Lecture 10 C.

Espresso : Unate Complement

①

What does Espresso do with these cubes?

given F and D (on-set and don't care set)
we need to find R (off-set)

$R = (F \cup D)'$ //off set is everything not in onset or dc-set

we need to somehow complement

2

Can we just switch all 1/0's in cube?

$$F_1 = ab + b'c \quad \not\longrightarrow \quad F_1' = a'b' + bc'$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 2 \\ 2 & 1 & 0 \end{bmatrix}$$

$$\begin{aligned} F' &= (ab + b'c)' \\ &= (ab)' \cdot (b'c)' \\ &= (a' + b')(b + c') \\ &= a'b + a'c' + b'b + b'c' \\ &= a'b + a'c' + b'c' \end{aligned}$$

No! Remember DeMorgan?

$$\begin{bmatrix} 0 & 1 & 2 \\ 0 & 2 & 2 \\ 2 & 0 & 0 \end{bmatrix}$$

multiple product terms make it harder to find the complement.

what now?

3

Espresso uses a divide-and-conquer method to keep simplifying function until it's easy to find the complement

$$F = x F_x + x' F_{x'}$$

← use shannon expansion to simplify equation

Espresso has found they could develop good heuristics for complementation if they are restricted to <u>unate functions</u>

<u>IDEA!</u> use shannon expansion until cofactor becomes unate.

What makes a function unate?

- Function is unate if all of it's variables are unate
- Variable is unate if it is monotone increasing OR montone decreasing
- Variable is monotone increasing (decreasing) if changing the variable from 0 to 1 causes all outputs of F that change to also change from 0 to 1 (1 to 0)

Example 1 Determine if $F(a,b,c) = ab' + b'c$ is unate

a	b	c	F	
0	0	0	0	0→1 change
0	0	1	1	no change
0	1	0	0	no change
0	1	1	0	no change
1	0	0	1	
1	0	1	1	
1	1	0	0	
1	1	1	0	

a is monotone increasing

a	b	c	F	
0	0	0	0	no change
0	0	1	1	1→0 change
0	1	0	0	
0	1	1	0	
1	0	0	1	1→0 change
1	0	1	1	1→0 change
1	1	0	0	
1	1	1	0	

b is monotone decreasing

a	b	c	F	
0	0	0	0	0→1 change
0	0	1	1	
0	1	0	0	no change
0	1	1	0	
1	0	0	1	no change
1	0	1	1	
1	1	0	0	no change
1	1	1	0	

c is monotone increasing

all variables monotone increasing / decreasing → thus all variables unate

all variables unate → thus function is unate!

Example 2 Determine if $F(a,b) = ab + a'b'$ is unate

a	b	F	
0	0	1	1→0 change
0	1	0	0→1 change
1	0	0	
1	1	1	

a is not monotone increasing or monotone decreasing

a is not unate

F is not unate

Is there an easier way to detect if a function is unate?

Yes! Each variable in the function can only appear as true or complemented.

$F(a,b,c) = ab' + b'c$ ← UNATE

a only appears as true (a)
b only appears complemented (b')
c only appears as true (c)

$F(a,b) = ab + a'b'$ ← NOT UNATE

a appears both true (a) and complemented (a')

a is not unate

Example 3 is $F(a,b,c,d) = ab + bc' + c'd$ unate?

YES. a appears only as true
b appears only as true
c appears only as complemented
d appears only as true

Example 4 is the following cover unate?

$$F = \begin{bmatrix} 1 & 0 & 2 & 4 \\ 2 & 0 & 1 & 4 \end{bmatrix}$$

these variables are monotone increasing
column only contains 1's (appears true) or 2's (does not appear at all)

this variable is monotone decreasing
column only contains 0's (appears complemented) or 2's (does not appear at all)

Yes! cover is unate.

Example 5 is the following cover unate?

$$F = \begin{bmatrix} 1 & 1 & 0 & 4 \\ 2 & 0 & 2 & 4 \end{bmatrix}$$

Not unate. variable appears as both true and complemented

we can check our work with a truth table

$F = abc' + b'$

a b c	F	
0 0 0	1	no change
0 0 1	1	no change
0 1 0	0	0→1
0 1 1	0	no change
1 0 0	1	
1 0 1	1	
1 1 0	1	
1 1 1	0	

a is monotone increasing

a b c	F	
0 0 0	1	1→0
0 0 1	1	1→0
0 1 0	0	
0 1 1	0	
1 0 0	1	no change
1 0 1	1	1→0
1 1 0	1	
1 1 1	0	

b is montone decreasing

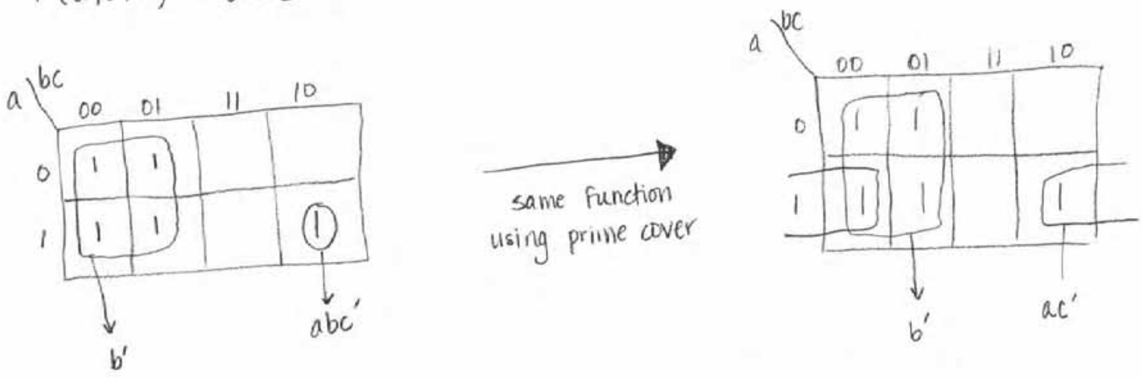
a b c	F	
0 0 0	1	no change
0 0 1	1	
0 1 0	0	no change
0 1 1	0	
1 0 0	1	no change
1 0 1	1	
1 1 0	1	1→0
1 1 1	0	

c is monotone decreasing

∴ F is unate! What happened?

A function that is unate may have a cover that is non unate.
<u>However</u>, a prime cover of a unate function is unate

$F(a,b,c) = abc' + b'$



this is not a prime cover

$F(a,b,c) = ac' + b$

this function cover is unate.

if we have a prime cover that is <u>not</u> unate then we know the function is not unate.

Let's take a step back → why do we care if the function is unate?

- We want to implement complement ($R = \overline{F \cup D}$)
- To complement quickly / efficiently we will use divide-and-conquer to simplify a function until it is unate.
- once function is unate, espresso has a complement function we can use

```
UNATE_COMPLEMENT(F)
begin
  M ← M(F)                            //compute personality matrix of F
  V ← MONOTONE(F)                     // determine if monotone increasing/decreasing
  M̄ ← PERS_UNATE_COMPLEMENT(M)        // complement M
  R ← TRANSLATE(M̄, V)                 // complement of F is determined by M̄ and V
end
```

$M \leftarrow M(F)$ // compute personality matrix of F

How is this done? definition below

↓

Personality Matrix

Let F be the matrix representation of a single output unate cover with k cubes and n input variables. The matrix M is the Boolean matrix defined as follows

$$M_{ij} = \begin{cases} 1 \text{ if } F_{ij} = 0 \text{ or } 1 \\ 0 \text{ if } F_{ij} = 2 \end{cases} \quad \text{for } i=1, \ldots, k,\ j=1, \ldots, n$$

Example 6 compute the personality matrix of F

$$F = \begin{matrix} 0 & 1 & 2 & 2 \\ 0 & 2 & 2 & 2 \\ 2 & 1 & 2 & 1 \\ 2 & 1 & 0 & 2 \\ 2 & 1 & 0 & 1 \end{matrix} \qquad M = \begin{matrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{matrix}$$

Basically don't cares become a 0 in the personality matrix

a 1/0 becomes a 1 in the personality matrix

$V \leftarrow$ MONOTONE(F) // determine if variables are monotone increasing/decreasing

Example 7 which variables are monotone increasing or monotone decreasing?

$$F = \begin{matrix} 0 & 1 & 2 & 2 \\ 0 & 2 & 2 & 2 \\ 2 & 1 & 2 & 1 \\ 2 & 1 & 0 & 2 \\ 2 & 1 & 0 & 1 \end{matrix}$$

1/2 only x_4 is monotone increasing

0/2 only x_3 is monotone decreasing

1/2 only x_2 is monotone increasing

0/2 only x_1 is monotone decreasing

$V = [0 \ 1 \ 0 \ 1]$

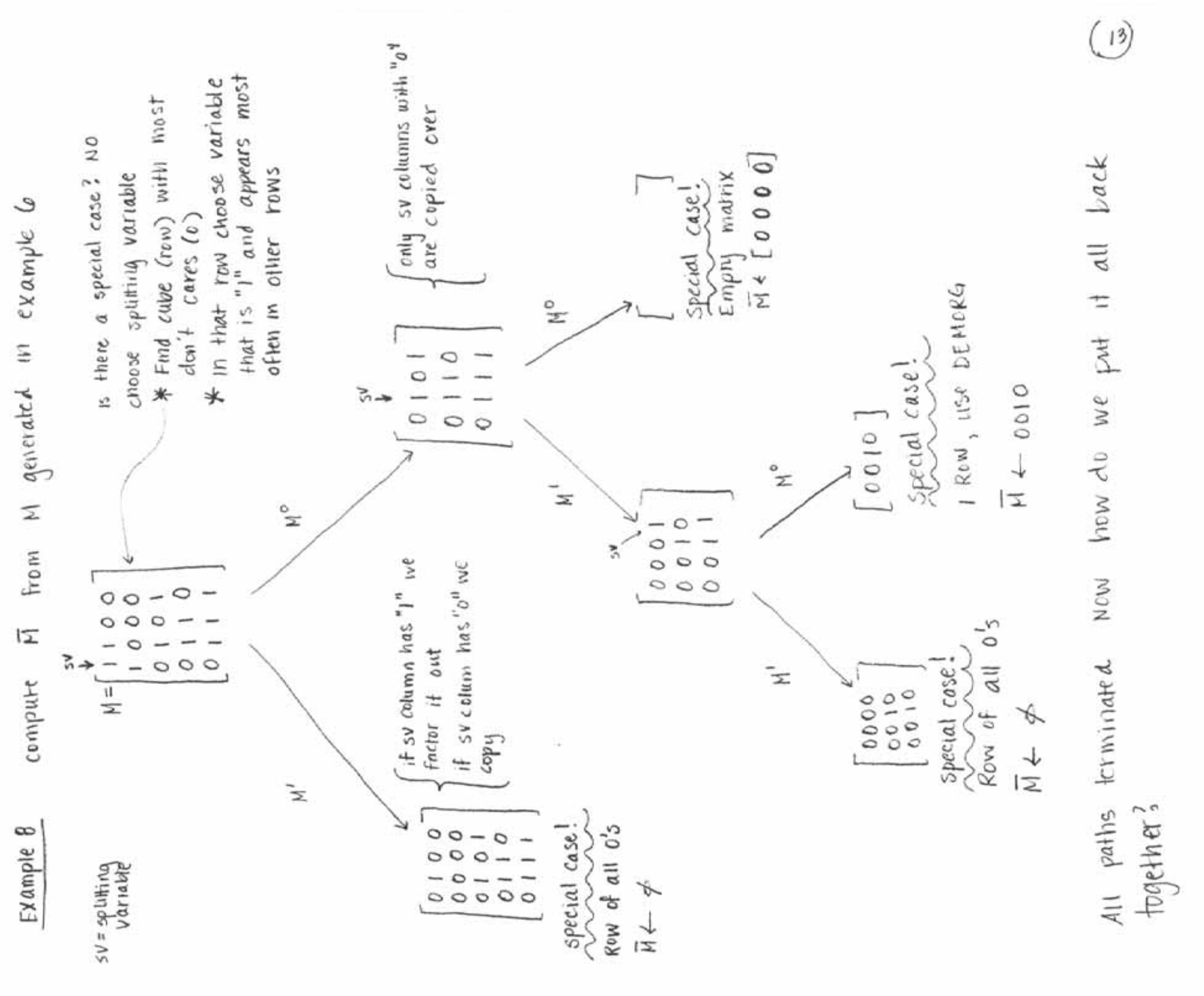
$M \leftarrow$ PERS-COMPLEMENT (M) // complement the personality matrix

How?

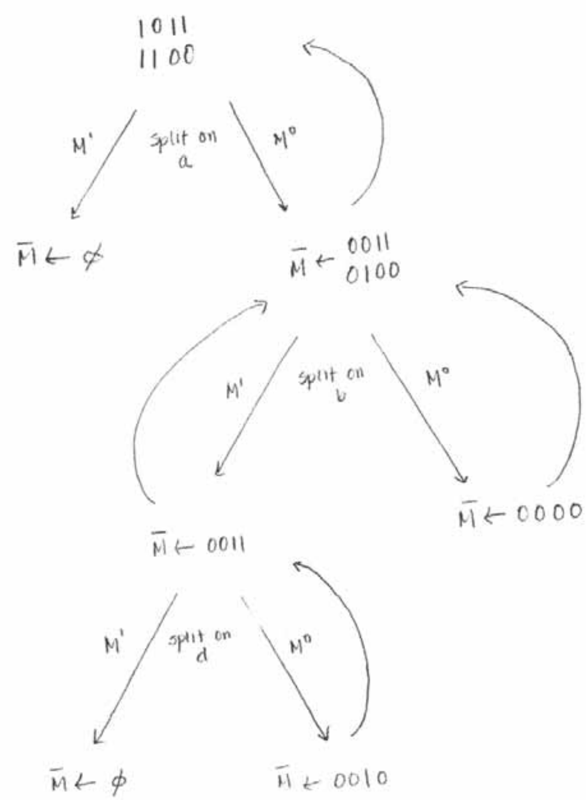
```
PERS_UNATE_COMPLEMENT(M)
begin
  M̄ ← Φ                                      // initialize
  (M̄, T) ← SPECIAL_CASES(M)                  // look for special cases
  if (T=1) return (M̄)                        // found a special case

  ĵ ← UCOMP_SELECT(M)                        // select a splitting variable
  (M¹, M⁰) ← PERS_COFACTORS(M, ĵ)            // computes M of the cofactors w.r.t x_j
  M̄¹ ← PERS_UNATE_COMPLEMENT( M¹ )           // one branch of recursion
  M̄⁰ ← PERS_UNATE_COMPLEMENT( M⁰ )           // another branch of recursion
 Return ( ← MERGE( M̄¹, M̄⁰))                 // merging process essentially concatenates M̄¹ and M̄⁰
end
```

SPECIAL_CASES	Result	Return
There is a row of all 0's	Function is a tautology and the complement of the function is empty	$T \leftarrow 1$, $\bar{M} \leftarrow \Phi$
M is empty	The complement is a tautology	$T \leftarrow 1$, $\bar{M} \leftarrow [0,\ldots,0]$ *
M has only 1 term	The complement is computed by DeMorgan's Law. $\bar{M}$ has one row.	$T \leftarrow 1$, $\bar{M} \leftarrow$ DEMORG
None of the above	Return indication of this.	$T \leftarrow 0$ $\bar{M} \leftarrow \Phi$

* remember a 0 in the personality matrix denotes a don't care



Example 8 CONT'



$$\boxed{\bar{M} = \begin{matrix} 1011 \\ 1100 \end{matrix}}$$

we decompose with shannon Expansion $F = xF_x + x'F_{x'}$

want F', so we can complement whole function

$$F' = (xF_x + x'F_{x'}) = x\overline{F_x} + x'\overline{F_{x'}} \qquad (3.1.8)$$

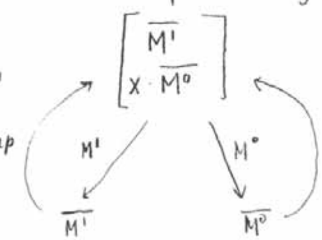
But in unate functions we can simplify this to

$$F' = \bar{x}\,\overline{F_{x'}} + \overline{F_x} \qquad \text{if } x \text{ is monotone increasing} \quad (3.5.1)$$

$$F' = x\overline{F_x} + \overline{F_{x'}} \qquad \text{if } x \text{ is montone decreasing} \quad (3.5.2)$$

How does this work with a personality matrix?

cofactor on M' side left alone as we go back up



cofactor on M^0 side gets a "1" in the split var position

1 in personality matrix mean variable = 0 or 1 (in F)

0 in personality matrix means variable = 2 (in F)

if variable = 0 in F, the variable is monotone decreasing and we use

$$F' = x\overline{F_x} + \overline{F_{x'}}$$

when reassembling leave the cofactor alone

when reassembling the other subtree, variable gets added back in

variable <u>NOT</u> a don't care → 1 is used in M

if variable = 1 in F, the variable is monotone increasing and we use

$$F' = \bar{x}\,\overline{F_{x'}} + \overline{F_x}$$

when reassembling leave the cofactor alone

when reassembling the other subtree, variable gets added back in

variable <u>NOT</u> a don't care → 1 is used in M

(16)

R ← TRANSLATE ($\bar{M}$, V) // complement of F is determined with $\bar{M}$ and V

Example 9 — Determine F' given $\bar{M}$ and V

$$\bar{M} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix} \qquad V = \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}$$

↓ 0's in the personality matrix are don't cares so in F' they remain don't cares

$$\bar{F} = \begin{bmatrix} & & 2 & 2 \\ & 2 & & \end{bmatrix}$$

↓ the 1's in the personality matrix represent a 1 or 0 which one? V tells us original value so in F' these values are flipped

$$\bar{F} = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 1 & 2 & 1 & 0 \end{bmatrix}$$

Done! $\bar{F} = \begin{bmatrix} 1 & 0 & 2 & 2 \\ 1 & 2 & 1 & 0 \end{bmatrix}$

(17)

Did this actually work? Let's check with a K-map

F = 0 1 2 2 // a'b
0 2 2 2 // a'
2 1 2 1 // bd
2 1 0 2 // bc'
2 1 0 1 // bc'd

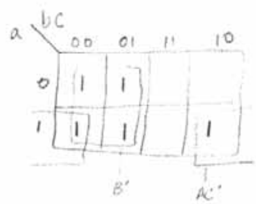
ab \ cd	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	
10				

F' = 1 0 2 2 // ab'
1 2 1 0 // acd'

ab \ cd	00	01	11	10
00				
01				
11				1
10	1	1	1	1

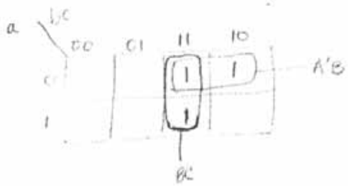
EXAMPLE 11: Find F' using unate complement

$F = B' + AC'$



using DeMorgan

$F' = (B' + AC')'$
$= (B')'(AC')'$
$= (B)(A' + C)$
$= A'B + BC$



using unate complement

① Determine Personality Matrix

$F = \begin{matrix} 2 & 0 & 2 \\ 1 & 2 & 0 \end{matrix}$ $M = \begin{matrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{matrix}$

② Determine V

$V = 1\ 0\ 0$

③ Find $\bar{M}$

split var

010
101

M' → 000 / 101 — special case: Row of all 0's, $\bar{M} \leftarrow \phi$

M^0 → 101 — special case: 1 Row, use DEMORG, $\bar{M} \leftarrow$ 100 / 001

$\bar{M} \leftarrow$ 110 / 011

if's empty we can ignore — M': $\bar{M} \leftarrow 0$

M^0: $\bar{M} \leftarrow$ 100 / 001

④ Translate $\bar{M}$ and V into F'

$\bar{M} = \begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{matrix}$ → $F' = \begin{matrix} ? & ? & 2 \\ 2 & ? & ? \end{matrix}$ → $\boxed{F' = \begin{matrix} 0 & 1 & 2 \\ 2 & 1 & 1 \end{matrix}}$

$V = 1\ 0\ 0$

0's in M become 2's in F'

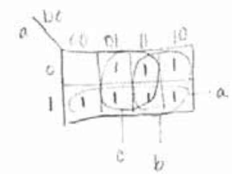
1's in M become opposite of value in V

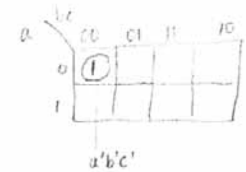
EXAMPLE 10: Find F' using unate complement

$F(a,b,c) = a + b + c$

using DeMorgan

$F' = (a + b + c)'$
$= a'b'c'$





using unate complement

① Determine Personality Matrix

$F = \begin{matrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{matrix}$ $M = \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$

② Determine V

$V = 1\ 1\ 1$

③ Determine $\bar{M}$

split var

100
010
001

M' → 000 / 010 / 001 — special case! Row of 0's, $\bar{M} \leftarrow \phi$

M^0 → split var 010 / 001

M' → 000 / 001 — special case! Row of 0's, $\bar{M} \leftarrow \phi$

M^0 → 001 — special case! 1 Row, use DEMORG, $\bar{M} \leftarrow 001$

$\bar{M} \leftarrow 111$

M': $\bar{M} \leftarrow \phi$ M^0: $\bar{M} \leftarrow 011$

M': $\bar{M} \leftarrow \phi$ M^0: $\bar{M} \leftarrow 001$

④ Translate $\bar{M}$ and V into F'

$\bar{M} \leftarrow 111$

$V = 111$

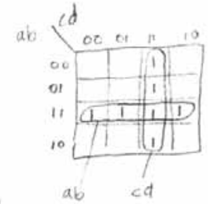
$\boxed{\bar{F} = 000}$

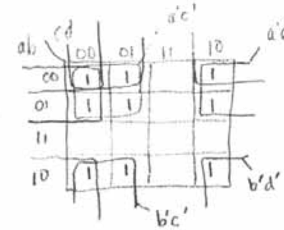
EXAMPLE 12: Find F' using unate complement

$F(a,b,c,d) = ab + cd$

$\boxed{\text{using DeMorgan}}$

$F' = (ab+cd)'$
$= (ab)'(cd)'$
$= (a'+b')(c'+d')$
$= a'c' + a'd' + b'c' + b'd'$





$\boxed{\text{using unate complement}}$

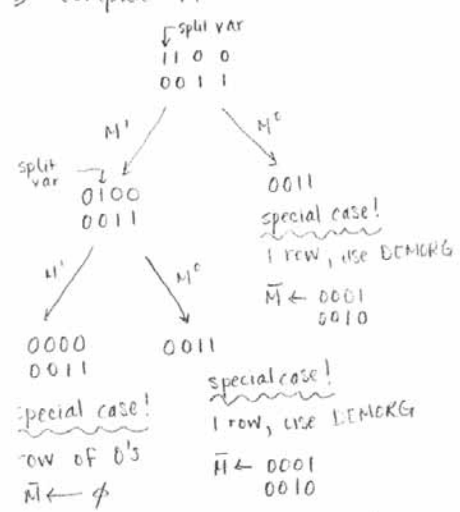
① Determine Personality Matrix

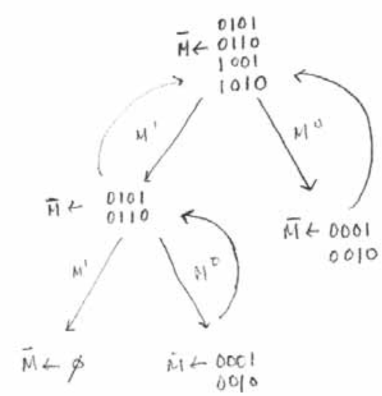
$F = \begin{matrix} 1122 \\ 2211 \end{matrix}$ $M = \begin{matrix} 1100 \\ 0011 \end{matrix}$

② Determine V

$V = 1111$

③ Compute $\overline{M}$





④ Translate $\overline{M}$ and V into F'

$\overline{M} \leftarrow \begin{matrix} 0101 \\ 0110 \\ 1001 \\ 1010 \end{matrix}$ $F' = \begin{matrix} 2?2? \\ 2??2 \\ ?22? \\ ?2?2 \end{matrix}$ $\boxed{F' = \begin{matrix} 2020 \\ 2002 \\ 0220 \\ 0202 \end{matrix}}$

$V = 1111$

0's in M become 2's in F'

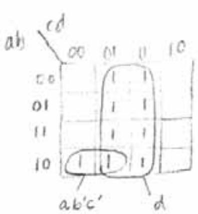
1's in M become opposite of value in V

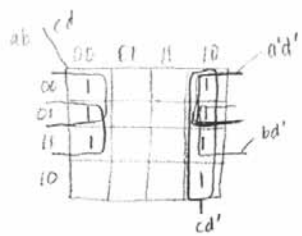
EXAMPLE 13: Find F' using unate complement

$F(a,b,c,d) = ab'c' + d$

$\boxed{\text{using DeMorgan}}$

$F' = (ab'c'+d)'$
$= (ab'c')'(d)'$
$= (a'+b+c)(d')$
$= a'd' + bd' + cd'$





$\boxed{\text{using unate complement}}$

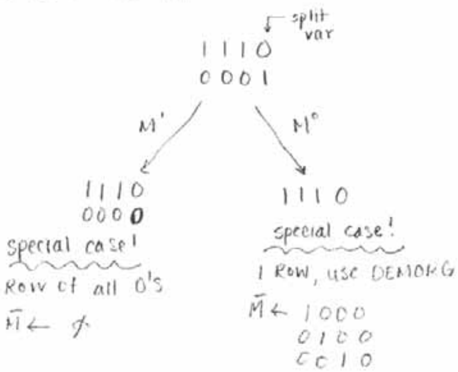
① Determine Personality Matrix

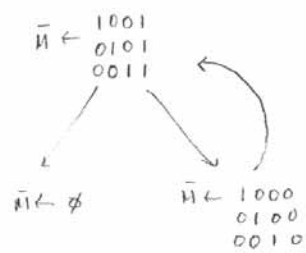
$F = \begin{matrix} 1002 \\ 2221 \end{matrix}$ $M = \begin{matrix} 1110 \\ 0001 \end{matrix}$

② Determine V

$V = 1001$

③ Determine $\overline{M}$





④ Translate $\overline{M}$ and V into F'

$\overline{M} \leftarrow \begin{matrix} 1001 \\ 0101 \\ 0011 \end{matrix}$ $\boxed{F' = \begin{matrix} 0220 \\ 2120 \\ 2210 \end{matrix}}$

$V = 1001$