ECE 274 Digital Logic – Fall 2008

Basic Logic Gates

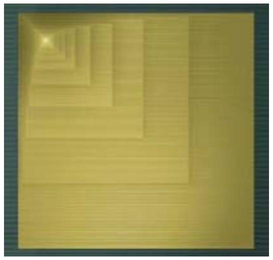
Digital Design 2.1 – 2.6

The University of Arizona
Tucson Arizona

Digital Design

Chapter 2:
Combinational Logic Design

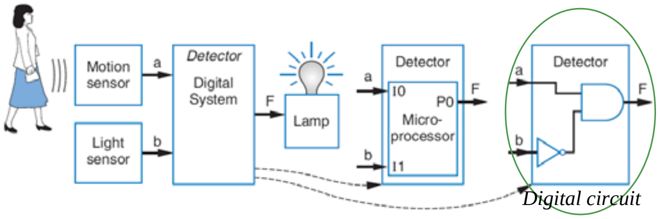
Slides to accompany the textbook *Digital Design*, First Edition,
by Frank Vahid, John Wiley and Sons Publishers, 2007.
http://www.ddvahid.com



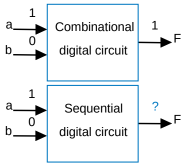
Digital Logic – Combinational Logic

Introduction

2.1



- Let's learn to design digital circuits
- We'll start with a simple form of circuit:
  - ***Combinational circuit***
    - A digital circuit whose outputs depend solely on the *present **combination** of the circuit inputs' values*

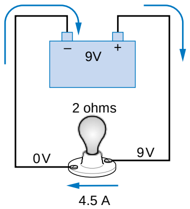


Digital Logic – Combinational Logic

Switches

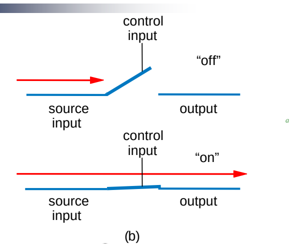
2.2

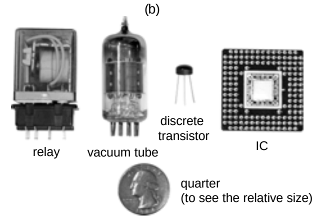
- Electronic switches are the basis of binary digital circuits
  - Electrical terminology
    - ***Voltage***: Difference in electric potential between two points
      - Analogous to water pressure
    - ***Current***: Flow of charged particles
      - Analogous to water flow
    - ***Resistance***: Tendency of wire to resist current flow
      - Analogous to water pipe diameter
    - V = I * R (Ohm's Law)



Digital Logic – Combinational Logic
Switches

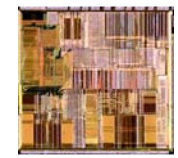
- A switch has three parts
  - Source input, and output
    - Current wants to flow from source input to output
  - Control input
    - Voltage that controls whether that current can flow
- The amazing shrinking switch
  - 1930s: Relays
  - 1940s: Vacuum tubes
  - 1950s: Discrete transistor
  - 1960s: Integrated circuits (ICs)
    - Initially just a few transistors on IC
    - Then tens, hundreds, thousands...





Digital Logic – Combinational Logic
Moore's Law

- IC capacity doubling about every 18 months for several decades
  - Known as "Moore's Law" after Gordon Moore, co-founder of Intel
    - Predicted in 1965 predicted that components per IC would double roughly every year or so
  - Today's ICs hold *billions* of transistors
    - The first Pentium processor (early 1990s) needed only 3 million

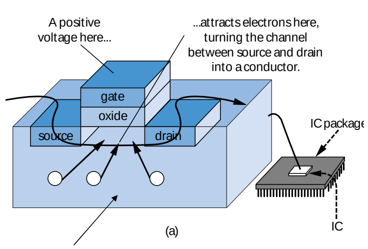


An Intel Pentium processor IC having millions of transistors

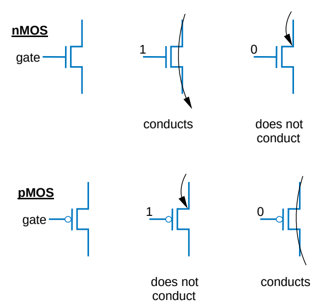
Digital Logic – Combinational Logic
CMOS Transistor

2.3

- CMOS transistor
  - Basic switch in modern ICs



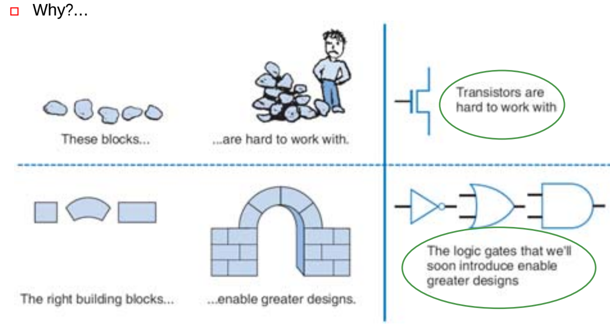
Silicon -- not quite a conductor or insulator: **Semiconductor**



Digital Logic – Combinational Logic
Boolean Logic Gates - Building Blocks for Digital Circuits

2.4

- "Logic gates" are better digital circuit building blocks than switches (transistors)
  - Why?...



Digital Logic – Combinational Logic
Boolean Algebra and its Relation to Digital Circuits

- To understand the benefits of "logic gates" vs. switches, we should first understand Boolean algebra
- "Traditional" algebra
  - Variable represent real numbers
  - Operators operate on variables, return real numbers
- ***Boolean Algebra***
  - Variables represent 0 or 1 only
  - Operators return 0 or 1 only
  - Basic operators
    - AND: *a AND b* returns 1 only when both a=1 and b=1
    - OR: *a OR b* returns 1 if either (or both) a=1 or b=1
    - NOT: *NOT a* returns the opposite of a (1 if a=0, 0 if a=1)

a	b	AND
0	0	0
0	1	0
1	0	0
1	1	1

a	b	OR
0	0	0
0	1	1
1	0	1
1	1	1

a	NOT
0	1
1	0

Digital Logic – Combinational Logic
Boolean Algebra and its Relation to Digital Circuits

- Developed mid-1800's by George Boole to formalize human thought
  - Ex: "I'll go to lunch if Mary goes OR John goes, AND Sally does not go."
    - Let F represent my going to lunch (1 means I go, 0 I don't go)
    - Likewise, m for Mary going, j for John, and s for Sally
    - Then **F = (m OR j) AND NOT(s)**
  - Nice features
    - Formally evaluate
      - m=1, j=0, s=1 --> F = (1 OR 0) AND NOT(1) = 1 AND 0 = **0**
    - Formally transform
      - F = (m and NOT(s)) OR (j and NOT(s))
        - Looks different, but same function
        - We'll show transformation techniques soon

a	b	AND
0	0	0
0	1	0
1	0	0
1	1	1

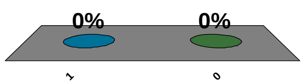
a	b	OR
0	0	0
0	1	1
1	0	1
1	1	1

a	NOT
0	1
1	0

Digital Logic – Combinational Logic
Evaluating Boolean Equations

- Evaluate the Boolean equation: F = (a AND b) OR (c AND d), where a=1, b=1, c=1, d=0. What is the value of F?

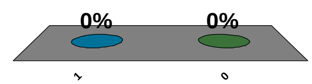
1. 1
2. 0



Digital Logic – Combinational Logic
Evaluating Boolean Equations

- Evaluate the Boolean equation: F = (a AND b) OR (c AND d), where a=0, b=1, c=0, d=1. What is the value of F?

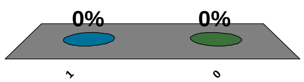
1. 1
2. 0



Digital Logic – Combinational Logic
Evaluating Boolean Equations

- Evaluate the Boolean equation: F = (a AND b) OR (c AND d), where a=1, b=1, c=1, d=1. What is the value of F?

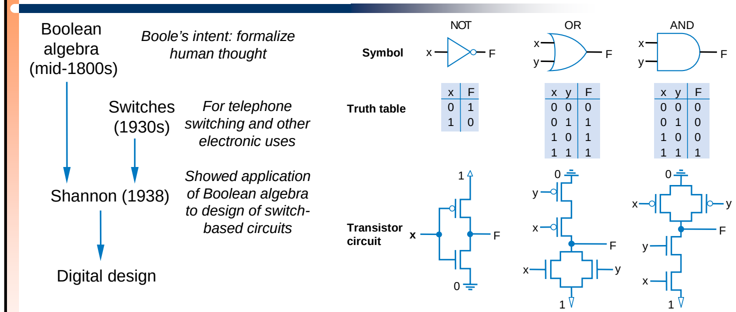
1. 1
2. 0



Digital Logic – Combinational Logic
Converting to Boolean Equations

- Convert the following English statements to a Boolean equation
  - Q1. a is 1 and b is 1.
    - Answer: F = a AND b
  - Q2. either of a or b is 1.
    - Answer: F = a OR b
  - Q3. both a and b are not 0.
    - Same as saying: both a and b are 1 (not 0)
    - Answer: F = a AND b
  - Q4. both a and b are not 1.
    - Answer: F = NOT(a) AND NOT(b)
  - Q5. a is 1 and b is 0.
    - Answer: F = a AND NOT(b)

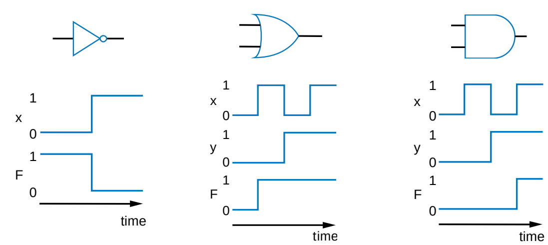
Digital Logic – Combinational Logic
Relating Boolean Algebra to Digital Design



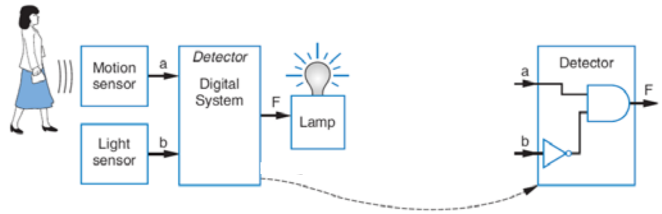
- Implement Boolean operators using transistors
  - Call those implementations ***logic gates***.
  - ***<u>Let's us build circuits by doing math</u>*** -- powerful concept

<u>Note</u>: These OR/AND implementations are inefficient; we'll show why, and show better ones later.

Digital Logic – Combinational Logic
NOT/OR/AND Logic Gate Timing Diagrams



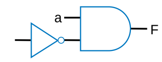
Digital Logic – Combinational Logic
Building Circuits Using Gates



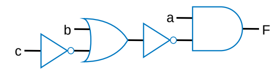
- Motion-in-dark Detector
  - Turn on lamp (F=1) when motion sensed (a=1) and no light (b=0)
  - F = a AND NOT(b)
  - Build using logic gates, AND and NOT, as shown
  - ***We just built our first digital circuit!***

Which circuit corresponds to the Boolean Equation:
F = a AND NOT(b OR NOT(c))

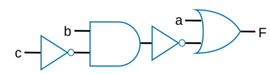
1. Circuit 1

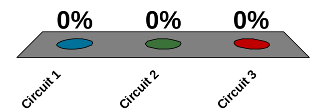


2. Circuit 2



3. Circuit 3

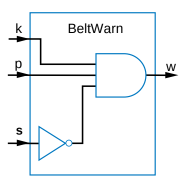




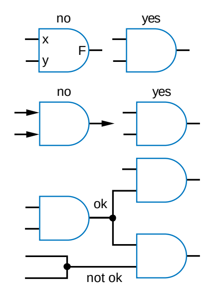
Digital Logic – Combinational Logic
Example: Seat Belt Warning Light System

- Design circuit for warning light
- Sensors
  - s=1: seat belt fastened
  - k=1: key inserted
  - p=1: person in seat
- Capture Boolean equation
  - person in seat, and seat belt not fastened, and key inserted
- Convert equation to circuit
- Notice
  - Boolean algebra enables easy capture as equation and conversion to circuit
    - How design with switches?
    - Of course, logic gates are built from switches, but we think at level of logic gates, not switches

w = p AND NOT(s) AND k



Digital Logic – Combinational Logic
Some Circuit Drawing Conventions



2.5

Digital Logic – Combinational Logic
Boolean Algebra

- By defining logic gates based on Boolean algebra, we can use algebraic methods to manipulate circuits
  - So let's learn some Boolean algebraic methods
- Start with notation: Writing a AND b, a OR b, and NOT(a) is cumbersome
  - Use symbols: a * b, a + b, and a' (in fact, a * b can be just ab).
    - Original: w = (p AND NOT(s) AND k) OR t
    - New: w = ps'k + t
      - Spoken as "w equals p and s prime and k, or t"
      - Or even just "w equals p s prime k, or t"
      - s' known as "complement of s"
    - While symbols come from regular algebra, *don't* say "times" or "plus"

Digital Logic – Combinational Logic
Boolean Algebra Operator Precedence

Boolean algebra precedence, highest precedence first.

Symbol	Name	Description
( )	Parentheses	Evaluate expressions nested in parentheses first
'	NOT	Evaluate from left to right
*	AND	Evaluate from left to right
+	OR	Evaluate from left to right

- Evaluate the following Boolean equations, assuming a=1, b=1, c=0, d=1.
  - F = ab + c.
    - Answer: the problem is identical to the previous problem, using the shorthand notation for *.
  - F = ab'.
    - Answer: we first evaluate b' because NOT has precedence over AND, resulting in F = 1 * (1') = 1 * (0) = 1 * 0 = 0.
  - F = (ac)'.
    - Answer: we first evaluate what is inside the parentheses, then we NOT the result, yielding (1*0)' = (0)' = 0' = 1.

Digital Logic – Combinational Logic
Boolean Algebra Terminology

- Example equation: F(a,b,c) = a'bc + abc' + ab + c
- ***Variable***
  - Represents a value (0 or 1)
  - Three variables: a, b, and c
- ***Literal***
  - Appearance of a variable, in true or complemented form
  - Nine literals: a', b, c, a, b, c', a, b, and c
- ***Product term***
  - Product of literals
  - Four product terms: a'bc, abc', ab, c
- ***Sum-of-products***
  - Equation written as OR of product terms only
  - Above equation is in sum-of-products form. "F = (a+b)c + d" is not.

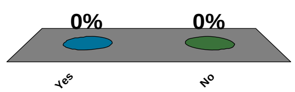
Digital Logic – Combinational Logic
Boolean Algebra Properties

- Commutative
  - a + b = b + a
  - a * b = b * a
- Distributive
  - a * (b + c) = a * b + a * c
  - a + (b * c) = (a + b) * (a + c)
    - (this one is tricky!)
- Associative
  - (a + b) + c = a + (b + c)
  - (a * b) * c = a * (b * c)
- Identity
  - 0 + a = a + 0 = a
  - 1 * a = a * 1 = a
- Complement
  - a + a' = 1
  - a * a' = 0
- *To prove, just evaluate all possibilities*

- Example: Show x + x'z equivalent to x + z.
  - Second distributive property
    - Replace x+x'z by (x+x')*(x+z).
  - Complement property
    - Replace (x+x') by 1,
  - Identity property
    - replace 1*(x+z) by x+z.

Digital Logic – Combinational Logic
Boolean Algebra

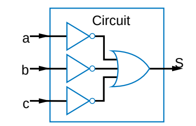
- Can xx' + xy(x'+y') ever evaluate to 1?
  1. Yes
  2. No

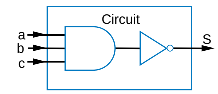


Digital Logic – Combinational Logic
Boolean Algebra Properties

- Null elements
  - a + 1 = 1
  - a * 0 = 0
- Idempotent Law
  - a + a = a
  - a * a = a
- Involution Law
  - (a')' = a
- DeMorgan's Law
  - (a + b)' = a'b'
  - (ab)' = a' + b'
  - Very useful!
- *To prove, just evaluate all possibilities*

- Aircraft lavatory sign example
  - Three lavatories, each with sensor (a, b, c), equals 1 if door locked
  - Light "Available" sign (S) if any lavatory available
- Equation and circuit
  - S = a' + b' + c'
- Transform
  - (abc)' = a'+b'+c' (by DeMorgan's Law)
  - S = (abc)'
- New equation and circuit
  - *Both are equivalent*





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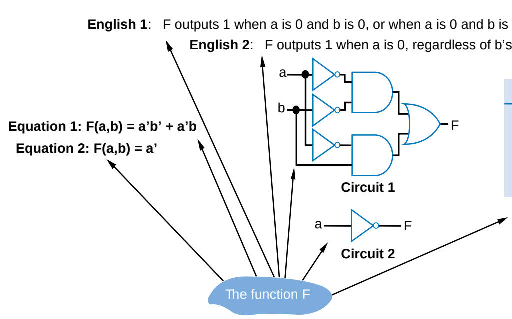
Digital Logic – Combinational Logic
Representations of Boolean Functions

- A function can be represented in different ways
  - Here are seven representations of the same function using four different methods: English, Equation, Circuit, and Truth Table



a	b	F
0	0	1
0	1	1
1	0	0
1	1	0

Digital Logic – Combinational Logic
Truth Table Representation of Boolean Functions

- Define value of F for each possible combination of input values
  - 2-input function: 4 rows
  - 3-input function: 8 rows
  - 4-input function: 16 rows
- Q: Use truth table to define function F(a,b,c) that is 1 when abc is 5 or greater in binary

a	b	F
0	0	
0	1	
1	0	
1	1	

(a)

a	b	c	F
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

(b)

a	b	c	d	F
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

(c)

a	b	c	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Digital Logic – Combinational Logic
Converting among Representations

- Can convert from any representation to any other
- Common conversions
  - Equation to circuit (we did this earlier)
  - Truth table to equation (which we can convert to circuit)
    - Easy -- just OR each input term that should output 1
  - Equation to truth table
    - Easy -- just evaluate equation for each input combination (row)
    - Creating intermediate columns helps

Q: Convert to truth table: F = a'b' + a'b

Inputs				Output
a	b	a'b'	a'b	F
0	0	1	0	**1**
0	1	0	1	**1**
1	0	0	0	**0**
1	1	0	0	**0**

Inputs		Outputs	Term
a	b	F	F = sum of
0	0	1	a'b'
0	1	1	a'b
1	0	0	
1	1	0	

F = a'b' + a'b

Q: Convert to equation

a	b	c	F	
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	0	
1	0	0	0	
1	0	1	1	ab'c
1	1	0	1	abc'
1	1	1	1	abc

F = ab'c + abc' + abc

Digital Logic – Combinational Logic
Standard Representation: Truth Table

- How can we determine if two functions are the same?
  - Is f = c'hp + c'hp' + c'h' the same as f = hc' + h'pc'?
    - Use algebraic methods
    - But if we failed, does that prove *not* equal? No.
- Solution: Convert to truth tables
  - Only ONE truth table representation of a given function
    - ***Standard*** representation -- for given function, only one version in standard form exists

f = c'hp + c'hp' + c'h'

f = c'h(p + p') + c'h'p

f = c'h(1) + c'h'p

f = c'h + c'h'p

(what if we stopped here?)

Q: Determine if F=ab+a' is same function as F=a'b'+a'b+ab, by converting each to truth table first

F = ab + 'a		
a	b	F
0	0	1
0	1	1
1	0	0
1	1	1

Same

F = a'b' + a'b + ab		
a	b	F
0	0	1
0	1	1
1	0	0
1	1	1

Digital Logic – Combinational Logic
Canonical Form -- Sum of Minterms

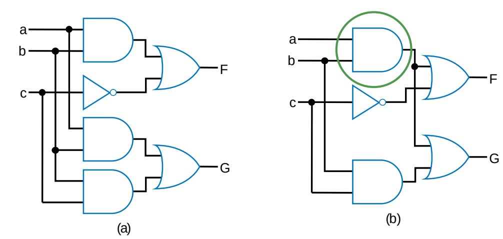
- Truth tables too big for numerous inputs
- Use standard form of equation instead
  - Known as ***canonical form***
  - Regular algebra: group terms of polynomial by power
    - $ax^2 + bx + c$ $(3x^2 + 4x + 2x^2 + 3 + 1 \text{ --> } 5x^2 + 4x + 4)$
  - Boolean algebra: create sum of minterms
    - ***Minterm***: product term with every function variable appearing exactly once, in true or complemented form
    - Just multiply-out equation until sum of product terms
    - Then expand each term until all terms are minterms

Q: Determine if F(a,b)=ab+a' is same function as F(a,b)=a'b'+a'b+ab, by converting first equation to canonical form (second already in canonical form)

F = ab+a' (already sum of products)
F = ab + a'(b+b') (expanding term)
F = ab + a'b + a'b' (SAME -- same three terms as other equation)

Digital Logic – Combinational Logic
Multiple-Output Circuits

- Many circuits have more than one output
- Can give each a separate circuit, or can share gates
- Ex: F = ab + c', G = ab + bc

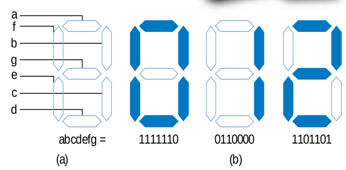


Option 1: Separate circuits   Option 2: Shared gates

Digital Logic – Combinational Logic
Multiple-Output Example: BCD to 7-Segment Converter

TABLE 2-4 4-bit binary number to seven-segment display truth table

w	x	y	z	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	0	0	0	0	0	0	0
1	0	1	1	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0	0	0
1	1	1	1	0	0	0	0	0	0	0



a = w'x'y'z' + w'x'yz' + w'x'yz + w'xy'z + w'xyz' + w'xyz + wx'y'z' + wx'y'z

b = w'x'y'z' + w'x'y'z + w'x'yz' + w'x'yz + w'xy'z' + w'xyz + wx'y'z' + wx'y'z

Digital Logic – Combinational Logic
In Class Exercise

- Convert the following Boolean equations to a digital circuit, sharing gates wherever possible.
  - F(a,b,c) = abc + a'b'c + bc'
  - G(a,b) = ab + a'b'