that this bound yields the capacity when in addition to $X \rightarrow Y \rightarrow Z$, the channel model is such that, $Y = f(X)$ for any deterministic function f. Note that when $Y = X$, we recover the result obtained in [3].

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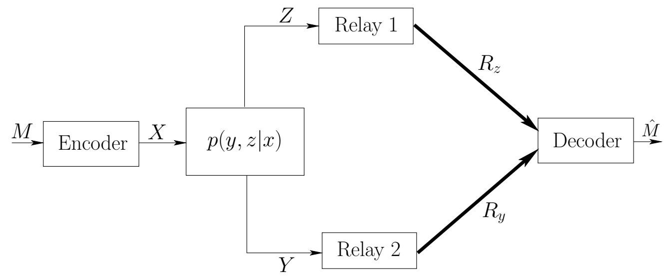

Fig. 1. The diamond channel.

We finally consider this class of diamond channel with partially separated relays, i.e., when the output of relay 2 is available to relay 1. This channel model is equivalent to the model when there is feedback from the receiver to relay 2. One of the main contributions of this paper is to establish the capacity of this model, when a) the broadcast channel is physically degraded, i.e, when $X \rightarrow Y \rightarrow Z$ forms a Markov chain, and b) the broadcast channel is semi deterministic, i.e., when $Y = f(X)$. For both these cases, we show that the capacity is given by the cut-set bound. These two results also show the fact that even feedback to one of the relays strictly increases the capacity of the diamond channel.

II. Diamond Channel

A diamond channel with a general broadcast channel and an orthogonal multiple access channel is described by an input alphabet $\mathcal{X}$, two output alphabets $\mathcal{Y}, \mathcal{Z}$, and transition probabilities $p(y,z|x)$.

A (n, f, f_1, f_2, g) code for this diamond channel is described by,

$$f : \{1, \ldots, \mathcal{M}\} \rightarrow \mathcal{X}^n \quad (1)$$
$$f_1 : \mathcal{Z}^n \rightarrow \{1, \ldots, |f_1|\} \quad (2)$$
$$f_2 : \mathcal{Y}^n \rightarrow \{1, \ldots, |f_2|\} \quad (3)$$
$$g : \{1, \ldots, |f_1|\} \times \{1, \ldots, |f_2|\} \rightarrow \{1, \ldots, \mathcal{M}\} \quad (4)$$

where f is the encoding function at the transmitter, f_1 is the encoding function at relay 1, f_2 is the encoding function at relay 2, and g is the decoding function at the receiver.

The transmitter sends $X^n = f(M)$ as the input to the broadcast channel, where $M \in \{1, \ldots, \mathcal{M}\}$ and the message M is decoded as $\hat{M} = g(f_1(Z^n), f_2(Y^n))$. The probability of error is defined as $P_e = \Pr(M \neq \hat{M})$. A rate triple (R, R_y, R_z) is achievable if for every $0 < \epsilon < 1$, $\eta > 0$, and sufficiently large n, there exists a (n, f, f_1, f_2, g) code such that $P_e \leq \epsilon$, and,

$$\frac{1}{n}\log\mathcal{M} \geq R - \eta \tag{5}$$

$$\frac{1}{n}\log|f_1| \leq R_z + \eta \tag{6}$$

$$\frac{1}{n}\log|f_2| \leq R_y + \eta \tag{7}$$

The capacity $\mathcal{C}(R_y, R_z)$ is defined as the largest R such that (R, R_y, R_z) is achievable.

A. Deterministic Broadcast

In this section, we consider diamond channels with deterministic broadcast channel, i.e., when the channel outputs Y and Z are deterministic functions of X. We characterize the capacity of this class of diamond channels in the following theorem.

Theorem 1: The capacity of the diamond channel, $\mathcal{C}(R_y, R_z)$, with deterministic broadcast channel is given as,

$$\mathcal{C}(R_y, R_z) = \max_{p(x)} \min(H(Y,Z), R_y + R_z, R_y + H(Z), R_z + H(Y)) \tag{8}$$

The converse follows from the cut-set upper bound [4]. To prove the achievability, we will make use of the capacity region of the deterministic broadcast channel without common messages [5], [6]. The capacity region of a deterministic broadcast channel without common messages is given as the set of rate pairs (R_1, R_2) satisfying,

$$R_1 \leq H(Z) \tag{9}$$

$$R_2 \leq H(Y) \tag{10}$$

$$R_1 + R_2 \leq H(Y,Z) \tag{11}$$

Now, for any input distribution $p(x)$, consider the expression,

$$G(p(x)) = \min(H(Y,Z), R_y + R_z, R_y + H(Z), R_z + H(Y)) \tag{12}$$

Depending on the value of (R_y, R_z), we have four cases (see Figure 2):

Case A: If (R_y, R_z) are such that,

$$R_y \leq H(Y) \tag{13}$$

$$R_z \leq H(Z) \tag{14}$$

$$R_y + R_z \leq H(Y,Z) \tag{15}$$

then, we have $G(p(x)) = R_y + R_z$ and we can achieve a rate of $R_y + R_z$ for the diamond channel by using a broadcast channel code with the rates,

$$R_1 = R_z, \quad R_2 = R_y \tag{16}$$

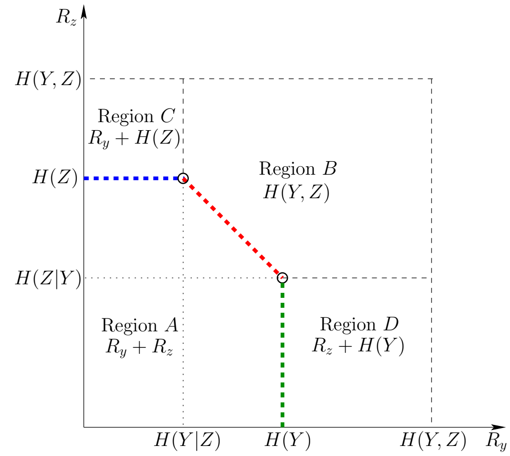

Fig. 2. Achievability for the diamond channel with deterministic broadcast.

Case B: If (R_y, R_z) are such that,

$$R_y \geq H(Y|Z) \tag{17}$$

$$R_z \geq H(Z|Y) \tag{18}$$

$$R_y + R_z \geq H(Y,Z) \tag{19}$$

then, we have $G(p(x)) = H(Y,Z)$ and we can achieve a rate of $H(Y,Z)$ for the diamond channel by using a broadcast channel code with the rates,

$$R_1 = H(Z), \quad R_2 = H(Y|Z) \tag{20}$$

or alternatively,

$$R_1 = H(Z|Y), \quad R_2 = H(Y) \tag{21}$$

Case C: If (R_y, R_z) are such that,

$$R_y \leq H(Y|Z) \tag{22}$$

$$R_y + R_z \leq H(Y,Z) \tag{23}$$

then, we have $G(p(x)) = R_y + H(Z)$ and we can achieve a rate of $R_y + H(Z)$ for the diamond channel by using a broadcast channel code with the rates,

$$R_1 = H(Z), \quad R_2 = R_y \tag{24}$$

Case D: If (R_y, R_z) are such that, $G(p(x)) = R_z + H(Y)$, then, similar to Case C, we can achieve a rate of $R_z + H(Y)$ for the diamond channel by using a broadcast channel code with the rates,

$$R_1 = R_z, \quad R_2 = H(Y) \tag{25}$$

We remark here that the achievability is counterintuitive since one might have expected to use the general broadcast channel code with common messages [6], [7], but as our result shows this is not necessary. We also note here that the cut-set bound continues to hold when relays are partially separated, i.e., the encoded output of relay 2 is available to both relay 1 and the decoder. Our result also shows that the capacity of this diamond channel remains the same even if the relays are partially separated.

B. Physically Degraded Broadcast

In this section, we consider diamond channels with physically degraded broadcast channel, i.e., when the channel $p(y,z|x)$ is such that

$$p(y,z|x) = p(y|x)p(z|y) \tag{26}$$

In the following theorem, we provide a new upper bound on the capacity $\mathcal{C}(R_y, R_z)$.

Theorem 2: The capacity of the diamond channel, $\mathcal{C}(R_y, R_z)$, with physically degraded broadcast channel is upper bounded by the maximum R such that,

$$R \le I(U;Z) + I(X;Y|U) \tag{27}$$
$$R_y \ge H(Y|U,V) - H(Y|X) \tag{28}$$
$$R_z \ge I(Z;V|U,Y) \tag{29}$$
$$R_y + R_z \ge R + I(Z;V|U,Y) \tag{30}$$

for joint distributions of the form,

$$p(x,u,y,z,v) = p(u,x)p(y|x)p(z|y)p(v|z,u) \tag{31}$$

where $|\mathcal{U}| \le |\mathcal{X}| + 4$, $|\mathcal{V}| \le |\mathcal{X}||\mathcal{Z}| + 4|\mathcal{X}| + 3$.
Alternatively, the capacity of the diamond channel is upper bounded as,

$$\begin{aligned}\mathcal{C}(R_y, R_z) \le \max\min(&I(U;Z) + I(X;Y|U),\\ &R_y + R_z - I(Z;V|U,Y)) \qquad (32)\\ \text{such that } &R_y \ge H(Y|U,V) - H(Y|X),\\ &R_z \ge I(Z;V|U,Y)\end{aligned}$$

We next have the following theorem.

Theorem 3: The capacity of the diamond channel, $\mathcal{C}(R_y, R_z)$, with degraded broadcast channel, when $Y = f(X)$, is given by the maximum R such that,

$$R \le I(U;Z) + I(X;Y|U) \tag{33}$$
$$R_y \ge H(Y|U,V) \tag{34}$$
$$R_z \ge I(Z;V|U,Y) \tag{35}$$
$$R_y + R_z \ge R + I(Z;V|U,Y) \tag{36}$$

for joint distributions of the form,

$$p(x,u,y,z,v) = p(u,x)p(y|x)p(z|y)p(v|z,u) \tag{37}$$

As a corollary, by setting $Y = X$ in Theorem 3, we recover the capacity result obtained in [3]. The proofs of Theorem 2 and 3 are omitted here due to space limitations and will be provided in the journal version of this paper [8].

III. Diamond Channel with Partially Separated Relays

We will now consider a variation of the diamond channel, where the relays are partially separated. In other words, the output of relay 2 is available to relay 1 (see Figure 3).

A (n, f, f_1, f_2, g) code for the diamond channel with partially separated relays is described by,

$$f : \{1, \ldots, \mathcal{M}\} \to \mathcal{X}^n \tag{38}$$

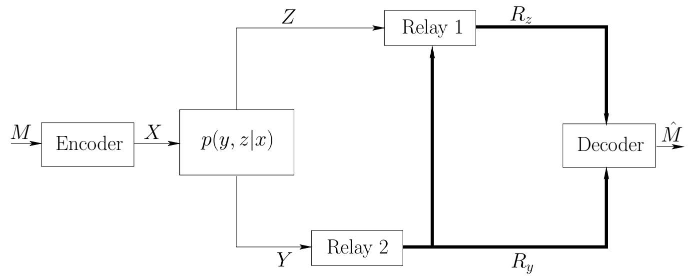

Fig. 3. The diamond channel with partially separated relays.

$$f_2 : \mathcal{Y}^n \to \{1, \ldots, |f_2|\} \tag{39}$$
$$f_1 : \mathcal{Z}^n \times \{1, \ldots, |f_2|\} \to \{1, \ldots, |f_1|\} \tag{40}$$
$$g : \{1, \ldots, |f_1|\} \times \{1, \ldots, |f_2|\} \to \{1, \ldots, \mathcal{M}\} \tag{41}$$

where f is the encoding function at the transmitter, f_1 is the encoding function at relay 1, f_2 is the encoding function at relay 2, and g is the decoding function at the receiver.

The transmitter sends $X^n = f(M)$ as the input to the broadcast channel, where $M \in \{1, \ldots, \mathcal{M}\}$ and the message M is decoded as $\hat{M} = g(f_1(Z^n, f_2(Y^n)), f_2(Y^n))$. The probability of error is defined as $P_e = \Pr(M \neq \hat{M})$. A rate triple (R, R_y, R_z) is achievable if for every $0 < \epsilon < 1$, $\eta > 0$, and sufficiently large n, there exists a (n, f, f_1, f_2, g) code such that $P_e \le \epsilon$, and,

$$\frac{1}{n}\log\mathcal{M} \ge R - \eta \tag{42}$$
$$\frac{1}{n}\log|f_1| \le R_z + \eta \tag{43}$$
$$\frac{1}{n}\log|f_2| \le R_y + \eta \tag{44}$$

The capacity $\mathcal{C}^{PS}(R_y, R_z)$ is defined as the largest R such that (R, R_y, R_z) is achievable.

A. Physically Degraded Broadcast

In this section, we consider the case when the broadcast channel of the diamond channel is physically degraded, i.e., when, $p(y,z|x) = p(y|x)p(z|y)$. In the following theorem, we characterize the capacity of this class of channels.

Theorem 4: The capacity of the diamond channel, $\mathcal{C}^{PS}(R_y, R_z)$, with physically degraded broadcast channel and partially separated relays is given as,

$$\mathcal{C}^{PS}(R_y, R_z) = \max_{p(x)} \min(I(X;Y), R_y + R_z, R_y + I(X;Z)) \tag{45}$$

The converse follows from the cut-set upper bound [4]. We will prove the achievability as follows. Fix an input distribution $p(x)$ and consider the function,

$$G(p(x)) = \min(I(X;Y), R_y + R_z, R_y + I(X;Z)) \tag{46}$$

Figure 4 shows all possible cases for the pair (R_y, R_z). It suffices to show that reliable transmission is possible at the rate $\min(I(X;Y), R_y + R_z, R_y + I(X;Z))$ at the three corner points P_1, P_2 and P_3.

Reliable transmission is possible at a rate $I(X;Y)$ at the corner point P_1, when $R_y = I(X;Y)$ and $R_z = 0$ by using a single-user channel code for relay 2 at a rate $I(X;Y)$. Reliable transmission is possible at a rate $I(X;Z)$ at the corner point P_3, when $R_y = 0$ and $R_z = I(X;Z)$, by using a single-user channel code for relay 1 at a rate $I(X;Z)$.

Therefore, to complete the achievability, we need to show that reliable transmission is possible at the rate $I(X;Y)$ when,

$$R_y = I(X;Y|Z) \tag{47}$$
$$= I(X;Y) - I(X;Z) \tag{48}$$
$$R_z = I(X;Z) \tag{49}$$

The encoder generates $2^{nI(X;Y)}$ x sequences, $x(w)$, according to $\prod_{i=1}^{n} p(x_i(w))$, where, $w = 1, \ldots, 2^{nI(X;Y)}$ and bins these sequences in $2^{n(I(X;Y)-I(X;Z))}$ bins uniformly and independently. Denote the bin index of $x(w)$ as $b_j(x(w))$, where $j = 1, \ldots, 2^{nI(X;Y|Z)}$ and the sub-index number of $x(w)$ as $l_s(x(w))$, where $s = 1, \ldots, 2^{nI(X;Z)}$. To transmit the message w, the encoder puts $x(w)$ as the input to the channel.

Relay 2 can reliably decode the message w with high probability. Relay 2 transmits the bin index, $b_j(x(\hat{w}))$ of the decoded codeword. Relay 1 uses the channel output sequence z and the bin index $b_j(x(\hat{w}))$ to decode the message w. Relay 1 can decode the correct message w with high probability since the number of x sequences in each bin is at most $2^{nI(X;Z)}$. Relay 1 transmits the sub-index number $l_s(x(\hat{w}))$ of the decoded message.

Decoder receives the bin index $b_j(\hat{w})$ from relay 2 and the sub-index number $l_s(x(\hat{w}))$ from relay 1. The decoder decodes the sub-index $l_s(x(\hat{w}))$ in the received bin $b_j(\hat{w})$ as the correct message.

We remark here that this achievability scheme is closely related to the scheme for successive encoding of correlated sources [9]. It was shown in [9] for the case of lossless source coding with partially connected encoders, that the rate-region can be strictly improved upon the case of separated encoders [10], [11]. It is also evident that due to the fact that relays are partially separated, we can achieve the cut-set upper bound which is not always achievable when the relays are separated.

B. Semi-Deterministic Broadcast

We will now consider the case when the broadcast channel of the diamond channel is such that $Y = f(X)$ for any deterministic function f. In the following theorem, we characterize the capacity of this class of diamond channels.

Theorem 5: The capacity of the diamond channel, $\mathcal{C}^{PS}(R_y, R_z)$ with semi-deterministic broadcast channel and partially separated relays is given as,

$$\mathcal{C}^{PS}(R_y, R_z) = \max_{p(x)} \min(I(X;Z) + H(Y|Z), R_y + R_z, R_y + I(X;Z), R_z + H(Y)) \tag{50}$$

The converse follows from the cut-set upper bound [4]. We will prove the achievability as follows.

Figure 5 shows all possible cases for the pair (R_y, R_z). It suffices to show that reliable transmission is possible at the

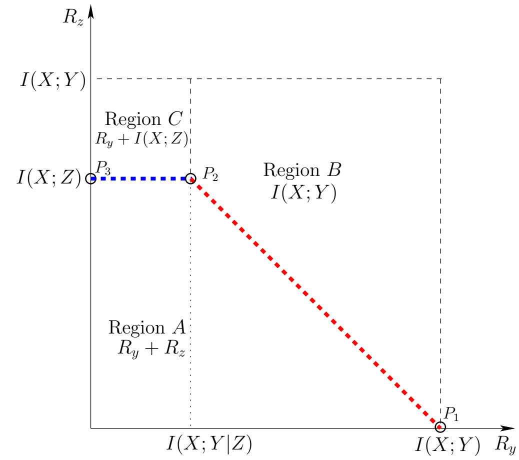

Fig. 4. Achievability for the diamond channel with degraded broadcast.

rate

$$\min(I(X;Z) + H(Y|Z), R_y + R_z, R_y + I(X;Z), R_z + H(Y)) \tag{51}$$

at the four corner points P_1, P_2, P_3 and P_4.

Reliable transmission is possible at a rate $I(X;Z)$ at the corner point P_1, when $R_z = I(X;Z)$ and $R_y = 0$ by using a single-user channel code for relay 1 at a rate $I(X;Z)$. Reliable transmission is possible at a rate $H(Y)$ at the corner point P_2, when $R_z = 0$ and $R_y = I(X;Y) = H(Y)$, by using a single-user channel code for relay 2 at a rate $I(X;Y)$.

Now, consider the corner point P_3, where we have,

$$R_y = H(Y|Z) \tag{52}$$
$$R_z = I(X;Z) \tag{53}$$
$$= I(X;Z|Y) + I(Z;Y) \tag{54}$$
$$= [I(X;Z,Y) - I(X;Y)] + I(Z;Y) \tag{55}$$

where (54) follows from the fact that $Y = f(X)$.

The encoder generates $2^{nI(X;Y,Z)}$ $x(w)$ sequences, where $w = 1, \ldots, 2^{nI(X;Y,Z)}$. The encoder also bins the $x(w)$ sequences in $2^{nI(X;Z|Y)}$ bins, where the bin index of the sequence $x(w)$ is denoted as $b_j(x(w))$, where $j = 1, \ldots, 2^{nI(X;Z|Y)}$. To transmit the message w, the encoder puts $x(w)$ as the input to the channel.

Upon observing the channel output y, relay 2 compresses the y sequences at a rate $H(Y|Z)$ with Z as side-information and transmits the compression bin-index, where the bin index of y sequence is denoted as $b_Y(y)$. The rate needed by relay 2 is $H(Y|Z)$.

Upon observing the z sequence from the channel and the bin index $b_Y(y)$ from relay 2, relay 1 first estimates the y sequence. It can estimate the correct y sequence with high probability since the number of y sequences in each bin is at most $2^{nI(Z;Y)}$. Let the sub-index of the estimated sequence y in the bin $b_Y(y)$ be denoted as $l_Y(b_Y(y), z)$. Relay 1 then proceeds to decode the message by decoding x by using z and the estimated y sequence. Relay 1 transmits the bin index of the decoded x sequence, $b_j(x(\hat{w}))$ and the sub-index of the decoded y sequence, $l_Y(b_Y(y), z)$. The total rate needed by

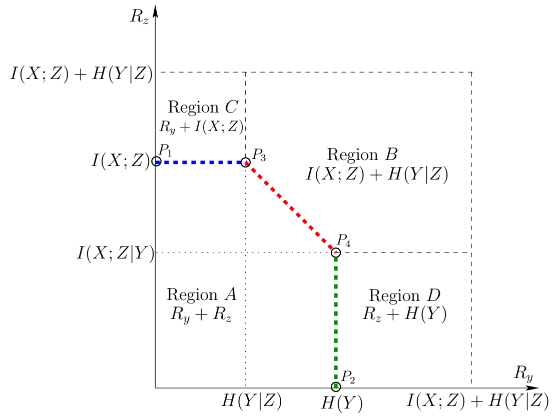

Fig. 5. Achievability for the diamond channel with semi-deterministic broadcast.

relay 1 is $I(X;Z|Y) + I(Z;Y) = I(X;Z)$.

Upon observing $b_Y(y)$ from relay 2 and the pair $(l_Y(b_Y(y), z), b_j(x(\hat{w})))$ from relay 1, the decoder first finds the correct y sequence as the $l_Y(b_Y(y), z)$th sub-index in the bin $b_Y(y)$. It next decodes the message by searching for a unique $x(w)$ in the bin $b_j(x(\hat{w}))$ such that $(x(w), y)$ are jointly typical. This is possible since the number of x sequences in each x-bin is approximately $2^{nI(X;Y,Z)}/2^{nI(X;Z|Y)} = 2^{nI(X;Y)}$. Therefore, the decoder can decode the message and reliable transmission is possible at a rate $I(X;Z) + H(Y|Z)$.

Now, consider the corner point P_4, where we have,

$$R_y = H(Y) \tag{56}$$

$$R_z = I(X;Z|Y) = I(X;Z,Y) - I(X;Y) \tag{57}$$

For this case, relay 2 can describe the y sequence to both relay 1 and the decoder. Relay 2 uses z and y to correctly decode the message and transmits the bin-index, $b_j(x(\hat{w}))$ of the decoded x sequence. The total rate needed by relay 1 is $I(X;Z|Y)$. Upon receiving y sequence from relay 2 and $b_j(x(\hat{w}))$ from relay 1, the decoder decodes the message by searching for a unique $x(w)$ in the bin $b_j(x(\hat{w}))$ such that $(x(w), y)$ are jointly typical. This is possible since the number of x sequences in each x-bin is approximately $2^{nI(X;Y,Z)}/2^{nI(X;Z|Y)} = 2^{nI(X;Y)}$. Therefore, the decoder can decode the message and reliable transmission is possible at a rate $I(X;Z) + H(Y|Z)$.

We remark here, that the main idea behind achievability of the rate $I(X;Z) + H(Y|Z)$ at the corner point P_3 is to use compress-and-forward at relay 2, where relay 2 compresses its output by using relay 1 output as the side information [12]. This approach of compress-and-forward to achieve the cut-set bound is different than that we have seen for the case of physically degraded relay channel, where both relay 1 and relay 2 are able to decode the message.

The capacity of the diamond channel with separated relays when $Y = X$ and Z is a noisy function of X was obtained in [3]. We note that this channel falls in the class of diamond channels with semi-deterministic broadcast component, since $Y = f(X)$. Moreover, this channel also falls in the class of diamond channels with physically degraded broadcast

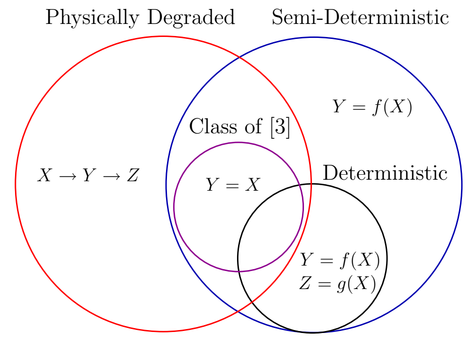

Fig. 6. Illustration of some classes of diamond channels.

component, since $X \to Y \to Z$ forms a Markov chain. To observe these inclusions, see Figure 6. Now, note that for this channel, it was shown in [3] that the cut-set upper bound is strictly sub-optimal when the relays are separated. On the other hand, when the relays are partially separated, we have from Theorems 4 and 5, that the cut-set upper bound is optimal. Since the case of partially separated relays is equivalent to having feedback from the decoder to relay 2, our results therefore show that feedback to even one of the relays strictly improves the capacity of the diamond channel.

IV. Conclusions

We considered several variations of the diamond channel with an orthogonal multiple access component. We established the capacity for the case when the broadcast channel is deterministic. We next provided an upper bound on the capacity when the broadcast channel is physically degraded. This upper bound was shown to be tight for a sub-class of such channels. We next considered the variation of diamond channel where the relays are partially separated and established the capacity when the broadcast channel is a) physically degraded and b) semi-deterministic. For both of these cases, we showed that the cut-set bound is tight.

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