Cluster-based Control Channel Allocation in Opportunistic Cognitive Radio Networks

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APPENDIX 1

Lemma 1: Let a vertex $x \in \mathcal{A}$ of a bipartite graph $\mathcal{G}(\mathcal{A} \cup \mathcal{B}, \mathcal{E})$ be connected to all vertices in the set $\mathcal{B}$. Then, x belongs to the maximum-edge biclique $Q^*(X^*, Y^*)$.

Proof: We prove Lemma 1 by contradiction. Let $x \in \mathcal{A}$ be a vertex of a bipartite graph $\mathcal{G}(\mathcal{A} \cup \mathcal{B}, \mathcal{E})$ such that there exists an edge (x, y), $\forall y \in \mathcal{B}$. Let $Q^*(X^*, Y^*)$ be the maximum edge biclique, and assume that $x \notin X^*$. By adding x to the graph Q^*, we obtain graph $Q'(X^* \bigcup x, Y^*)$, which is still a biclique since x is connected to every vertex in $\mathcal{B}$, and hence, every vertex in Y^*. The number of edges of the biclique Q' is $(|X^*| + 1) \times |Y^*| > |X^*| \times |Y^*|$. This contradicts our initial assumption that Q^* is a maximum-edge biclique. The same result can be shown for any vertex $y \in \mathcal{B}$ that is connected to all vertices in $\mathcal{A}$. □

APPENDIX 2

Lemma 2: Any $x \in A_i$ with $C_i \subseteq C_x$ will be included in the biclique $Q_i^*(X_i^*, Y_i^*)$ computed by Algorithm 1.

Proof: Let $x \in \mathcal{A}_i$ be a vertex of a bipartite graph $\mathcal{G}_i(\mathcal{A}_i \cup \mathcal{B}_i, \mathcal{E}_i)$. Suppose that there exists an edge (x, y) $\forall y \in \mathcal{B}_i$. Assume that the maximum edge biclique $Q_i^*(X_i^*, Y_i^*)$ is computed during the jth iteration of Algorithm 1. Then any CR added to X_i in the previous iterations will be part of Q_i^*. Hence, it is sufficient to show that x will be added to X_i^* before or during the jth iteration. If $Y_i^* = C_i$ then $x \in X_i^*$, since the addition of x increases the number of edges of Q_i^* by $|C_i|$. If $Y_i^* \subset C_i$, there exists some $x' \in X_i^*$ such that $C_{x'} \bigcap C_i \subset C_i$. Since on initialization $Y_i = C_i$ and $C_x \bigcap C_i = C_i$ according to line 4 of Algorithm 1, x will be added to X_i^* before x'. Hence, Q_i^* must contain x. □

APPENDIX 3

Lemma 3: If $\text{CR}_i \in X_j^2$ and $\text{CR}_j \in X_i^2$, then $Q_i^2 = Q_j^2$.

Proof: After step 1, CR_i and CR_j will have received the updates of their neighbors. Suppose that CR_i selects $Q_i^2 = Q_k^1$, where CR_k is a neighbor of CR_i, or is CR_i itself. Given that $\text{CR}_j \in X_i^2$, then $\text{CR}_j \in X_k^1$, and hence, CR_j is a neighbor of CR_k. Following a similar argument, we can show that for the decision $Q_j^2 = Q_m^1$ to be made, the selected Q_j^2 must be constructed by a node $\text{CR}_m \in \text{NB}_i$, given that $\text{CR}_i \in X_j^2$. Because CR_k and CR_m are neighbors of both CR_i and CR_j, CR_i and CR_j must have received both Q_k^1 and Q_m^1 in step 1, before updating their own bicliques. Due to the imposed total ordering, CR_i concludes that $Q_m^1 < Q_k^1$, and CR_j concludes that $Q_k^1 < Q_m^1$. This is true only if $k = m$. □

APPENDIX 4

Lemma 4: Suppose that for three nodes CR_i, CR_j, and CR_k, we have $\text{CR}_k \in X_i^2$ and $\text{CR}_k \in X_j^2$ with $Q_i^2 = Q_j^2$. Then if $\text{CR}_i \notin X_k^2$, it must also hold that $\text{CR}_j \notin X_k^2$.

Proof: Lemma 4 can be proved by contradiction. Assume that $\text{CR}_j \in X_k^2$. Because $\text{CR}_j \in X_k^2$ and $\text{CR}_k \in X_j^2$, then $Q_j^2 = Q_k^2$ by Lemma 3. However, by assumption we also have $Q_i^2 = Q_j^2$, and hence $Q_i^2 = Q_k^2$. Since $\text{CR}_k \in X_i^2$ and $Q_i^2 = Q_k^2$, this also means that $\text{CR}_i \in X_k^2$, which leads to a contradiction. Hence, $\text{CR}_j \notin X_k^2$. □

APPENDIX 5

Theorem 1: For any $\text{CR}_j \in X_i^3$, $Q_i^3 = Q_j^3$.

Proof: Q_i^3 is a pruned version of Q_i^2, i.e., $X_i^3 \subseteq X_i^2$. Therefore, any $\text{CR}_j \in X_i^3$ must also be a member of X_i^2. Also for any $\text{CR}_j \in X_i^3$, we have $\text{CR}_i \in X_j^2$, since otherwise, CR_j would have been removed from X_i^3. Using Lemma 3, it follows that $Q_i^2 = Q_j^2$. Now consider any $\text{CR}_k \in X_i^2$ that is removed from X_i^2 in step 3, i.e., $\text{CR}_k \notin X_i^3$. This happens only if $\text{CR}_i \notin X_k^2$, which also means (by Lemma 2) that $\text{CR}_j \notin X_k^2$, and CR_k will also be removed from X_j^2 in step 3. Hence, every CR that is removed from X_i^2 will also be removed from X_j^2, making $X_i^3 = X_j^3$. For two bicliques with the same membership, it follows that $Y_i^3 = Y_j^3$, and hence $Q_i^3 = Q_j^3$. □

APPENDIX 6

Lemma 5: In every cluster produced by SOC, at least one CR is one-hop away from all other CRs of that cluster.

Proof: Consider a cluster that is represented by the biclique $Q_i^3(X_i^3, Y_i^3)$. According to Theorem 1, all $\text{CR}_j \in X_i^3$ converge to the same cluster membership in step 3. For any CR_i and $\text{CR}_j \in X_i^3$, it holds that $\text{CR}_i \in X_j^2$ and $\text{CR}_j \in X_i^2$. Otherwise, CR_i would have removed CR_j from X_i^2 in step 2, and similarly CR_j would have removed CR_i from X_j^2. According to Lemma

3, if $\mathrm{CR}_i \in X_j^2$ and $\mathrm{CR}_j \in X_i^2$, it holds that $Q_i^2 = Q_j^2$. This means that all members of a cluster formed after step 3 must have computed the same biclique in step 2. However, the biclique Q_i^2 of any CR in step 2 is the best biclique Q_j^1 with $\mathrm{CR}_j \in \mathrm{NB}_i$ or $j = i$. Hence, the only way that all CRs of a cluster would choose Q_j^1 as the best biclique in step 2 is if CR_j is a neighbor to all. Therefore, at least one CR is one hop away from all CRs of the cluster. $\square$