PRACTICE PROBLEM SET 7
Espresso Logic Optimization

1. Implement F(a, b, c) = ∑m(2, 4, 5) + ∑d(1, 6)
   a. as a Boolean n-space cube
   b. in compact cubical form (show the on-set F^{ON}, the off-set F^{OFF}, and the don't care set F^{DC})

2. Calculate c ∩ d for

$$c = \begin{matrix} 0 & 0 & 0 & 3 & 4 \\ 2 & 2 & 1 & 4 & 3 \end{matrix} \qquad d = \begin{matrix} 2 & 2 & 1 & 4 & 3 \\ 1 & 0 & 1 & 4 & 3 \\ 2 & 0 & 2 & 3 & 4 \end{matrix}$$

3. Calculate c ∪ d for the following matrices.

$$c = \begin{matrix} 0 & 0 & 0 & 3 & 4 \\ 2 & 2 & 1 & 4 & 3 \end{matrix} \qquad d = \begin{matrix} 2 & 2 & 1 & 4 & 3 \\ 1 & 0 & 1 & 4 & 3 \\ 2 & 0 & 2 & 3 & 4 \end{matrix}$$

4. Using a truth table, determine if F(a, b, c) = ab + ac + bc is unate.

5. Determine if F(a, b, c, d) = a'c'd' + a'c'd + a'cd + ac'd + ab'c is unate (hint: use a K-map to check the cover provided).

6. Given the prime cover of F, F(c), is the function F unate?

$$F(c) = \begin{matrix} 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \\ 2 & 2 & 0 & 0 \\ 1 & 1 & 2 & 0 \end{matrix}$$

7. Given the prime cover of F, F(c), is the function F unate?

$$F(c) = \begin{matrix} 0 & 0 & 0 & 2 \\ 0 & 1 & 2 & 1 \\ 2 & 2 & 1 & 0 \\ 1 & 0 & 2 & 0 \end{matrix}$$

$$F(c) = \begin{matrix} 1 & 2 & 1 & 2 \\ 0 & 2 & 2 & 1 \\ 1 & 1 & 1 & 2 \\ 1 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{matrix}$$

8. Calculate the cofactor of F ($F_{AB'}$), with respect to the cube c = 1 0 2 2 (AB').

$$F = \begin{matrix} 2 & 2 & 1 & 2 \\ 0 & 2 & 1 & 0 \\ 1 & 1 & 1 & 2 \\ 1 & 2 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{matrix}$$

9. Calculate the cofactor of F, F_c, with respect to the cube c = 1 0 2 2 (AB').

10. If F^{ON}={[1011], [2111], [0102], [0112]} and F^{DC} = {[0012], [1201]}, what is F^{OFF}?

    (a) F^{OFF} = {[0022], [1202], [1220]}

    (b) F^{OFF} = {Φ}

    (c) F^{OFF} = {[2202]}

    (d) F^{OFF} = {[1220], [0002]}

    (e) none of the above

11. Given c = {[1 2 2], [2 1 2]} and d = {[0 2 2], [2 0 0], [2 1 1]}, which of the following are equivalent to c ∩ d?

    (a) c ∩ d = {Φ}

    (b) c ∩ d = {[2 2 2]}

    (c) c ∩ d = {[1 0 0], [2 1 1], [0 1 2]}

    (d) c ∩ d = {[1 0 0], [1 1 1], [2 1 1]}

    (e) none of the above

12. Which of the following statements are true?

    (a) If a non-prime cover is unate, the function is unate.

    (b) If a non-prime cover is NOT unate, the function is NOT unate.

    (c) If a prime cover is unate, the function is unate.

    (d) If a prime cover is NOT unate, the function is NOT unate.

    (e) Only a truth table can indicate whether a function is unate.

13. Given the function cover F (shown in Figure 1), what is the cofactor with respect to B?

    (a) F_B = { Φ }

    (b) F_B = {[1 1 1 1], [0 2 1 2]}

    (c) F_B = {[1 2 1 1], [0 2 1 2]}

    (d) F_B = {[0 0 2 2], [2 0 1 0]}

    (e) none of the above

Figure 1: Function cover used in Problem 13

$$F = \begin{matrix} 0 & 0 & 2 & 2 \\ 0 & 2 & 1 & 2 \\ 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 0 \end{matrix}$$

14. Is the function depicted in Figure 2 unate? (Hint: Convert to a Boolean expression.)

    (a) Yes.

    (b) No.

    (c) I don't have enough information to tell.

Figure 2: Truth table used in Problem 14

a	b	c	d	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

15. If the constraint matrix is empty, what does this tell us?
    (a) Expansion of the cube will not intersect with the offset.
    (b) A complete column covering of B has been achieved.
    (c) All variables have been assigned to either the lowering set (L) or the raising set (RA).
    (d) No further cubes can (or need to) be covered.
    (e) Everything that could have been added to the raising set (RA) is already there.

16. Using the blocking matrix in Figure 3, which of the following are valid column coverings?
    (a) L = {1, 2, 3}
    (b) L = {1, 2}
    (c) L = {1, 3}
    (d) L = {2, 3}
    (e) L = {3}

Figure 3: Truth table used in Problem 16

B =

1	*2*	*3*
1	1	0
0	1	1
1	0	1
0	0	1

17. F(a, b, c, d) = ab' + b'c' + c'd + ad, calculate F' using espresso's UNATE_COMPLEMENT algorithm. Check your answer with a K-map. (Be sure to show your work)

18. Provided F(a, b, c, d) = ab + bc' + ad', calculate F' using espresso's UNATE_COMPLEMENT algorithm. Check your answer with a K-map. (Be sure to show your work)

19. Using Espresso's COMPLEMENT function, compute Z'

Z =

0	2	1	0
2	0	1	0
2	0	2	1
1	2	2	1

20. Using Espresso's COMPLEMENT function, compute Z'

Z =

1	2	0	1
2	1	1	1
1	1	2	1
0	2	2	1

21. Using espresso's COMPLEMENT algorithm find F' given F(a, b, c, d, e) = abe + a'ce + a'b'd'e + a'cd'e.

22. Using Espresso's EXPAND function, expand F

$$F = \begin{matrix} 0 & 0 & 0 & 2 \\ 0 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{matrix} \qquad R = \begin{matrix} 1 & 2 & 0 & 2 \\ 2 & 1 & 0 & 0 \end{matrix}$$

23. Using Espresso's EXPAND function, expand F

$$F = \begin{matrix} 1 & 1 & 0 & 2 \\ 0 & 0 & 2 & 1 \\ 0 & 1 & 2 & 0 \\ 1 & 0 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{matrix} \qquad R = \begin{matrix} 0 & 1 & 2 & 1 \\ 2 & 0 & 2 & 0 \end{matrix}$$

24. Using espresso's expand algorithm, find the prime implicants of F.

$$F = \begin{matrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 0 & 2 \end{matrix} \qquad R = \begin{matrix} 0 & 2 & 2 & 0 \\ 1 & 0 & 1 & 2 \end{matrix}$$

25. Is a prime cover required to guarantee F is unate? Provide a ***short*** explanation (1 paragraph max).

26. Why does BINATE_SELECT function only consider "1" or "0" entries but ignores "2" entries when choosing a splitting variable?

```
Procedure BINATE_SELECT(𝒢)

/* Given a cover 𝒢 = {𝒢^k}, selects the "most" binate variable
/* x_ĵ for splitting. The number of variables is n.

  Begin
  for (j = 1,..., n)
      Begin
      p0(j) ← | {c^i ∈ 𝒢 | c^i_j = 0} |      /* Count the number of
                                              /* cubes with a 0
                                              /* in the j^th input position.
      p1(j) ← | {c^i ∈ 𝒢 | c^i_j = 1} |      /* Count the number of
                                              /* cubes with a 1
                                              /* in the j^th input position.
      End
  if ( max  min {p0(j), p1(j)} = 0) return (U ← True, 0)   /* U = True indicates 𝒢 was
      j                                                    /* unate and no ĵ was chosen.
  else
      Begin
      J ← {j | min (p0(j), p1(j)) > 0}
      ĵ ← argmax {p0(j) + p1(j)}               /* Select ĵ in the set of maximizers
          j∈J                                  /* of p0(j) + p1(j), i.e. the
      return (U ← False; ĵ)                    /* most binate variable.
      End
  End
```

27. Espresso's COMP1 function needs to determines if a cover is unate, if so the UNATE_COMPLEMNT function is called. Write pseudo code that determines whether the function is unate.