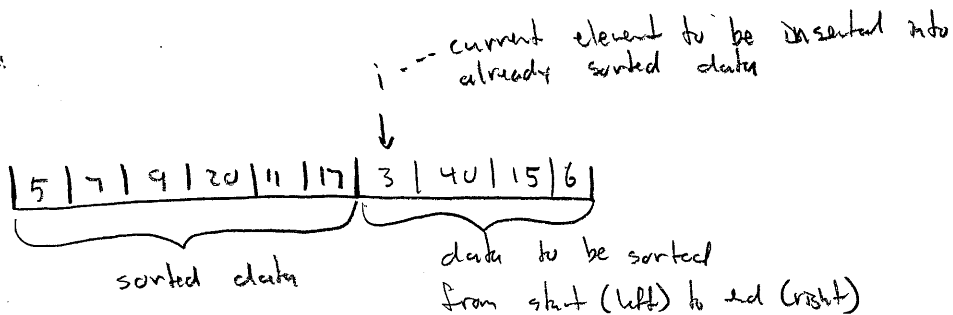
Sorting

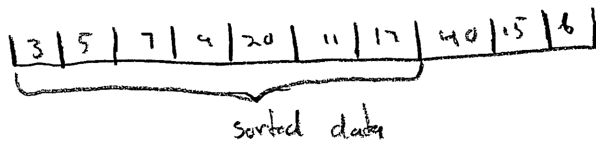
- Often need efficient algorithms to order data within your program
- Comparison sorts rely on comparing elements to determine the order for those elements
- Data can be sorted in ascending order or descending order

Insertion Sort: Data is sorted by examining each element from the beginning to end, where each element is inserted into the correct location within the already sorted data

Example:



⇓ after inserting i into sorted data



Insertion Sort Pseudocode:

1. for $j=1$ to size-1 } loops over n-1 elements
   a) $i = j-1$
   b) key = a[j]
   c) while $i >= 0$ and $a[i] > key$
      I) $a[i+1] = a[i]$
      II) i--
   d) $a[i+1]$=key

} in worst case you must examine all previous elements

Run time $T(n) = \sum_{i=1}^{n-1} i$

$= 1 + 2 + 3 + 4 + 5 + 6 + 7 + \dots + n-1$

$$T(n) = \frac{n(n+1)}{2} - n$$

$$O\left(\frac{n(n+1)}{2} - n\right) = \underline{\underline{O(n^2)}}$$

Benefits of Insertion Sort: Inserting a new item into a sorted array using insertion sort has a complexity of $O(n)$

↳ + let's see bubblesort do that!

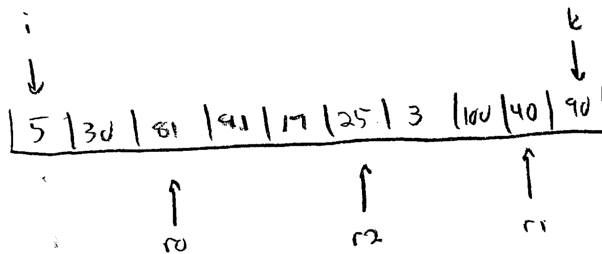
Quicksort: Recursively partitions data and sorts each partition

- Partitioning is done by selecting an element (often called the pivot) in the unsorted data. All elements "greater" than pivot will be moved to the right of pivot. All elements less than pivot will be moved to the left of pivot.

- How to choose the pivot?
  - ↳ many options exist (purely random, median, median-of three)
  - ↳ Median of three:
    1. pick three random locations within array
    2. choose the median of the three

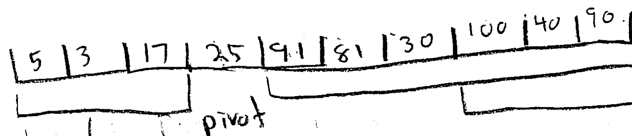
- After partitioning we need to know the location of the pivot.

Example:



median = 25

⇓ after partitioning



pivot

↳ pivot is in the final sorted position

→ all values right of pivot are greater than pivot, but need to be sorted

↳ all values to the left of pivot are less than pivot, but need to be sorted

Pseudocode for Partition operator:

```
int partition (int * data, int i, int k)
  ① r0 = (rand() % (k-i+1)) + i        ] find three random locations between
  ② r1 = (rand() % (k-i+1)) + i        ] i and k inclusive
  ③ r2 = (rand() % (k-i+1)) + i        ]
  ④ pval = median (data[r0], data[r1], data[r2])      *this is the pivot value

  ⑤ i--
  ⑥ k++

  ⑦ while (1) {
       a) do
            [i] k--
       b) while (a[k] > pval)

       c) do
            [i] i++
       d) while (a[i] < pval)

       e) if i < k
            [i] swap (a[i], a[k])
       f) else return k
```

*Note: this method works for data without duplicates.

Quicksort Pseudocode

```
qksort(int *data, int i, int k)

① if (i < k)
   a) j = partition(data, i, k)
      qksort(data, i, j-1)
      qksort(data, j+1, k)
```

Initial call to Quicksort: qksort(data, 0, size-1)

Runtime Complexity:

① Worst-case: Occurs when result of each partitioning has one subproblem of size 0 and one subproblem of size n-1

↳ Partitioning will repeat n times each with a subproblem 1 smaller

⇓

* Note: This is similar to how insertion sort works

$T(n) = O(n) + T(n-1)$

$= n + n-1 + n-2 + n-3 + \dots + 1$

$\Theta(n^2)$

② Best Case: Occurs when as even a split possible is achieved, resulting in two subproblems of size $\frac{n}{2}$

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n)$$

$$= n + 2\left(\frac{n}{2}\right) + 2\left(\frac{n}{4}\right) + 2\left(\frac{n}{8}\right) + \cdots + 1$$

$O(n \log n)$

③ Average Case: $O(n \log n)$

Mergesort: Sorting method that divides data in half at each stage until a single node is found (which is therefore sorted), and merges the data within two halves into a sorted array

+ Merging process can be performed efficiently as a single pass over the data

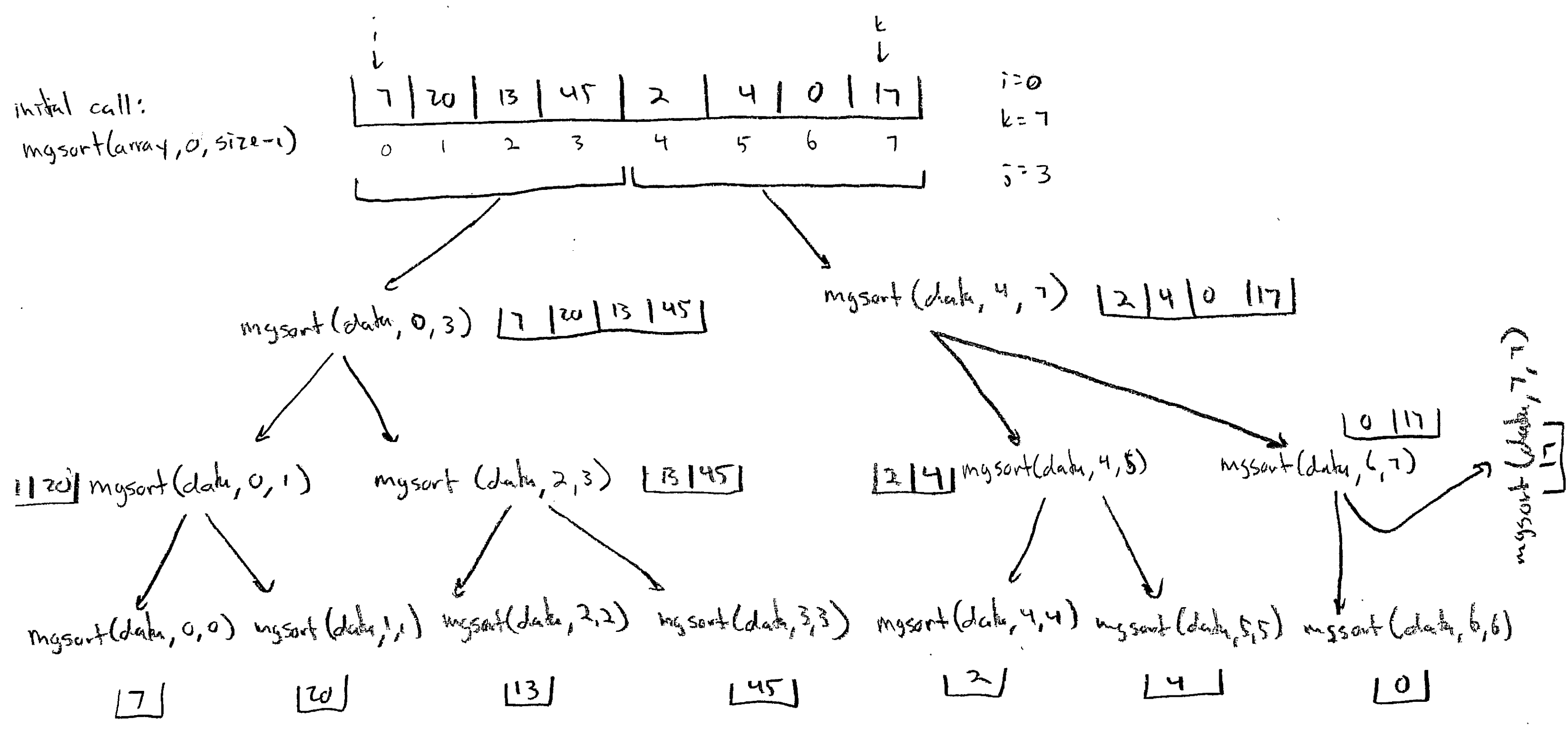
+ Efficient algorithm with worst case complexity of $O(n \log n)$

+ Potential drawback is the need for additional memory as the merge cannot be performed in place

Pseudocode for Mergesort:

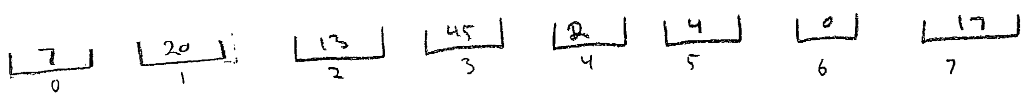
```
int mgsort(int *data, int i, int k)
  ① if i < k          * we have at least two items to sort
     a) j = (i+k-1)/2
     b) mgsort(data, i, j)
     c) mgsort(data, j+1, k)
     d) merge(data, i, j, k)
```

* i and k are the low and high indices within the array currently being sorted



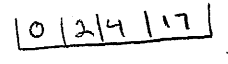
Pseudocode for merge process

```
merge (ind * data, int i, int j, int k)

  (1) ipos = i            * next location in left half
  (2) jpos = j+1          * next location in right half
  (3) mpos = 0            * next location in merged array

  (4) allocate storage for merge elements
        m = (int *) malloc ( sizeof(int) * (k - i + 1) )

  (5) while ( ipos <= j || jpos <= k )

      a) if ipos > j              * no elements left in left half
           I) while jpos <= k
                i    m[mpos] = data[jpos]
                ii   jpos++
                iii  mpos++
           II) break
      b) else if jpos > k         * no elements left in left half
           I) while ipos <= j
                i    m[pos] = data[ipos]
                ii   ipos++
                iii  mpos++
           II) break
      c) if data[ipos] < data[jpos]
           I) m[mpos] = data[ipos]
           II) ipos++
           III) mpos++
      d) else
           I) m[mpos] = data[jpos]
           II) jpos++
           III) mpos++

  (6) copy m back to data
        memcpy (& data[i], m, sizeof(int) * (k - i + 1))

  (7) free m

  (8) return success
```

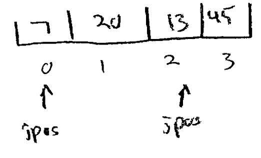


merge(data, 0, 0, 1)   merge(data, 2, 2, 3)   merge(data, 4, 4, 5)   merge(data, 6, 6, 7)

merge(data, 0, 1, 3)   merge(data, 4, 5, 7)



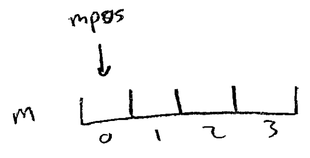
before:



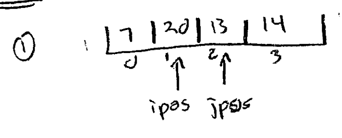
ipos=0

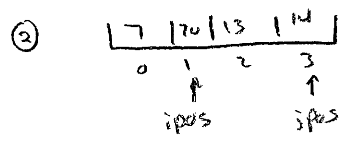
jpos=2

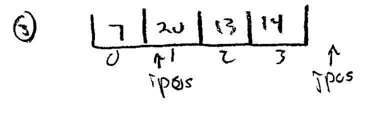
mpos=0

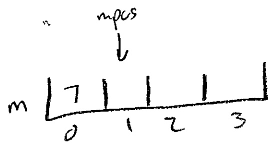


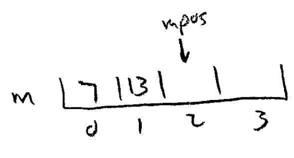
merging:

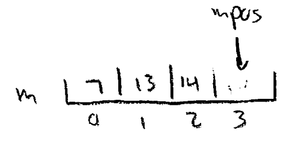


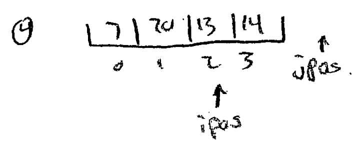








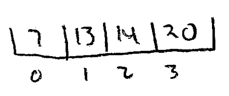


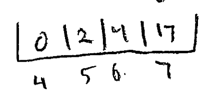


m | 7 | 13 | 14 | 20 |

* m is now sorted merger of left and right halves.

copy m back:





merge(data, 0, 3, 7)