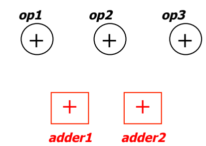
ECE 474A/57A
Computer-Aided Logic Design

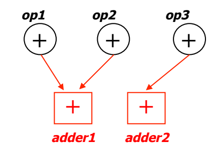
Behavioral Synthesis
Resource Sharing & Binding

Sharing vs. Binding

- Resource Sharing
  - Assignment of a resource to more than one operation
  - Goal – reduce area by allowing multiple non-concurrent operations to share the same hardware operator
- Resource Binding
  - Explicit mapping between operations and resources



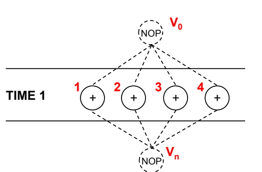
Resource Sharing
- We have 3 add operations and 2 adder units



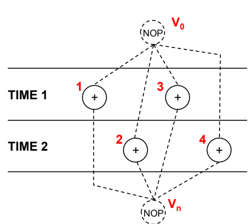
Resource Binding
- Add op1 and op2 executes on adder unit 1
- Add op3 executes on adder unit2

Resource Binding

- Resource binding can be applied to scheduled or non-scheduled sequencing graphs
  - Scheduled sequencing graphs provides limitation on possible sharing



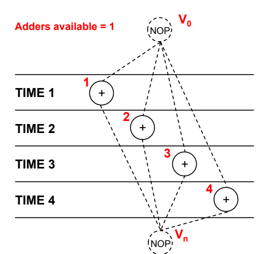
Requires 4 adders to meet the time constraint (upper bound = 1)



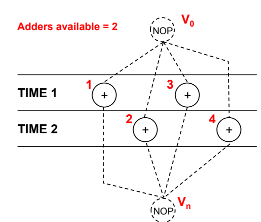
Requires 2 adders to meet the time constraint (upper bound = 2)

Resource Binding

- Resource binding can be applied to scheduled or non-scheduled sequencing graphs
  - Scheduled sequencing graphs provides limitation on possible sharing
  - Non-scheduled sequencing graphs, the limitation of resource sharing effects the latency by limiting the concurrency of operations (LIST_L scheduling)



At most 1 add operation can be executed in a time slice, latency = 4

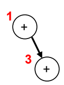


At most 2 add operations can be executed in a time slice, latency = 2

Sharing and Binding for Resource Dominated Circuits

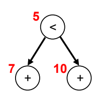
- We are interested in the set of vertices of the sequencing graph (omit source/sink nodes)
- How much sharing is possible?
- Two or more operations can be bound to the same resource if the are **compatible**
  - Not concurrent
  - Can be implemented with the same resource type

a = b + c
e = a + 5



v1 and v3 are not concurrent

if (a < b)
c = 5 + f
else
c = 5 + g

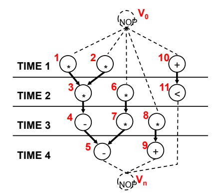


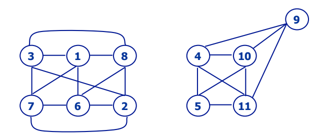
v7 and v10 are not concurrent

Two operations are NOT concurrent if
- Either one starts after the other has finished
- Alternative choices (mutually exclusive) of a branching decision

Resource Compatibility Graph

- Graph whose set of vertices is a one-to-one correspondence with operations in the sequencing graph and whose edges denotes the compatible operations pairs



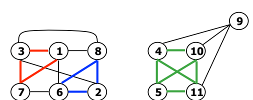


3, 1 – Same op, 3 starts after 1
3, 2 – Same op, 3 starts after 2
3, 4 – Different ops
3, 5 – Different ops
3, 6 – Same op, BUT neither starts after the other and not alternative choices of branch1
3, 7 – Same op, 3 starts in Time 2 and 7 starts in Time 3
3, 8 – Same op, 3 starts in Time 2 and 8 starts in Time 3
3, 9 – Different ops
3, 10 – Different ops
3, 11 – Different ops

Repeat for each node

Compatibility Graph Shows Resource Sharing

- As many disjoint (no common elements) components as resource types
  - A multiply operations is not compatible with an add operation
- Clique - group of mutually compatible operations correspond to subset of vertices that are mutually connected
  - Each vertices connected to every other vertices
- Maximal set of mutually compatible operations are represented by maximal clique



Maximize size of cliques, must ensure all vertices included

{1, 3, 7}
{2, 6, 8}
{1, 8}
{4, 5, 10, 11}
{9}

- The optimum resource sharing is on that minimizes the number of required resource instances
  - Resource instance relates to cliques
  - Partitioning graph into minimum number of cliques yields optimal sharing

 # Resources = # cliques

We need 2 adders, 2 multipliers

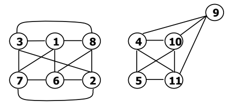
Clique Partitioning

```
CLIQUE_PARTITION( G(v, e) ){
    Π = Φ                                  // initial set of partitions to empty
    while( G(v,e) not empty ) do{          // while the graph is not empty, keep iterating
        C = MAX_CLIQUE( G(v,e) )           // compute a maximal clique in graph
        Π = Π U C                          // add max clique to set of partitions
        delete C from G(v,e)               // remove max clique from graph
    }
}

MAX_CLIQUE( G(v, e) ){
    C = vertex with largest degree
    repeat {
        repeat {
            U = { v Є V : v ∉ C and adjacent to all vertices of C}
            if ( U ≠ Φ ){                  // no such vertices exist
                return C
            }
            else{
                select v Є U               // pick one
                C = C U v                  // add to clique
            }
        }
    }
}
```

Clique Partitioning
Example 1

Π = Φ // set of partitions is initially empty



Is G empty? No.

Find max clique

C = 1 // vertex with largest degree, anything with 4 will do

U = {3, 7, 6, 8} // these vertices are connected to 1

V = 3

C = {1} U {3} = {1, 3}

U = {7, 8} // these vertices are connected to 1and 3

C = {1, 3} U {7} = {1, 3, 7}

U = { Φ } // no others vertices connect to 1, 3, and 7

Return {1, 3, 7}

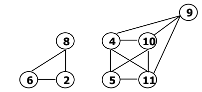
Π = {1, 3, 7}

Remove {1, 3, 7} from G

Vertices	Degree
1	4
2	4
3	4
4	4
5	3
6	4
7	4
8	4
9	3
10	4
11	4

Clique Partitioning
Example 1

Π = {1, 3, 7}



Is G empty? No.

Find max clique

C = 4 // vertex with largest degree, anything with 4 will do

U = {5, 9, 10, 11} // these vertices are connected to 4

V = 5

C = {4} U {5} = {4, 5}

U = {10, 11} // these vertices are connected to 4 and 5

C = {4, 5} U {10} = {4, 5, 10}

U = {11} // these vertices are connected to 4, 5, and 10

C = {4, 5, 10} U {11} = {4, 5, 10, 11}

U = { Φ } // no others vertices connect to 4, 5, 10, and 11

Return {4, 5, 10, 11}

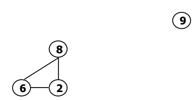
Π = {1, 3, 7}, {4, 5, 10, 11}

Remove {4, 5, 10, 11} from G

Vertices	Degree
2	2
4	4
5	3
6	2
8	2
9	3
10	4
11	4

Clique Partitioning
Example 1

Π = {1, 3, 7}, {4, 5, 10, 11}



Is G empty? No.

Find max clique

C = 2 // vertex with largest degree, anything with 2 will do

U = {6, 8} // these vertices are connected to 2

V = 6

C = {2} U {6} = {2, 6}

U = {8} // these vertices are connected to 2 and 6

C = {2, 6} U {8} = {2, 6, 8}

U = { Φ } // no others vertices connect to 2, 6, and 8

Return {2, 6, 8}

Π = {1, 3, 7}, {4, 5, 10, 11}, {2, 6, 8}

Remove {2, 6, 8} from G

Vertices	Degree
2	2
6	2
8	2
9	0

Clique Partitioning
Example 1

Π = {1, 3, 7}, {4, 5, 10, 11}, {2, 6, 8}

Is G empty? No.

Find max clique

C = 9

U = {9}

U = { Φ } // no others vertices connect to 9

Return {9}

Π = {1, 3, 7}, {4, 5, 10, 11}, {2, 6, 8}, {9}

Remove {9} from G

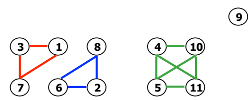
Vertices	Degree
9	0

Clique Partitioning
Example 1

Π = {1, 3, 7}, {4, 5, 10, 11}, {2, 6, 8}, {9}

Is G empty? Yes!

- What does clique partition tell us?
  - {1, 3, 7} – multiplier
  - {4, 5, 10, 11} – alu
  - {2, 6, 8} – multiplier
  - {9} – alu
- We know how much sharing AND binding



Clique Partitioning
Example 2

Π = Φ

Is G empty? No.

Find max clique

C = 4

U = {1, 2, 3, 5, 6}

V = 1

C = {4} U {1} = {1, 4}

U = {5, 6}

C = {1, 4} U {5} = {1, 4, 5}

U = {6}

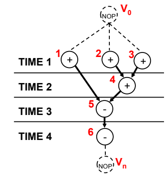
C = {1, 4, 5} U {6} = {1, 4, 5, 6}

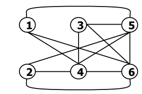
U = { Φ }

Return {1, 4, 5, 6}

Π = {1, 4, 5, 6}

Remove {1, 4, 5, 6} from G





Vertices	Degree
1	3
2	3
3	3
4	5
5	5
6	5

Clique Partitioning
Example 2

Π = {1, 4, 5, 6}

Is G empty? No.

Find max clique

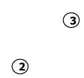
C = 2

U = {Φ}

Return {2}

Π = {1, 4, 5, 6}, {2}

Remove {2} from G



Vertices	Degree
2	0
3	0

Π = {1, 4, 5, 6}, {2}

Is G empty? No.

Find max clique

C = 3

U = {Φ}

Return {3}

Π = {1, 4, 5, 6}, {2}, {3}

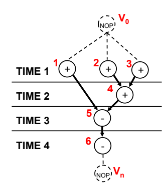
Remove {3} from G

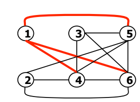
Vertices	Degree
3	0

Clique Partitioning
Example 2

Π = {1, 4, 5, 6}, {2}, {3}

Is G empty? Yes

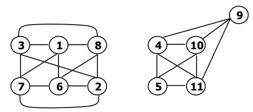




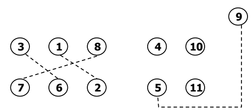
- Need 3 ALUs
  - ALU 1 executes ops 1, 4, 5, 6
  - ALU 2 executes op 2
  - ALU 3 executes op 3

Resource Conflict Graph

- Instead of compatibility we can instead look at conflicts
  - May simplify the graph
- Resource conflict graph
  - Graph whose set of vertices is a one-to-one correspondence with operations in the sequencing graph and whose edges denotes the ***conflicting*** operations pairs
  - To simplify graph, we consider conflicts between each resource type independently



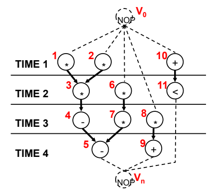
resource compatibility graph

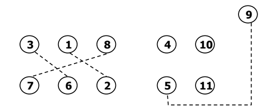


resource conflict graph

Building Resource Conflict Graph

- To simplify graph, we consider conflicts between each resource type independently





Consider Multipliers (1, 2, 3, 6, 7, 8)

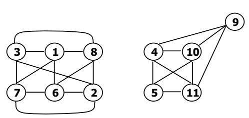
1, 2 – concurrent
3, 6 – concurrent
7, 8 – concurrent

Consider ALUs (4, 5, 9, 10, 11)

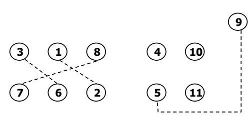
5, 9 – concurrent

Building Resource Conflict Graph

- Conflict graph is the complement of the compatibility graph
- In conflict graph, looking for set of mutually compatible operations
  - Subset of vertices that are NOT connected by edges
  - Also called independent set of G



resource compatibility graph



resource conflict graph

Graph Coloring

```
VERTEX_COLOR ( G(v, e) ){
    for( i=1 to | V | ){
        C = 1                          // use number to represent color
        while( there exists a vertex adjacent to vi with color c) do{
            C = C + 1
        }
        label vi with C
    }
}
```

- Use graph coloring to find independent sets
  - Each color represents a resource instance (two adders will be represented by two different colors)
- Optimal resource sharing corresponds to vertex coloring with minimal amount of colors

Graph Coloring
Example 1

i = 1   // look at vertex 1

C = c1 // represents first color

Is there any adjacent vertices with color = 1? No.

v_1 = c1

i = 2   // look at vertex 2

C = c1

Is there any adjacent vertices with color = 1? Yes.

C = c2

Is there any adjacent vertices with color = 2? No.

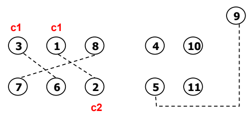
v_2 = c2

i = 3   // look at vertex 3

C = c1 // represents first color

Is there any adjacent vertices with color = 1? No.

v_3 = c1



Graph Coloring
Example 1

i = 4   // look at vertex 4

C = c1 // represents first color

Is there any adjacent vertices with color = 1? Yes – remember conflict is implied across different resource types.

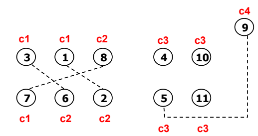
C = c2

Is there any adjacent vertices with color = 2? Yes.

C = c3

Is there any adjacent vertices with color = 3? No.

v_4 = c3



Similarly repeat for remaining

- Four colors required – need four resources
  - c1 is used for multiply
  - c2 is used for multiply
  - c3 is used for alu
  - c4 is used for alu

Graph Coloring
Example 2

i = 1

C = c1

Adjacent vertices with color = 1? No.

v_1 = c1

i = 2

C = c1

Adjacent vertices with color = 1? Yes.

C = c2

Adjacent vertices with color = 2? No.

v_2 = c2

i = 3

C = c1

Adjacent vertices with color = 1? No.

v_3 = c1

i = 4

C = c1

Adjacent vertices with color = 1? Yes.

C = c2

Adjacent vertices with color = 2? No.

v_4 = c2

i = 5

C = c1

Adjacent vertices with color = 1? Yes.

C = c2

Adjacent vertices with color = 2? Yes.

C = c3

Adjacent vertices with color = 3? No.

v_5 = c3

i = 6

C = c1

Adjacent vertices with color = 1? Yes.

C = c2

Adjacent vertices with color = 2? Yes.

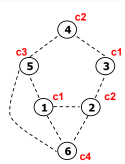
C = c3

Adjacent vertices with color = 3? Yes.

C = c4

Adjacent vertices with color = 4? No.

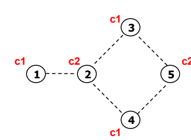
v_6 = c4



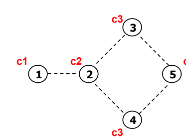
- **Four colors required – need four resources**
  - **c1 for node 1, 3 op**
  - **c2 for node 2, 4 op**
  - **c3 for node 5 op**
  - **c4 for node 6 op**

Graph Coloring

- VERTEX_COLOR algorithm sensitive to ordering of vertices explored - variety of modifications available
  - Switching pair assignment of colors
  - Backtracking to switching larger number of vertices



- Node ordering 1, 2, 3, 4, 5
- Requires 2 colors



- Node ordering 1, 5, 2, 3, 4
- Requires 3 colors

Conclusion

- Considered several types ways to find resource sharing and binding
  - Compatibility Graph / Max Clique
  - Conflict Graph / Vertex color
- Again, many other methods available
  - Golumbic's algorithm
  - Left-edge algorithm
  - ILP formulation
- Idea of sharing and binding not limited to adders and multipliers
  - Registers
  - Determining minimal number of memory ports
  - Bus sharing