Throughput-optimal Sequential Channel Sensing and Probing in Cognitive Radio Networks

Tao Shu and Marwan Krunz
Department of Electrical and Computer Engineering
University of Arizona

Technical Report
TR-UA-ECE-2008-4
August 2008

Abstract

In a dynamic spectrum sharing system, a cognitive radio (CR) is provided with more channels than what it can use. So it is important for the CR to select the right channels for its transmission. To avoid interfering with incumbent primary radios, the existing schemes are based on channel sensing, which use the busy/idle status of a channel as the criterion to select channels. Such schemes in general do not provide good throughput performance for CRs. In this paper, we study a throughput-optimal joint sensing/probing scheme for CRs that uses the channel quality as a second criterion in selecting channels. The difficulty of this problem comes from the fact that a CR cannot first scan all channels and then pick the best one. This is because the total number of channels might be large, while a CR senses and probes channels sequentially due to its power and hardware limitations. After sensing and probing a channel, the CR needs to make a decision about whether to terminate the scan and use the underlying channel or to skip it and scan the next one. The optimal use-or-skip decision strategy that maximizes the CR's average throughput is one of our primary concerns in this study. This optimal strategy is derived by formulating the above sequential channel sensing/probing/access process as an infinite-horizon rate-of-return problem, which we solve using optimal stopping theory. We then further look into the structure of this strategy to conduct a second-round optimization over the operational parameters, such as the sensing and probing times. We show through numerical examples that when these operational parameters are properly set, significant throughput gain (e.g., about 100%) can be achieved by our joint sensing/probing scheme over the conventional one that uses sensing alone.

1 Introduction

The benefit of *dynamic spectrum sharing* (DSS) as a means of improving spectrum utilization is now well recognized [18]. DSS aims at opening the under-utilized portions of the spectrum for secondary re-use, provided that the transmissions of secondary radios do not cause harmful interference to the licensed radios (a.k.a., primary radios (PRs)). Because now a secondary radio is provided with more channels than what it can use, a critical challenge in DSS is to select in real-time the channels that the secondary radios should use. Such a selection should provide a secondary radio with the maximum possible throughput under the premise that PRs will not be negatively affected by this selection. For scalability purposes, a distributed selection algorithm is also desirable.

The *cognitive radio* (CR) is regarded as the enabling technology for DSS [13]. The conventional way for a CR to select channels distributedly is to scan (sense) channels and access those channels that are deemed to be idle. Although this approach guarantees a safe (secondary) access to spectrum for CRs, it generally does not give optimal throughput performance. This is because the CR does not account for the quality of the idle channel. As a result, transmitting over channels of poor conditions comprises the CR's throughput.

In this paper, we study a *joint sensing/probing* mechanism for cognitive radio networks (CRNs) to improve the throughput. This mechanism uses the channel quality information as a second criterion for channel selection. Specifically, a channel-probing component is added right after channel sensing to decide the maximum data rate supported by the probed channel. Among idle channels, a CR will use only good channels that support relatively high data rates.

Although the use of probing has been comprehensively studied in the past for general wireless systems [14], the problem is new in the context of CRNs. First, unlike previous work, channel selection in a CRN is sequential. Due to the large number of channels and the CR's power/hardware limitations, it is not possible for a CR to first scan all channels simultaneously and then pick the best one. A CR can only sense and probe channels sequentially. After sensing and probing a channel, the CR needs to decide whether to terminate the scan and use the underlying channel or to skip it and scan the next one. Furthermore, to avoid collisions with PRs, a CR cannot recall (use) a channel it previously skipped, because of the staleness of that sensing outcome (the channel may have been taken by other CR or PR transmissions). This non-recall use of channel along with the lack of knowledge about the conditions of those un-probed channels make such a sequential decision making non-trivial. Second, the decision making process becomes even more difficult when the CR's sensing and probing overheads need to be accounted for in each step. Empirical data shows that sensing one channel takes tens of ms and probing one new channel takes from 10 to 133 ms, depending on the association and capture speed between the transmitter and receiver after each channel hopping [1]. At the same time, to reduce collisions with newly activated PRs, a CR's transmission time over an idle channel must be restricted, e.g., in the order of hundreds of ms or at most few seconds. Therefore, the accumulated overhead after sequentially sensing/probing several channels becomes comparable with or even greater than the CR's transmission time. When these overheads are concerned, to find a slightly better channel may not justify continuing the channel search process. As will become clear shortly, these new aspects of a CRN require a totally new formulation for the problem.

In this paper, we address the following key issues that are aimed at making the CRN's sensing/probing/access scheme operationally efficient. First, we derive the throughput-optimal decision strategy for the sequential channel sensing/probing process. It turns out that this optimal strategy has a threshold structure, which basically says whether the channel is good or bad. To set this threshold properly, we need to consider the tradeoff between the achievable data rate brought by good channels and the time cost (and consequently, throughput reduction) for searching for good channels. Second, we derive the maximum acceptable channel probing time that guarantees a positive throughput gain for the proposed method over a scheme that does not utilize probing. This knowledge is important because the accumulated probing time may be so significant that it cancels out gains achieved by selecting good channels. Third, we optimize the channel sensing time. In realistic systems, this sensing time determines the accuracy of the channel sensing process. A shorter sensing time reduces the scanning time of each channel at the expense of increasing the sensing false alarm rate, making the CR miss more spectrum opportunities. This in turn increases the number of channels the CR needs to sense and probe, leading to possibly longer overall channel search time. We exploit the tradeoff between the sensing time and sensing accuracy to minimize the total channel search time (or equivalently, maximize the throughput). Our work is the first to incorporate the relationship between the sensing time and sensing accuracy in a multi-rate setting.

The above contributions are achieved by performing two rounds of optimizations. In the first round, we treat the sensing and probing times as parameters, and derive the parametric optimal probing strategy. This is achieved by formulating the sensing/probing/access process as an infinite-horizon maximum rate-of-return problem in the optimal stopping theory [3], with the number of bits that the CR is able to send in one transmission as the return, and the overall channel search plus transmission times as the time cost. Next, we look into the particular structure of the optimal probing strategy and perform a second round of optimization over the operational parameters, such as the sensing and probing times, aiming at maximizing the outcome of the first-round optimization.

Besides the above optimization considerations, we are also interested in the aggregate throughput performance when a network of CRs coexist with PRs, and each CR reacts according to the sensing/probing/access scheme in a distributed way. A Markov-chain model is developed for our performance analysis, whereby the contention between CRs, the sensing strategies employed (random channel sensing and collaborative channel sensing), and probing threshold settings at individual CRs are all accounted for. Our results show that when the sensing/probing parameters are properly set, the addition of probing can significantly improve the CRN's throughput, e.g., over 100% gains are observed in our simulations.

The remainder of this paper is organized as follows. Section 2 describes the system model and its maximum rate of return formulation. Based on optimal-stopping theory, we solve the optimization problem in Section 3. Section 4 studies the performance for multi-CR case. Numerical results are presented in Section 5. Section 6 reviews related works and Section 7 concludes the paper. All proofs of the theorems are given in the appendix.

2 Model Description and Problem Formulation

2.1 System Model

We consider a set of C licensed channels. The status of a channel is modeled as a continuous-time random process that alternates between two states: IDLE and BUSY. A BUSY (IDLE) state indicates that some (no) PR user is transmitting over the channel. Denote the average IDLE and BUSY durations by α and β, respectively. When the channel is observed at an arbitrary time, its idle and busy probabilities are given by $P_I = \frac{\alpha}{\alpha+\beta}$ and $P_B = \frac{\beta}{\alpha+\beta}$, respectively. Here we focus on the homogeneous channel utilization scenario, i.e., we assume that the states of different channels are driven by homogeneous and independent random process. This may correspond to the scenario that all channels belong to the same licensed network. The channel selection problem under *heterogeneous* channel utilization is actually trivial, because in that case a CR should select the channel with the lowest utilization.

Along with the PR users, the spectrum is opportunistically shared with a number of CRs.

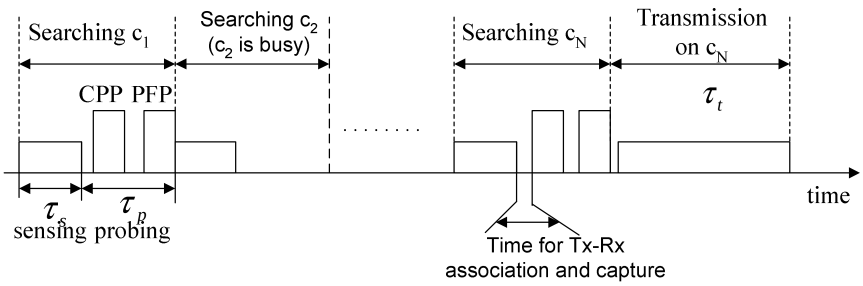

Figure 1: Sequential channel sensing and probing before transmission.

To simplify the exposure, we ignore for the time being the CR-to-CR contention issue related to having multiple CRs. This allows us to focus on the channel sensing/probing/access process of a pair of CR transmitter and receiver, with the goal of optimizing this process. We also assume that some synchronization mechanism (e.g., a random-number-generater-based one) is in place so that the CR transmitter and receiver are always sensing and probing the same channel at the same time. We will account for the contention issue in Section 4 when we study the multi-CR scenario.

When a CR wants to transmit, it starts searching for channels sequentially, as shown in Figure 1. Specifically, at the beginning, the CR randomly picks channel c_1, $1 \leq c_1 \leq C$, and samples it for τ_s time. Then the CR decides whether channel c_1 is idle or busy. If it is busy, the CR randomly selects the next channel c_2, $1 \leq c_2 \leq C$, to sample, and so on. Suppose that in the nth step, channel c_n is determined as idle. Then the CR transmitter begins to probe that channel by sending a channel probing packet (CPP) over channel c_n using a predefined power. The CR receiver measures the strength of the received CPP and decides the maximum achievable data rate (MADR), r_n, that can be supported by the current channel. The value of r_n is selected from a set of discrete rates: $\{R_k, k = 0, 1, \ldots, K\}$, where R_k increases with k and $R_0 = 0$. This MADR value is then embedded into a probing feedback packet (PFP), which is sent back from the receiver to the transmitter over channel c_n. The time spent on one CPP/PFP exchange plus the preceding time for association and capture between the transmitter and receiver is called the channel probing time τ_p. After receiving the PFP, the transmitter decides whether to use this channel or not. This is done by comparing r_n with some channel quality threshold, r^*. If $r_n \geq r^*$, then the transmitter terminates the channel search and transmits at rate r_n over channel c_n for τ_t duration of time (τ_t should be short enough such that collisions with newly activated PRs for this amount of time are deemed acceptable). If $r_n < r^*$, the CR will skip this channel and continue to sense the next one. Because the CR receiver also has knowledge of r^* (e.g., this information can be embedded into the CPP), there is no need for the transmitter to notify the receiver about its decision. Note that if the channel is busy during the sensing phase, no probing packets should be exchanged between the CR transmitter and receiver, to avoid interfering with PRs. However, the receiver still has to wait for τ_p time to realize that the channel must be busy. Therefore, whether or not the channel is idle, the time cost for one step of channel search is $\tau_s + \tau_p$.

Remark: If a CR can transmit over J idle channels at a time, then J parallel channel searching/access instances can be initiated and maintained by the CR. Each instance will independently search and use one idle channel according to the above sequential process. The optimizations over each instance are identical and independent. Therefore we only need to focus on one such instance in our treatment.

Channel sensing is modeled as a binary hypothesis test, where H_0 indicates an idle channel and H_1 indicates an occupied channel. Let $x(t)$ be the sample collected by the CR. Then,

$$x(t) = \begin{cases} n(t), & H_0 \text{ (idle)} \\ s(t) + n(t), & H_1 \text{ (occupied)} \end{cases} \tag{1}$$

where $n(t)$ is the AWGN and $s(t)$ is the received PR's signal at the CR. Regarding this sensing process, the probabilities of false alarm P_{fa} and miss detection P_{md} are defined as follows

$$P_{fa}(\tau_s) = \Pr\{\text{CR decides the channel is busy}|H_0\} \tag{2}$$

$$P_{md}(\tau_s) = \Pr\{\text{CR decides the channel is idle}|H_1\}. \tag{3}$$

Note that these two probabilities are functions of the sensing time τ_s.

The unconditional probabilities that a CR decides a channel is idle (Q_I) or busy (Q_B) are given by

$$Q_I = P_B P_{md} + P_I(1 - P_{fa}) \approx P_I(1 - P_{fa}) \tag{4}$$

$$Q_B = P_B(1 - P_{md}) + P_I P_{fa} \approx P_B + P_I P_{fa} \tag{5}$$

The approximation in the last steps is due to the practical requirement that $P_{md} \ll 1$ (e.g., 1% is a typical value), which ensures a secondary role for the CRs.

2.2 Problem Formulation

The throughput-optimal sequential channel sensing/probing/access process can be formulated as an optimal stopping problem. We first briefly describe the definition of an optimal stopping problem and then present our formulation.

An optimal stopping problem is defined by the following two components [3]:

1. A sequence of random variables $X_1,\ X_2, \ldots,$ whose joint distribution is assumed to be known.

2. A sequence of real-valued reward functions, $y_0,\ y_1(x_1),\ y_2(x_1, x_2), \ldots,\ y_\infty(x_1, x_2, \ldots)$.

The sequence $X_1, X_2, \ldots$, can be observed sequentially (one variable at a time) for as long as needed. For each observation instance $n = 1, 2, \ldots$, after observing $X_1 = x_1,\ X_2 = x_2, \ldots, X_n = x_n$, one may stop and receive the known reward $y_n(x_1, \ldots, x_n)$, or one may continue to observe X_{n+1}. If the decision is not to take any observations, the received reward is the constant y_0. If the observer never stops, the received reward is $y_\infty(x_1, \ldots,)$. The goal is to choose a rule to stop such that the expected reward at the stopping time N, $E\{y_N\}$, is maximized. According to this framework, the optimal-stopping formulation of our problem is as follows.

First, we define the sequence of observations. For the nth sensing and probing step, $n \geq 1$, the MADR value of the channel, $r_n \in \{0, R_1, \ldots, R_K\}$, can be obtained. Depending on the fading and shadowing effects on the channel, let the distribution of r_n be $p_k = \Pr\{r_n = R_k\}$, $k = 0, 1, \ldots, K$ (we assume the fluctuations on different channels are i.i.d.). We define X_n as the outcome of the nth-step sensing and probing: $X_n = 0$ if the channel is busy and $X_n = r_n$ if the channel is idle. The distribution of X_n can be calculated as follows.

$$q_0 \overset{\text{def}}{=} \Pr\{X_n = 0\} = Q_B + Q_I p_0 \tag{6}$$

$$q_k \overset{\text{def}}{=} \Pr\{X_n = R_k\} = Q_I p_k, \quad \text{for } 1 \leq k \leq K. \tag{7}$$

Next, we define the sequence of rewards: The reward of stopping after observing X_n is defined as the throughput achieved by transmitting over channel c_n and with the entire time span (i.e., from the beginning of observing X_1 until the end of transmission over channel c_n) taken into account. Recall that the use of channel must be non-recall. Mathematically, the reward for transmitting over channel c_n is given by

$$y_n(X_1, \ldots, X_n) \overset{\text{def}}{=} y_n(X_n) = \frac{B_{eff}(n)}{T_{tot}(n)} = \frac{X_n \tau_t (1 - P_{loss})}{n(\tau_s + \tau_p) + \tau_t} \tag{8}$$

where $B_{eff}(n)$ is the number of collision-free data bits that can be transmitted over channel c_n, $T_{tot}(n)$ is the total time cost including channel search and transmission times, and P_{loss} is the probability that channel c_n is re-occupied by some returning PR during the CR's transmission, and thus a collision occurs and the CR's transmission is void. Defining the moment of sensing as the reference point, denote the forward recurrence time of the channel's IDLE period by the random variable $\tilde{\tau}_0$ and its pdf by $\tilde{f}_0$. We can calculate P_{loss} as follows

$$P_{loss} = \Pr\{\tilde{\tau}_0 < \tau_t\} = \int_0^{\tau_t} \tilde{f}_0(t)dt. \tag{9}$$

Following standard renewal theory analysis:

$$\tilde{f}_0(t) = \frac{1 - \int_0^t f_0(\tau)d\tau}{\int_0^\infty \tau f_0(\tau)d\tau} \tag{10}$$

where f_0 is the pdf of the channel's IDLE period. For example, if f_0 is an exponential distribution with mean α, then $\tilde{f}_0 = f_0$ and P_{loss} can be calculated as $P_{loss} = 1 - e^{-\frac{\tau_t}{\alpha}}$.

Define $\Psi = \{N : N \geq 1, E[T_{tot}(N)] < \infty\}$ as the set of all possible stopping rules. Our problem is to find an optimal stopping rule $N^* \in \Psi$ that maximizes the following rate-of-return objective function:

$$\text{maximize}_{N\in\Psi} \ \frac{E\{B_{eff}(N)\}}{E\{T_{tot}(N)\}}. \tag{11}$$

Clearly, because the CR decides after each observation whether or not to stop (according to some rule), the final stopping time N becomes a random variable. Therefore, the number of bits that can be effectively transmitted at the stopping point, $B_{eff}(N)$, together with the time cost $T_{tot}(N)$, are both random variables related to N. This is in contrast to the $B_{eff}(n)$ and $T_{tot}(n)$ in (8), where n is a constant.

The reason we wish to maximize the ratio in (11) rather than the true expected average $E\left\{\frac{B_{eff}(N)}{T_{tot}(N)}\right\}$ is that if the problem is repeated independently Z times with a fixed stopping rule leading to i.i.d. stopping times, $N_1, \ldots, N_Z$ and i.i.d. returns $B_{eff}(N_1), \ldots, B_{eff}(N_Z)$, then the overall average return per unit time is the ratio $(B_{eff}(N_1) + \ldots + B_{eff}(N_Z))/(T_{tot}(N_1) + \ldots + T_{tot}(N_Z))$. As $Z \to \infty$, the limit of the expectation of the above ratio (if it exists) must converge to $E\{B_{eff}(N)\}/E\{T_{tot}(N)\}$ by the law of large numbers [3]. Therefore, our objective function can be interpreted as the long-term average throughput provided by the stopping rule.

3 Optimal Stopping Rule and Optimization Considerations

In this section, we first solve the maximum-rate-of-return problem (11) using optimal stopping theory. We then further examine the structure of our solution to address the optimization issues raised in Section 1.

3.1 Throughput-optimal Stopping Rule

The solution to (11) heavily hinges on the optimal stopping theory [3]. Specifically, according to [3], in order to solve problem (11), we can first consider a transformed version of the problem, whose reward sequence is defined by

$$\begin{aligned} w_n &= B_{eff}(n) - \lambda T_{tot}(n) \\ &= X_n \tau_t (1 - P_{loss}) - \lambda[n(\tau_s + \tau_p) + \tau_t]. \end{aligned} \tag{12}$$

When the parameter λ is chosen such that the optimal expected reward of the transformed problem, i.e., $V^* \stackrel{\text{def}}{=} sup_{N\in\Psi} E\{B_{eff}(N) - \lambda T_{tot}(N)\}$, becomes zero, the optimal stopping rule N^* of this transformed problem is also the optimal stopping rule of the original problem (11). In addition, the solution of λ that makes $V^* = 0$ hold, denoted as λ^*, is the maximum throughput in (11) achieved by the optimal stopping rule N^*. Applying this philosophy, we present the following results regarding the existence and solution of the optimal stopping rule for problem (11).

Theorem 1: An optimal solution to (11) exists. The maximum throughput λ^* that is achieved by this optimal stopping rule is the solution of the equation: $E\{\max(X_n\tau_t(1-P_{loss})-\lambda^*\tau_t, 0)\} = \lambda^*(\tau_s + \tau_p)$. The optimal stopping rule is given by $N^* = \min\{n \geq 1 : X_n \geq \frac{\lambda^*}{1-P_{loss}}\}$.

All proofs of theorems are presented in Appendix. Regarding the calculation of the optimal throughput and the optimal stopping rule of (11), we have the following theorem:

Theorem 2: λ^* has a unique solution.

For the particular discrete-rate CRs considered in our work, a fast numerical algorithm can be developed to calculate the exact λ^* in at most $\mathcal{O}(K)$ time, where K is the number of rates supported by the CR. Such an algorithm is based on the following observations. First, for the multi-rate system $X_n \in (R_0, R_1, \ldots, R_K)$, where $R_0 = 0 < R_1 < \ldots < R_K$, define k^* to be the minimum integer that satisfies $R_{k^*} \geq \frac{\lambda^*}{1-P_{loss}}$. Obviously, $1 \leq k^* \leq K$. Using this notation, the equation $E\{\max(X_n\tau_t(1 - P_{loss}) - \lambda^*\tau_t, 0)\} = \lambda^*(\tau_s + \tau_p)$ can be written as

$$\begin{aligned} g(\lambda^*) \stackrel{\text{def}}{=} \sum_{k=k^*}^{K} (R_k\tau_t(1 - P_{loss}) - \lambda^*\tau_t)q_k = \lambda^*(\tau_s + \tau_p), \\ \text{given that } R_{k^*-1} < \frac{\lambda^*}{1 - P_{loss}} \leq R_{k^*}. \end{aligned} \tag{13}$$

This gives a candidate solution for λ^*

$$\lambda^* = \frac{\tau_t(1-P_{loss})\sum_{k=k^*}^{K} R_k q_k}{\tau_s + \tau_p + \tau_t \sum_{k=k^*}^{K} q_k}, \quad \text{if } R_{k^*-1} < \frac{\lambda^*}{1-P_{loss}} \leq R_{k^*} \tag{14}$$

The range of values for k^* is from 1 to K. Therefore, one can first enumerate all candidates of λ^* according to (14), and then pick the one that satisfies the condition $R_{k^*-1} < \frac{\lambda^*}{1-P_{loss}} \leq R_{k^*}$. Theorem 2 guarantees that there is only one candidate satisfying this condition. The particular R_{k^*} under which the right λ^* is obtained is the threshold rule that determines whether an idle channel is good enough to be used.

3.2 Optimization Considerations

3.2.1 Impact of Probing Overhead

In this section, we evaluate the relationship between λ^* and the operational parameters τ_s and τ_p. We first look into the structure of the optimal solution (14). It turns out that λ^* can be written as a segmented function. Specifically, for the jth segment, $1 \leq j \leq K$, the value of λ^* satisfies $R_{j-1}(1-P_{loss}) < \lambda^* \leq R_j(1-P_{loss})$. To satisfy this condition, it must be the case that

$$R_{j-1} < \frac{\tau_t \sum_{k=j}^{K} R_k q_k}{\tau_s + \tau_p + \tau_t \sum_{k=j}^{K} q_k} \leq R_j. \tag{15}$$

After some mathematical manipulations, (15) leads to the following condition:

$$\frac{\sum_{k=j}^{K} R_k q_k - R_j \sum_{k=j}^{K} q_k}{R_j} \leq \frac{\tau_s + \tau_p}{\tau_t} < \frac{\sum_{k=j}^{K} R_k q_k - R_{j-1} \sum_{k=j}^{K} q_k}{R_{j-1}}. \tag{16}$$

The ratio $\eta \stackrel{\text{def}}{=} \frac{\tau_s+\tau_p}{\tau_t}$ represents the efficiency of the channel sensing/probing/access scheme. All the time factors in (16) are separated from the upper and lower bounds of the segment, allowing a neat partition of segments based on η. Following this thread, λ^* can be explicitly written in the following segmented form:

$$\lambda^* = \begin{cases} \lambda_1^*(\eta) & \text{for } \phi_1 \leq \eta < \Phi_1 \\ \vdots & \\ \lambda_j^*(\eta) & \text{for } \phi_j \leq \eta < \Phi_j \\ \vdots & \\ \lambda_K^*(\eta) & \text{for } \phi_K \leq \eta < \Phi_K \end{cases} \tag{17}$$

where $\lambda_j^*(\eta) \stackrel{\text{def}}{=} \frac{(1-P_{loss})\sum_{k=j}^{K} R_k q_k}{\eta + \sum_{k=j}^{K} q_k}$, $\phi_j \stackrel{\text{def}}{=} \frac{\sum_{k=j}^{K} R_k q_k - R_j \sum_{k=j}^{K} q_k}{R_j}$, and $\Phi_j \stackrel{\text{def}}{=} \frac{\sum_{k=j}^{K} R_k q_k - R_{j-1} \sum_{k=j}^{K} q_k}{R_{j-1}}$, $j = 1, \ldots, K$. Because we are interested in determining whether the inclusion of probing leads to better performance, we can assume τ_s and τ_t to be fixed (we will discuss the case when τ_s is an optimization variable shortly), and analyze the structure of λ^* as a function of τ_p. In this sense, η becomes a one-to-one image of τ_p.

Theorem 3: Given τ_s and τ_t, the function λ^* defined in (17) is a continuous and strictly mono-decreasing segmented function over the entire domain of η.

When probing is not used, the average throughput, denoted by λ', can be derived as follows. First, the number of channels that are sensed until an idle channel is found follows a geometric distribution with parameter Q_I. So the average time cost for finding an idle channel is given by $T_s \stackrel{\text{def}}{=} \tau_s/Q_I$. Once an idle channel is found, the average data rate supported by that channel is $\bar{R} = \sum_{k=1}^{K} R_k p_k$. So λ' is calculated as

$$\lambda' = \frac{(1 - P_{loss})\tau_t \sum_{k=1}^{K} R_k p_k}{T_s + \tau_t} = \frac{(1 - P_{loss}) \sum_{k=1}^{K} R_k q_k}{\eta' + Q_I} \tag{18}$$

where $\eta' \stackrel{\text{def}}{=} \frac{\tau_s}{\tau_t}$. Given τ_s and τ_t, λ' is a constant. The equation $\lambda^*(\eta) = \lambda'$ must have a unique solution. This is because when $\tau_t = 0$, the sensing/probing/access scheme is at least as good as the sensing/access scheme, while $\tau_t \to \infty$, $\lambda^*(\infty) = 0 < \lambda'$. The property presented in Theorem 3 guarantees the existence of a unique intersection between $\lambda^*(\eta)$ and λ'. Therefore, the maximum acceptable τ_t that guarantees a throughput gain for sensing/probing/access scheme is given by

$$\tau_p^{\max} = \tau_t \lambda^{*-1}(\lambda') - \tau_s \tag{19}$$

where $\lambda^{*-1}(\cdot)$ denotes the inverse function of $\lambda^*(\eta)$. The significance of (19) is that it dictates when probing should be used for a given set of sensing/probing/access parameters.

3.2.2 Impact of Sensing Time

In this section, we are interested in the impact of τ_s on the optimal throughput. It is well known that for a given sensing/access CR system, throughput is a concave function of τ_s [11]. So there exists an optimal sensing time that maximizes the throughput. However, our finding in this section reveals that in general the concavity of the throughput is not preserved when probing is included, largely due to the more complicated structure of the multi-rate system. The encouraging side of our finding is that when τ_s is the variable, the throughput maintains its segmented structure. Treating a segment as our evaluation unit, the trend of the throughput is concave over the segments of τ_s. Based on this property, we can derive a closed range $\mathbf{T}_s^o$ that contains the optimal sensing time τ_s^o. The value of this range is that for any $\tau_s \in \mathbf{T}_s^o$, it leads to a throughput greater than what can be achieved under any $\tau_s \notin \mathbf{T}_s^o$. The range $\mathbf{T}_s^o$ is also provably efficient, i.e., any value inside this range can achieve at least a provable fraction of the maximum throughput achieved with τ_s^o. As a result, achieving provably near-optimal performance is still guaranteed.

Our analysis involves evaluating the partition points of each segment defined in (17). In total, there are $K+1$ distinct partition points: $\phi_0 \stackrel{\text{def}}{=} \Phi_1 = \infty > \phi_1 > \phi_2 > \ldots > \phi_{K-1} > \phi_K = 0$, where the new notation ϕ_0 is defined for presentation convenience. For $1 \leq j \leq K-1$, ϕ_j can be written as

$$\phi_j = (1 - P_{fa}(\tau_s))C_j \tag{20}$$

where $C_j \stackrel{\text{def}}{=} \frac{P_I \sum_{k=j}^{K}(R_k - R_j)p_k}{R_j}$ is a channel-dependent quantity that does not depend on τ_s. We consider an energy detector for the channel sensing, for which the channel false alarm probability is approximated by [11]

$$P_{fa}(\tau_s) = Q\left(\left(\frac{\epsilon}{\sigma_u^2} - 1\right)\sqrt{\tau_s f_s}\right) \tag{21}$$

where $\frac{\epsilon}{\sigma_u^2}$ is the decision threshold for sensing and f_s is the bandwidth of the channel. Given a minimum sensing time $\tau_s^{\min}$ and a desired miss detection probability $\bar{P}_{md}$, the decision threshold should be chosen such that for any $\tau_s \geq \tau_s^{\min}$, we have $P_{md}(\tau_s) \leq \bar{P}_{md}$, i.e., [11]

$$\frac{\epsilon}{\sigma_u^2} = Q^{-1}(1 - \bar{P}_{md})\sqrt{2\gamma + 1}\tau_s^{\min} f_s + \gamma + 1 \tag{22}$$

where γ is the received signal-to-noise ratio of the PR signal at the CR. The relationship between P_{fa} and τ_s in (21) is not in closed-form and thus is hard to manipulate. Given the parameters γ, $\tau_s^{\min}$, P_{md}, and f_s, we suggest an exponential curve fitting for (21), yielding $P_{fa}(\tau_s) \approx e^{-b\tau_s}$. Mathematically, this fitting is inspired by the well-known approximation [16] $\text{erfc}(x) \leq e^{-x^2}$. Numerically, we found that this exponential fitting achieves good accuracy. Figure 2 shows an example when $\gamma = 0.01$, $\tau_s^{\min} = 0.1$ ms, $P_{md} = 1\%$, and $f_s = 1$ MHz. The average fitting error in this case is less than 8%.

Applying the exponential fitting of $P_{fa}(\tau_s)$ and treating τ_s as the variable, the domain of the jth segment defined in (17) now becomes:

$$(1 - e^{-b\tau_s})C_j\tau_t < \tau_s + \tau_p \leq (1 - e^{-b\tau_s})C_{j-1}\tau_t. \tag{23}$$

The above partition is not in explicit form of τ_s because τ_s appears on both sides of each inequality. To get the explicit partitions, we need to solve the following series of equations of

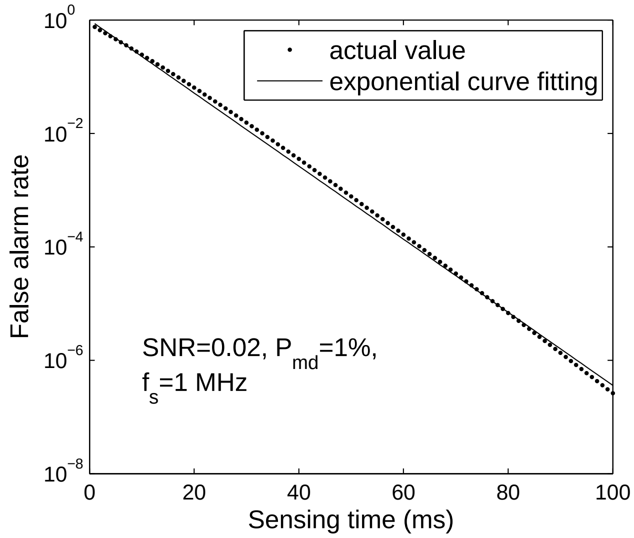

Figure 2: Exponential curve fitting of the false alarm rate.

τ_s:

$$\tau_s = (1 - e^{-b\tau_s})C_j\tau_t - \tau_p, \quad 1 \leq j \leq K. \tag{24}$$

For each equation, if a non-negative solution exists, then it gives a partition point over τ_s. The difficulty here is that such a solution does not always exist.

Theorem 4: The following four statements specify the existential condition and structure of the solutions to (24):

1. Existential condition: An equation in (24) has solutions if and only if $C_j\tau_t - \frac{1}{b} - \frac{1}{b}\ln bC_j\tau_t \geq 0$;

2. Number of solutions: Each equation in (24) can have at most two solutions. At most one equation can have exactly one solution;

3. Sign of solutions: If an equation has two solutions, then both solutions are positive (or negative) if $\frac{1}{b}\ln bC_j\tau_t$ is positive (or negative). In other words, it is impossible to have one positive solution and one negative solution for the same equation;

4. Structure of solutions: If the jth equation has two positive solutions, denoted as $\tau_s^{(j,\text{high})}$ and $\tau_s^{(j,\text{low})}$, where $\tau_s^{(j,\text{high})} \geq \tau_s^{(j,\text{low})} > 0$, then the $(j-1)$th equation must have two positive solutions, which satisfy the condition $\tau_s^{(j-1,\text{high})} > \tau_s^{(j,\text{high})} \geq \tau_s^{(j,\text{low})} > \tau_s^{(j-1,\text{low})} > 0$.

According to Theorem 4, the structure of the solutions to (24) and the resulted partition points over τ_s are illustrated in Figure 3, where the functions $h_j(\tau_s) \stackrel{\text{def}}{=} \left(1 - e^{-b\tau_s}\right) C_j\tau_t - \tau_p - \tau_s$, and $h_j(\tau_s) = 0$, $1 \leq j \leq K$, are equivalent equation set to (24). The optimization of τ_s is based on examining the structure of this segmentation. Specifically, counting down from $j = K$ to 1 in (24), let the j^*th equation be the first one that has positive solution(s). The segmented function

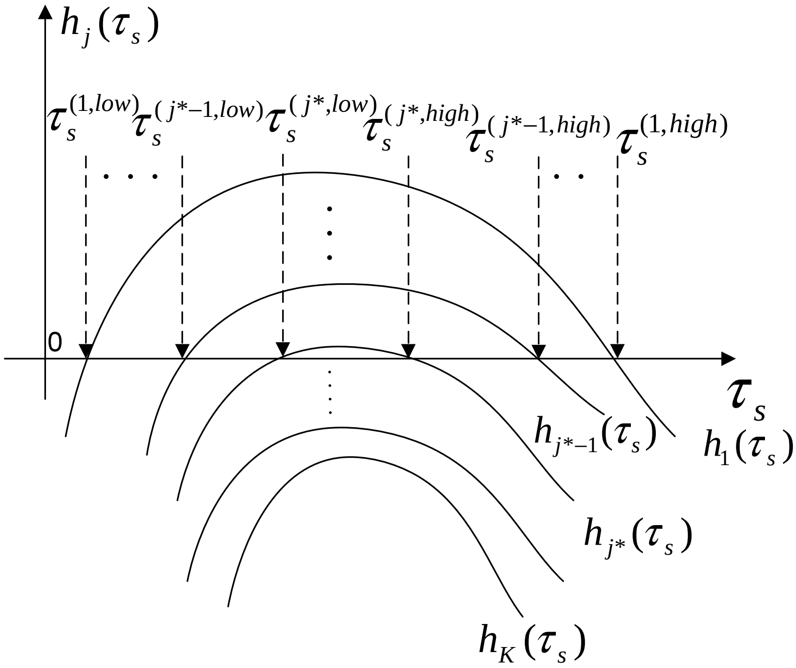

Figure 3: Structure of the solutions to (24) and the partition points over τ_s.

$\lambda^*(\tau_s)$ is described as follows: 1. The total number of segments is $2j^* + 1$. 2. The domain of these segments are given from left to right by $[0, \tau_s^{(1,\text{low})})$, $[\tau_s^{(1,\text{low})}, \tau_s^{(2,\text{low})})$, ..., $[\tau_s^{(j^*,\text{low})}, \tau_s^{(j^*,\text{high})})$, $[\tau_s^{(j^*,\text{high})}, \tau_s^{(j^*-1,\text{high})})$, ..., $[\tau_s^{(1,\text{high})}, \infty)$. 3. Recalling the condition (15) required for λ^*, for the jth left-most and the jth right-most segments, where $1 \leq j \leq j^*$, the corresponding λ^* must satisfy $(1 - P_{loss})R_{j-1} < \lambda^* \leq (1 - P_{loss})R_j$. In addition, the specific value of λ^* in these two segments is given by

$$\lambda^* = \frac{(1 - e^{-b\tau_s})\tau_t P_I (1 - P_{loss}) \sum_{k=j}^{K} R_k p_k}{\tau_s + \tau_p + (1 - e^{-b\tau_s})\tau_t P_I \sum_{k=j}^{K} p_k}. \tag{25}$$

Three properties regarding this segmentation can be observed: 1. Inside each segment, the relationship between λ^* and τ_s, i.e., (25), is no longer convex or monotonic. So λ^* is in general neither convex nor monotonic for the entire domain $\tau_s \geq 0$. 2. Interestingly, the trend of λ^* is concave if we treat segment as our observation unit: Starting from the left-most segment, $[0, \tau_s^{(1,\text{low})})$, any τ_s in the next segment gives greater λ^* than what any τ_s in the previous segment gives. This trend is valid until reaching the segment in the middle, $[\tau_s^{(j^*,\text{low})}, \tau_s^{(j^*,\text{high})})$. Starting from this segment and until the last one, any τ_s in the next segment gives smaller λ^* than what any τ_s in the previous segment gives. 3. The segment in the middle gives λ^*s that are greater than in any other segments. In other words, we can define $\mathbf{T}_s^o \stackrel{\text{def}}{=} \{\tau_s : \tau_s^{(j^*,\text{low})} \leq \tau_s \leq \tau_s^{(j^*,\text{high})})\}$. $\mathbf{T}_s^o$ is a closed range that contains the optimal τ_s^o. In addition, any $\tau_s \in \mathbf{T}_s^o$ achieves greater throughput than any $\tau_s' \notin \mathbf{T}_s^o$, and its throughput is bounded by $(1 - P_{loss})R_{j^*} \leq \lambda^* \leq (1 - P_{loss})R_{j^*+1}$. Therefore, for the sensing/probing/access scheme, even though we cannot find τ_s^o explicitly, we can still decide a good range for τ_s that gives provably near-optimal performance.

Lemma 1: The closed range $\mathbf{T}_s^o$ is $\frac{R_{j^*}}{R_{j^*+1}}$-optimal, i.e., any $\tau_s \in \mathbf{T}_s^o$ can achieve at least $\frac{R_{j^*}}{R_{j^*+1}}$ fraction of the maximum throughput, where j^* denotes the id of the first equation that has positive solution(s) when counting down from the Kth to the first equation defined in (24).

4 Throughput Analysis for CRNs

In this section, we study the aggregate network throughput when several CRs share the same spectrum, each being driven by its own sensing/probing/access process discussed before. An important factor we need to consider in this scenario is collisions between CRs, i.e., more than one pair of CR transmitter/receiver may be sensing the same channel at the same time, so none of them can use the channel even if this channel is idle and is of a good quality.

We consider two sensing strategies for the CRs: random channel sensing and collaborative channel sensing. In random channel sensing, each CR pair randomly selects a channel to sense in each step. There is no information exchange between different CR pairs. For collaborative sensing, CRs exchange their channel-hopping information in every step to avoid multiple CRs hopping to the same channel at the same time.

A discrete-time Markov-chain model is used to analyze the throughput of the CRN. Time is divided into slots with slot length $\tau_s + \tau_p$. So for a CR, each step of channel sensing/probing takes exactly one slot and each transmission takes $L \stackrel{\text{def}}{=} \lceil \frac{\tau_t}{\tau_s + \tau_p} \rceil$ slots. We assume that CRs are synchronized, i.e., the slots of different CRs are aligned. Let the number of CR transmitter/receiver pairs be M. To simplify the presentation, here we only consider the fundamental case when each CR link can only sense, probe, and transmit over one channel at a time. The case that a CR link can simultaneously use $J > 1$ channels can be treated as J independent one-channel *virtual* CR links and analyzed accordingly. To evaluate the CRN's capability of harvesting the spectrum, we are interested in a saturated traffic scenario, i.e., there is always backlogged traffic at each CR link. The state of the Markov chain is defined as a tuple $(x_1, \ldots, x_M)$, where each element $x_m \in \{0, 1\}$ stands for the activity of the mth CR link in the current slot: $x_m = 0$ means that CR link m is sensing and probing a channel; $x_m = 1$ denotes

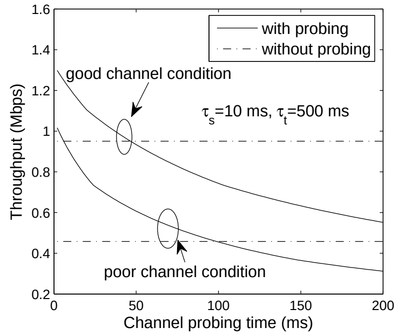

Figure 4: Throughput vs. probing time.

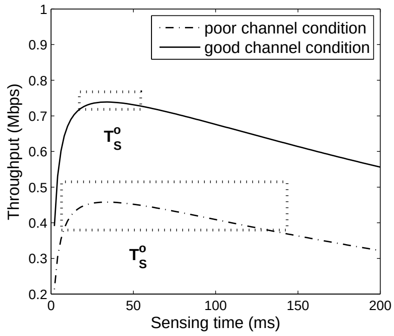

Figure 5: Throughput vs. sensing time.

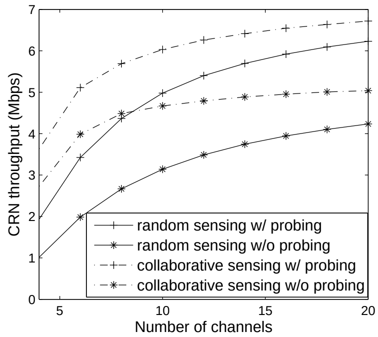

Figure 6: CRN throughput vs. number of channels.

an ongoing transmission by that link. The CR links' activities in the current slot are mutually independent, but are related to all CR links' activities in the previous slot, $(x'_1, \ldots, x'_M)$, i.e., the transition probability of the chain has the following property

$$\Pr(x_1, \ldots, x_M | x'_1, \ldots, x'_M) = \prod_{m=1}^{M} \Pr(x_m | x'_1, \ldots, x'_M) \tag{26}$$

This property is reasonable, because a CR's activity in the current slot should depend only on its observation of other CRs' activities in the previous slot.

Without loss of generality, we consider the transition probability of CR link 1. We first consider the random sensing strategy. The calculation includes the following four cases:

Case 1: $\Pr(0|0, x'_2, \ldots, x'_M)$

In this case, the transition probability contains four components

$$\begin{aligned} \Pr(0|0, x'_2, \ldots, x'_M) &= P_{cr\_occupied} + P_{cr\_collision} + P_{pr\_occupied} \\ &\quad + P_{poor\_channel} \end{aligned} \tag{27}$$

where $P_{cr\_occupied}$ denotes the probability that the channel sensed/probed by CR link 1 in the previous slot was being used (transmitted over) by other CR links; $P_{cr\_collision}$ denotes that even though the situation described in $P_{cr\_occupied}$ did not happen, the channel sensed/probed by CR link 1 in the previous slot was also sensed/probed by at least another CR links in the same slot; $P_{pr\_occupied}$ denotes that even though the two situations in $P_{cr\_occupied}$ and $P_{cr\_collision}$ did not happen, the channel sensed/probed by CR link 1 in the previous slot was being occupied by PRs; $P_{poor\_channel}$ denotes that even though the three situations in $P_{cr\_occupied}$, $P_{cr\_collision}$, and $P_{pr\_occupied}$ did not happen, the channel sensed/probed by CR link 1 in the previous slot was having a bad channel condition, i.e., its MADR is below the channel quality threshold.

To calculate these four probabilities, we note that a collision of multiple CRs sensing the same channel can be detected during the probing operation. After detecting a collision, the CRs will sense/probe other channels in the next slot. Therefore a CR's transmission over some channel indicates this channel is collision-free between CRs. We define the following notations: $M_1^{(m)} \stackrel{\text{def}}{=} \sum_{i=1;i\neq m}^{M} x_i'$ and $M_0^{(m)} \stackrel{\text{def}}{=} (M-1) - M_1^{(m)}$. $M_1^{(m)}$ denotes the number of transmitting CR links in the previous slot, not counting the mth link; $M_0^{(m)}$ is the number of sensing/probing channels in the previous slot, not counting the mth link. Then we can calculate

$$P_{cr\_occupied} = \frac{M_1^{(1)}}{C} \tag{28}$$

$$P_{cr\_collision} = \left(1 - \frac{M_1^{(1)}}{C}\right)\left[1 - \left(\frac{C-1}{C}\right)^{M_0^{(1)}}\right] \tag{29}$$

$$P_{pr\_occupied} = \left(1 - \frac{M_1^{(1)}}{C}\right)\left(\frac{C-1}{C}\right)^{M_0^{(1)}} Q_B \tag{30}$$

$$P_{poor\_channel} = \left(1 - \frac{M_1^{(1)}}{C}\right)\left(\frac{C-1}{C}\right)^{M_0^{(1)}} Q_I \sum_{k=0}^{k^*-1} p_k \tag{31}$$

Case 2: $\Pr\{1|0, x_2', \ldots, x_M'\}$

This transition probability is simply given by

$$\begin{aligned} & \Pr\{1|0, x_2', \ldots, x_M'\} = 1 - \Pr\{0|0, x_2', \ldots, x_M'\} \\ = & \left(1 - \frac{M_1^{(1)}}{C}\right)\left(\frac{C-1}{C}\right)^{M_0^{(1)}} Q_I \sum_{k=k^*}^{K} p_k \end{aligned} \tag{32}$$

Case 3: $\Pr\{0|1, x_2', \ldots, x_M'\}$

This case means that CR link 1 finished its transmission in the previous slot, so it starts looking for a new channel in the current slot. The transition probability is given by

$$\Pr\{0|1, x_2', \ldots, x_M'\} = 1/L \tag{33}$$

Case 4: $\Pr\{1|1, x_2', \ldots, x_M'\}$

This is simply calculated as

$$\Pr\{1|1, x_2', \ldots, x_M'\} = 1 - \Pr\{0|1, x_2', \ldots, x_M'\} = (L-1)/L. \tag{34}$$

Having obtained the transition probability matrix, the Markov chain's stationary distribution for the random vector $(x_1, \ldots, x_M)$ can be calculated using standard state-transition balance equations. Among all the states, those with $\sum_{m=1}^{M} x_i > C$ are infeasible, and therefore their stationary distribution probability is 0. The CRN's throughput is then calculated as

$$R_{tot} = \sum_{x_1=0}^{1} \cdots \sum_{x_M=0}^{1} \Pr\{x_1, \ldots, x_M\} \sum_{m=1}^{M} x_m \bar{R} \tag{35}$$

where $\bar{R} \stackrel{\text{def}}{=} \frac{(1-P_{loss})\sum_{k=k^*}^{K} R_k p_k}{\sum_{k=k^*}^{K} p_k}$ is the average throughput a CR link can achieve when it is transmitting.

In the above calculation, we have assumed a fully-distributed random channel sensing strategy. When a collaborative sensing strategy is used, then the above calculation should be modified as follows: the term, $\left(\frac{C-1}{C}\right)^{M_0^{(1)}}$, should be replaced by $\left(\frac{C-1}{C}\right)^{\max(0, M_0^{(1)}+1-C)}$ in (28) through (32). This is because under collaborative sensing, if the number of links that are sensing channels is not greater than the number of channels, then there is no collision. Otherwise the collision is only due to those links that exceed the number of channels.

5 Numerical Examples

We first consider the optimal throughput of a single CR link as a function of various operational parameters. We assume a discrete-rate CR system that supports four rates: 1, 2, 3, and 4 Mbps. We consider the rate distribution, $(p_0, p_1, p_2, p_3, p_4)$, under two channel conditions: a good channel condition with rate distribution of $(0.1, 0.1, 0.2, 0.2, 0.4)$ and a poor channel condition with rate distribution (0.4, 0.2, 0.2, 0.1, 0.1). We assume that the average idle and busy time of each channel is $\alpha = \beta = 500$ ms.

In Figure 4, we plot the throughput of the sensing/probing/access scheme as a function of the channel probing time. The channel sensing and transmission times are fixed at 10 ms and 500 ms, respectively. We assume that $P_{fa} = 0.1$. The throughput of the CR link when probing is not used is also plotted for comparison. It is clear that as long as τ_p is kept sufficiently small, i.e., $\tau_p < 45$ ms under good channel conditions or $\tau_p < 100$ ms under poor channel conditions, the use of probing leads to throughput gains over the scheme that does not use probing. Recalling that the reported per-channel probing time in current 802.11 WLAN systems ranges from 10 to 133 ms [1], the above probing time requirement is non-trivial, because there is still a reasonable chance under the current technology that the use of probing could actually undermine the throughput. In addition, the effect of probing is more significantly observed under poor channel conditions, e.g., the throughput gain reaches about 120% when $\tau_p = 10$ ms. At the same time, the maximum acceptable channel probing time becomes 100 ms. These results favor the use of probing when the channel condition is bad, which is in line with our intuition.

Figure 5 depicts the throughput as a function of the channel sensing time. Here we use the exponential curve fitting for the P_{fa}—τ_s relationship with parameter $b = 14.8349$. The concave trend of the throughput as function of τ_s can be clearly observed in this figure. More importantly, even though we cannot analytically derive the globally optimal τ_s^o that maximizes the throughput, our analysis in section 3 shows that they must be located in the ranges denoted by the dotted boxes, i.e., $\tau_s^o \in [15.1, 43.9]$ ms under good channel conditions, and $\tau_s^o \in [6.8, 140.5]$ ms under poor channel conditions. The sensing time in practical operations should be controlled within these ranges to achieve bounded near-optimal throughput.

Next, we use the optimal threshold derived for individual links to drive the operation of CRN. In Figures 6, we fix the number of CR links $M = 8$ and plot the CRN throughput as a function of number of channels, C. The rate distribution for each CR link is given by (0.2, 0.2, 0.2, 0.2, 0.2). We set $\tau_s = \tau_t = 10$ ms, and $\tau_t = 500$ ms. It is clear that the collaborative sensing strategy achieves greater throughput than the random sensing strategy, largely due to the smaller collision probability under collaborative sensing. In addition, it can be observed that under both strategies, the CRN throughput in general increases with C. However, the speed of increase is fast when C is small, and becomes slow when C is large. This trend is due to the smaller collision probability and the more likelihood of finding an idle channel when C is large. A third observation is that in the case of collaborative sensing, the slope of the throughput curve is less steep than in the random sensing case when C is large. This is because the collision probability under collaborative sensing has become 0 when C is large, and therefore the throughput increase is only due to the increased probability of finding a good idle channel.

6 Related Work

In existing literature on DSS schemes, sensing is the only operation performed by CRs to select channels. The problem of finding a good channel is reduced to finding an idle one. Based on the assumptions about the sensing overhead and sensing accuracy, these works can be classified into three types. The first type assumes negligible sensing time and an accurate sensing outcome (i.e., zero false alarm and miss detection rates). Under this highly idealized sensing model, these work usually focus on other aspects of DSS. For example, the work in [12], [7], and [17] study the collision issues between a transmitting CR and a returning PR by optimizing the CR's transmission time, the distribution of the CR's transmission time, and the CR's probability of transmission, respectively. The second body of works assumes a non-negligible sensing time and an accurate sensing outcome. Among them, the work in [10] minimizes the time cost of finding an idle channel by optimizing the sensing frequency and the sensing sequence of different channels. A similar problem is investigated in [19] under partial channel observability, whereby a CR can only sense a small subset of channels in order to find one available. The work in [9] considers the scenario that a CR can transmit over multiple channels simultaneously, but these channels have to be found in a sequential manner (one after another). The problem is to decide an optimal number of idle channels the CR should use such that the average throughput, which accounts for the aggregate rate provided by the combined channels and the time cost on finding these channels, is maximized. The third body of works assumes a realistic sensing model, whereby the sensing accuracy becomes a function of the sensing time. These works aim at exploiting the tradeoff between the sensing time and the sensing accuracy for the purpose of minimizing the time latency of finding an idle channel [4] or maximizing the throughput of a CR link [11][6]. Compared with these previous work, the use of probing significantly complicates the channel selection problem, because only finding idle channels is no longer sufficient; we need to find good idle channels. Furthermore, our work is the first to incorporate the relationship between the sensing time and sensing accuracy in a multi-rate setting.

Channel probing has been comprehensively studied for general wireless systems under the assumption that the probing overhead is negligible [14]. However, joint optimization of the reward obtained from channel selection and the cost incurred by channel probing only recently started to receive attention. There are a few related work, but all of them are for wireless systems with dedicated channels, e.g, [15, 2, 21, 20, 8, 5, 22]. The difference between our problem and the previous work includes the following: First, the problem of combined channel sensing and probing has not been considered in all existing works. The inclusion of sensing in the channel selection problem is non-trivial, because the sensing time can affect the throughput by non-linearly changing the channel sensing accuracy. Second, previous work on channel probing involves only a relatively small channel pool, e.g., a pool of 3 channels for 802.11

and 8 channels for 802.11a. For CRNs, this pool can be much larger. As a result, those algorithms designed for small channel pools, e.g., the finite-horizon stopping method in [15] and the tree-based searching algorithm in [5, 8], become practically infeasible in CRNs because of the prohibitive computational complexity when the number of channels is large. In our work, an infinite-horizon formulation is employed, which is particularly suitable for modeling large channel pools. Third, the ultimate concern of all previous work is the optimal probing strategy that maximizes the throughput. In this work, we are not only interested in the optimal probing strategy, but also in the particular structure of this strategy, with the objective of performing a second-round optimization over operational parameters such as the sensing time and the probing time. Fourth, we also study the aggregate throughput for a network of CRs, in which collisions between CRs during sensing and probing is possible. Nearly all related work has ignored this fact and only considered the single-link case in their analysis.

7 Conclusions

Our study has indicated that a carefully-designed joint channel sensing and probing scheme for CRNs can achieve significant throughput gains over the conventional mechanism that uses sensing alone. Our findings include: (1) The throughput-optimal probing strategy has a threshold structure, which basically judges whether a channel is good or bad when being probed, (2) To achieve throughput gain over the conventional sensing approach, the probing time has to be smaller than some explicit upper bound; otherwise using sensing alone can achieve better throughput, (3) When probing is used, the throughput in general is no longer a concave function of the sensing time, largely due to the more complicated multi-rate structure induced by the addition of probing. However, this function has a segmented structure. If we treat segments as our observation units, the trend of this function is concave. We exploited this property to derive a closed range for the sensing time that provides provably near-optimal throughput performance.

A. Proof of Theorem 1

Proof: 1. Existence. We need to prove that for any finite λ, an optimal stopping rule exists for the transformed problem (12). It follows from Theorem 1 in Chapter 3 of [3] that the optimal stoping rule exists if the following two conditions are satisfied:

1. $E\{\sup_n w_n\} < \infty$.

2. $\lim_{n\to\infty} \sup_n w_n = -\infty, a.s.$

By examining (12), condition 2 is clearly satisfied. Condition 1 can be proved by applying Theorem 1 in Chapter 4 of [3]. Specifically, it is easy to see that the random variable

$$X'_n \stackrel{\text{def}}{=} X_n \tau_t (1 - P_{loss}) - \lambda \tau_t \tag{36}$$

is only related to the random variable X_n. Because X_n's are i.i.d. for all $n = 1, \ldots$, X'_n's must also be i.i.d.. In addition, because X_n takes a finite number of values and λ is finite, X'_n must also be finite. Therefore, it holds that $E\{\max(X'_n, 0)\} < \infty$ and $E\{(\max(X'_n, 0))^2\} < \infty$. So according to Theorem 1 in Chapter 4 of [3], it holds that $E\{\sup_n w_n\} = E\{\sup_n X'_n - n\lambda(\tau_s + \tau_p)\} < \infty$. So condition 1 is also satisfied.

2. The optimal solution. w_n can be written as $w_n = X'_n - n\lambda(\tau_s + \tau_p)$, where X'_n is defined in (36). So X'_n can be considered as the reward for observing $w_1, \ldots, w_n$, and $\lambda(\tau_s + \tau_p)$ can be deemed as the cost for each observation. Applying the principle of optimality in Chapter 2 of [3], the optimal stopping rule of the transformed problem (12) is given by

$$N^* = \min\{n \geq 1 : X'_n \geq V^*\} \tag{37}$$

where V^* denotes the expected return from an optimal stopping rule; it satisfies the following optimality equation

$$V^* = E\{\max\{X'_n, V^*\}\} - \lambda(\tau_s + \tau_p). \tag{38}$$

Equivalently, the above equation can be written as

$$E\{\max(X'_n - V^*, 0)\} = \lambda(\tau_s + \tau_p). \tag{39}$$

Recalling the connection between the original problem and its transformed version, the value of λ that gives $V^* = 0$ is simply the solution of (11). With $V^* = 0$, we have the following equation:

$$E\{\max(X_n \tau_t (1 - P_{loss}) - \lambda^* \tau_t, 0)\} = \lambda^*(\tau_s + \tau_p) \tag{40}$$

According to Theorem 1 in Chapter 6 of [3], the solution of the above equation, λ^*, is the maximum objective function value for problem (11). At the same time, substituting $V^* = 0$ into (37), we derive the optimal stopping rule to problem (11):

$$N^* = \min\{n \geq 1 : X_n \geq \frac{\lambda^*}{1 - P_{loss}}\} \tag{41}$$

This concludes the proof of Theorem 1. ■

B. Proof of Theorem 2

Proof: The main idea is to show that the LHS of (40) is a mono-decreasing function in λ^* while the RHS is mono-increasing of λ^*. They must intersect at one and only one point. Particularly, we define $g(\lambda) = E\{\max(X_n\tau_t(1-P_{loss}) - \lambda\tau_t, 0)\}$. For the general case of a continuous random variable X_n with pdf $q(x)$, $g(\lambda)$ can be extended as

$$g(\lambda) = \int_{\frac{\lambda}{1-P_{loss}}}^{\infty} x\tau_t(1-P_{loss})q(x)dx - \int_{\frac{\lambda}{1-P_{loss}}}^{\infty} \lambda\tau_t q(x)dx. \tag{42}$$

The first-order derivative of g is given by

$$\begin{aligned} \frac{dg(\lambda)}{d\lambda} &= -\lambda\tau_t q\left(\frac{\lambda}{1-P_{loss}}\right) \\ &\quad - \left[\tau_t \int_{\frac{\lambda}{1-P_{loss}}}^{\infty} q(x)dx - \lambda\tau_t q\left(\frac{\lambda}{1-P_{loss}}\right)\right] \\ &= -2\lambda\tau_t q\left(\frac{\lambda}{1-P_{loss}}\right) - \tau_t \int_{\frac{\lambda}{1-P_{loss}}}^{\infty} q(x)dx. \end{aligned} \tag{43}$$

Clearly, both terms in (43) are strictly negative, and therefore $g(\lambda)$ is a strictly mono-decreasing function. In addition, $g(0) = \tau_t(1-P_{loss})E[X_n] < \infty$ and $g(\infty) = 0$. It is clear that the RHS of (40) is a strictly mono-increasing function of λ with function values of 0 and ∞ when $\lambda = 0$ and $\lambda = \infty$, respectively. For the case of discrete random variable X_n, it is easy to see that the above monotonic property does not change. Therefore, λ^* must have a unique solution. ■

C. Proof of Theorem 3

Proof: First, by evaluating $\lambda_j^*(\eta)$, it is clear that inside each segment, λ^* is continuous and strictly mono-decreasing. Next, it can be easily verified that $\Phi_j = \phi_{j-1}$ and $\lim_{\eta\to\Phi_j} \lambda_j^*(\eta) = \lambda_{j-1}^*(\phi_{j-1})$ for $j = 2, \ldots, K$. Therefore, λ^* is continuous and strictly mono-decreasing over the entire domain of η. ■

D. Proof of Theorem 4

Proof: 1. We first prove the existential condition. Define

$$h_j(\tau_s) = \left(1 - e^{-b\tau_s}\right) C_j\tau_t - \tau_p - \tau_s, \quad 1 \le j \le K. \tag{44}$$

This function is concave since its second-order derivative is:

$$\frac{d^2 h_j}{d\tau_s^2} = -b^2 C_j \tau_t e^{-b\tau_s} < 0. \tag{45}$$

Because of this concavity, it is clear that $h_j(\tau_s) = 0$ has solutions if and only if the function's maximum value is not smaller than 0. The maximum value is calculated as follows:

$$\frac{dh_j}{d\tau_s} = e^{-b\tau_s} bC_j\tau_t - 1 = 0. \tag{46}$$

From (46) we can get $\tau_s^o = \frac{1}{b}\ln bC_j\tau_t$. Accordingly, the maximum value of $h_j(\tau_s)$ is given by

$$h_j^{\max} \stackrel{\text{def}}{=} h_j(\tau_s^o) = C_j\tau_t - \frac{1}{b} - \frac{1}{b}\ln bC_j\tau_t \tag{47}$$

Then statement 1 follows.

2. The first half of Statement 2 is clear due to the concavity of $h_j(\tau_s)$. We prove the second half after Statement 4.

3. The proof is based on contradiction. We first consider the case when $\tau_s^o = \frac{1}{b}\ln bC_j\tau_t > 0$. From the concavity of h_j, it is clear that at least one solution must be positive. Now suppose the second solution is negative. Then $h_j(0) \geq 0$ must hold. However, from (44), when $\tau_s = 0$, $h_j(0) = -\tau_p < 0$. Similar contradiction can be established when $\tau_s^o = \frac{1}{b}\ln bC_j\tau_t < 0$. So Statement 3 holds.

4. From the definition of C_j in (20), it is clear that $C_j < C_{j-1}$. Now consider the solution $\tau_s^{(j,\text{high})}$ of the jth equation. From (24), we have

$$\tau_s^{(j,\text{high})} = (1 - e^{b\tau_s^{(j,\text{high})}})C_j\tau_t - \tau_p < (1 - e^{b\tau_s^{(j,\text{high})}})C_{j-1}\tau_t - \tau_p. \tag{48}$$

As $\tau_s \to \infty$, it holds that $\lim_{\tau_s\to\infty}(1 - e^{-b\tau_s})C_{j-1}\tau_t - \tau_p = C_{j-1}\tau_t - \tau_p < \infty$. So the function $(1 - e^{-b\tau_s})C_{j-1}\tau_t - \tau_p$ must intersect with the function τ_s between $\tau_s^{(j,\text{high})}$ and ∞ (the two boundaries not included). Applying a similar logic to $\tau_s^{(j,\text{low})}$, it is clear that the function $(1 - e^{-b\tau_s})C_{j-1}\tau_t - \tau_p$ also intersects with the function τ_s between 0 and $\tau_s^{(j,\text{low})}$ (the two boundaries not included). Note that the two domains $(\tau_s^{(j,\text{up})}, \infty)$ and $(0, \tau_s^{(j,\text{low})})$ do not overlap with each other. Accounting for Statements 2 and 3, it can be concluded that there are only two solutions to the $(j-1)$th equation, each of which are positive. One solution is located in $(\tau_s^{(j,\text{high})}, \infty)$ and the other located in $(0, \tau_s^{(j,\text{low})})$. So Statement 4 follows.

Based on Statement 4, the second half of Statement 2 is straightforward: Counting down from $j = K$ to 1, the first equation, say the j^*th one, that has solutions is the only one that can have exactly one solution. For all $j < j^*$, each equation must have two solutions. ■

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