ECE 474A/57A
Computer-Aided Logic Design

Logic Optimization: ESPRESSO

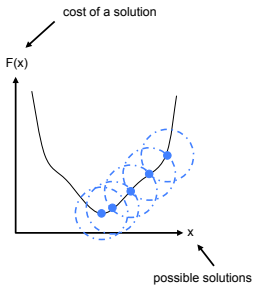
Some Problems are Hard

Using Exact Algorithms vs. Heuristics

- Quine-McCluskey
  - Calculated all prime implicants to derive the optimal solution(s)
  - Petrick's Method derives all covers to determine minimum cover set(s)
  - Number of prime implicants grow quickly -- solution space is huge!
  - Finding the minimum cover set in a class of NP complete problems
    - Determining optimal solution is difficult
- Move to heuristics
  - Look at generating a quality solution quickly (not necessarily optimal)

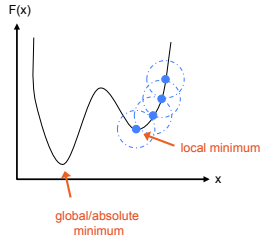
Local Search

- Don't generating all prime implicants and minterms
- Instead, ESPRESSO successively modify a given initial cover
  - This technique is called a *local search* algorithm
- Idea behind local search
  - Search space or solution space - set of all possible values and cost associated with solution
  - Start with an initial value
  - Search all points in neighborhood for a feasible point whose cost is less than current
    - Different problems have different neighborhood definitions
  - If one is found, start process over



Local Search

- Drawback of local searches is local optimality
  - Solution is locally optimal if its neighborhood does not contain any solutions with a lower cost
  - Locally optimal solution may not be the optimal solution
- Modify local search so we don't get stuck at the local minimum



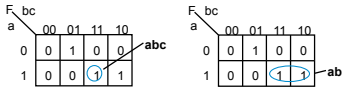
Espresso

- Espresso utilizes local search (keeping in mind local minimum problem)
  - Probably most popular minimization algorithm
  - Extremely efficient Boolean manipulation
- Composed of three main operations
  - EXPAND, REDUCE, IRREDUNDANT
- Other operations include
  - COMPLEMENT, ESSENTIAL PRIMES, LASTGASP, MAKESPARSE

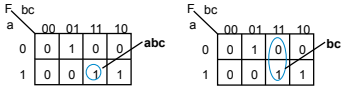
- Espresso Heuristic (in a nutshell)
  - Apply Expand and Irredundant operators to optimize the current function specification
  - Uses the reduce operator to get out of local minimum
  - Iterated until the solution converges

Espresso – Expand Operator Overview

- EXPAND
  - Deleting one (or more) of its literals
  - Check for validity



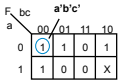
Expand abc by removing c (results in ab)
Is it valid? Yes.



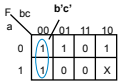
Expand abc by removing a (results in bc)
Is it valid? No.

Espresso – Expand Operator Overview

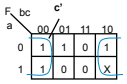
- Goal is to expand a non-prime implicants to prime with the least number of literals



Expand a'b'c' by removing a'
Is it valid? Yes.



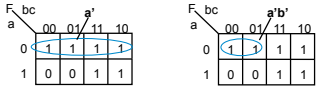
Expand b'c' by removing b'
Is it valid? Yes.



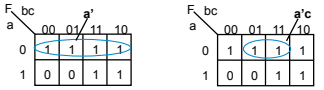
c' is an prime implicant

Espresso – Reduce Operator Overview

- REDUCE
  - Adding one or more literals
  - Check for validity



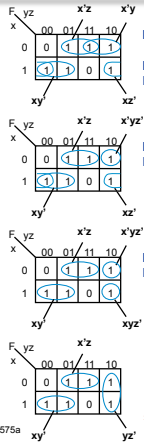
Reduce a' by adding b' (results in a'b')
Is it valid? Yes.



Reduce a' by adding c (results in ac)
Is it valid? Yes.

Espresso – Reduce Operator Overview

- Goal is to decrease the size of implicants such that expansion may lead to a better solution
  - Avoiding a local minimum



No implicant can be expanded

Reduce x'y to x'yz'
Is it valid? Yes.

Reduce xz' to xyz'
Is it valid? Yes.

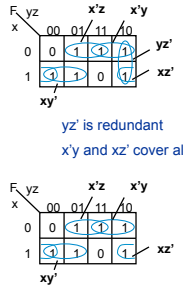
Expand x'yz' to yz'
Is it valid? Yes.

F = x'z + yz' + xy'

Reduction helped find a better solution!

Espresso – Irredundant Operator Overview

- IRREDUNDANT
  - Implicant in a cover is redundant if all the minterms covered by it are contained in other implicants in the cover

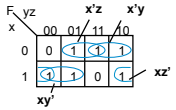


yz' is redundant

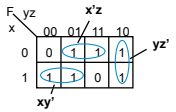
x'y and xz' cover all minterms contained in yz'

Espresso – Irredundant Operator Overview

- Irredundant cover is not the same as minimal cover



irredundant cover

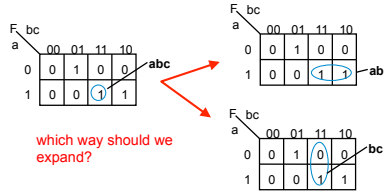


irredundant cover

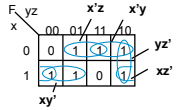
minimal cover

Espresso – Additional Concerns

- Additional concerns
  - Validity check operations
  - Which direction should the move make?



which way should we expand?



which implicant should we reduce?
which literal should we add?

Espresso

```
espresso(F,D) {                          <-- F is the on-set, D is the don't care set
    R = complement(F U D);
    F = expand(F,R);                     // initial expansion
    F = irredundant(F,D);                // initial irredundant cover
    E = essentials(F,D);                 // detect essential prime implicants
    F = F – E;                           // remove essential prime implicants from f
    D = D U E;                           // add essential prime implicants to D
    repeat {
        φ1 = |F |;
        F = reduce(F,D);
        F = expand(F,R);
        F = irredundant(F,D);
    } until (|F | ≥ φ1);
    F = F U E;
    D = D – E;
    RETURN F;
}
```

repeated application of REDUCE, EXPAND, IRREDUNDANT operations while cost keeps decreasing

ESPRESSO, to be continued…

- We've seen the high-level idea behind ESPRESSO
  - ESPRESSO performs extremely efficient Boolean manipulation
- How are these operations actually performed?
- How is data represented?