1. Let $X \sim U[0, 8]$ and let $Y = g(X)$ with $g$ as shown. Find
   a) $F_Y(0)$
   b) $F_Y(1)$
2. Let $X \sim N(0,1)$ and let $Y = g(X)$, where $g(x) = 7x + 2$. Find
   a) $\mu_Y$
   b) $\sigma_Y^2$
3. Let $X \sim N(0,1)$ and let $Y = X^2$. Find and sketch $f_Y$. 
4. Let $X$ and $Y$ be jointly uniform on the given region.

a) Find and sketch $f_X$

b) Find and sketch $f_Y$
5. Let $X$ and $Y$ be jointly uniform on the region shown below. Let $W = 2X + 2Y$ and $Z = X - Y$.

a) Find $f_{W,Z}$.
b) Sketch the region over which $f_{W,Z}$ is non-zero.
c) Are $X$ and $Y$ independent?
1. Let $X \sim U[0, 8]$ and let $Y = g(X)$ with $g$ as shown. Find
   
   a) $F_Y(0)$
   b) $F_Y(1)$

   \[
   F_Y(0) = P(Y \leq 0) = P(g(X) \leq 0) = P(3X \leq 0) = P(X \geq \frac{5}{3}) = \frac{5}{8}
   \]

   \[
   F_Y(1) = P(Y \leq 1) = P(3X \leq 1) = P(X \geq \frac{5}{3}) = \frac{13}{16}
   \]
2. Let $X \sim N(0,1)$ and let $Y = g(X)$, where $g(x) = 7x + 2$. Find

a) $\mu_Y$

b) $\sigma_Y^2$

\[
\begin{align*}
\mu_Y &= E[Y] = E[g(X)] = E[7X+2] = 7E[X] + 2 \\
&= 2 \\
\sigma_Y^2 &= E[(Y-2)^2] = E[(7X)^2] = 49E[X^2] \\
&= 49
\end{align*}
\]
3. Let $X \sim N(0,1)$ and let $Y = X^2$. Find and sketch $f_Y$.

\[
\frac{f_Y(y)}{\sqrt{2\pi}} = \frac{1}{\sqrt{2\pi}} e^{-\frac{x_1^2}{2}} + \frac{1}{\sqrt{2\pi}} e^{-\frac{x_2^2}{2}}
\]

where $x_1 = \sqrt{y}$, $x_2 = -\sqrt{y}$, and $g'(x) = 2x$. The graph of $f_Y(y)$ is shown with its domain and critical points.
4. Let $X$ and $Y$ be jointly uniform on the given region.
   
a) Find and sketch $f_X$
   
b) Find and sketch $f_Y$

\[
\begin{align*}
\int_{\mathbb{R}} f_X(x) &= \int_{-\infty}^{\infty} f_{x,y}(x,y) \, dy = \int_{0}^{\frac{1}{3}} \frac{1}{\frac{3}{2}} \, dy = \frac{2}{3}y \bigg|_0^{\frac{1}{3}} = \frac{2}{9} x I(\{x\}) \mathbb{I}[0, \frac{3}{2}] \\
\int_{\mathbb{R}} f_Y(y) &= \int_{-\infty}^{\infty} f_{x,y}(x,y) \, dx = \int_{\frac{3}{2}}^{3} \frac{1}{\frac{2}{3}} \, dx = \frac{2}{3}x \bigg|_{\frac{3}{2}}^{3} = 2(1-y) \mathbb{I}[0,1]
\end{align*}
\]
5. Let $X$ and $Y$ be jointly uniform on the region shown below. Let $W = 2X + 2Y$ and $Z = X - Y$.

a) Find $f_{W,Z}$.

b) Sketch the region over which $f_{W,Z}$ is non-zero.

c) Are $W$ and $Z$ independent?

\[
\begin{align*}
W &= 2X + 2Y \\
Z &= X - Y
\end{align*}
\]

\[
\begin{align*}
X &= \frac{1}{2}(W + Z) \\
Y &= \frac{1}{2}(W - Z)
\end{align*}
\]

\[
f_{W,Z}(w,z) = \frac{1}{4} \quad \text{for } \begin{pmatrix} w \\ z \end{pmatrix} \text{ in region above}
\]

when $(x,y)$ lie on the line $(y = x - 1)$

\[
\begin{align*}
xx &\quad \Rightarrow \frac{1}{2}(w - 2z) = y - x - 1 \quad \Rightarrow \quad x = \frac{1}{4}(w - 2z) + 1 \\
xx &\quad \Rightarrow \quad \frac{1}{2}(w + 2z) = \frac{1}{2}(w - 2z) + 1 \quad \Rightarrow \quad z = 1
\end{align*}
\]

Similarly, the other three boundary lines give $z = -1$, $w = 2$, $w = -2$.

\[
(\alpha, \beta) \quad \Rightarrow \quad f_{W,Z}(w,z) = \frac{1}{8} f_{X,Y}(w, z)
\]

\[
\begin{align*}
(0.75,0.75) &\quad \Rightarrow \quad f_X(0.75) \neq 0 \quad \text{and} \quad f_Y(0.75) \neq 0 \quad \{ \text{See figure} \}
\end{align*}
\]

\[
\begin{align*}
\text{But}, \quad f_{X,Y}(0.75,0.75) = 0
\end{align*}
\]

\[
\begin{align*}
\Rightarrow \quad f_{X,Y}(0.75,0.75) \neq f_X(0.75) f_Y(0.75)
\end{align*}
\]