Design and Evaluation of an Efficient Guard-band-aware Multi-channel Spectrum Sharing Mechanism

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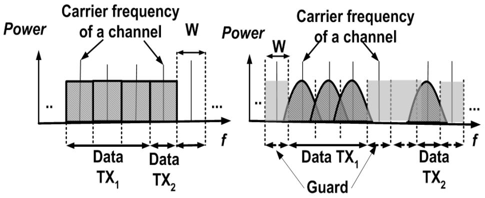
Abstract

Several spectrum access/sharing algorithms for cognitive radio networks (CRNs) have been designed assuming no adjacent-channel interference (i.e., no interference from neighboring CR transmissions operating over adjacent channels). However, in practice, such an assumption may not hold, and guard bands are realistically needed to prevent interference from neighboring CR transmissions operating on adjacent channels. Introducing guard bands is a restrictive constraint on the effective use of the spectrum. In this work, we investigate the problem of assigning channels/powers to CR transmissions, while accounting for such a constraint. To improve spectrum efficiency and avoid unnecessary blocking of CR transmissions, we propose a novel *guard-band-aware* channel assignment scheme. The proposed scheme reduces the number of required guard channels for a given transmission by exploiting the benefits of utilizing adjacent channels while considering the already reserved guard channels. We analytically formulate the channel access as a joint power control and channel assignment optimization problem, with the objective of minimizing the required spectrum resource for a given CR transmission. Because the optimization problem is found to be a binary linear program (BLP), which is known to be NP-hard in general, we present a near-optimal algorithm to solve this problem based on a sequential fixing procedure, where the binary variables are determined iteratively by solving a sequence of linear programs. Simulation results are provided, which verify the accuracy of the proposed algorithm and demonstrate the significant gain achieved by being guard-band-aware.

1 Introduction

The tremendous growth of wireless applications and services is straining the effectiveness of conventional static spectrum planning policies. Recent field studies conducted by the FCC and other agencies revealed vast temporal and geographical variations in the utilization of the licensed spectrum, ranging from 15% to 85% [1, 2]. Such studies prompted regulators to push for a more efficient and adaptive spectrum allocation policy. As a result, the FCC has recently revised its regulations to allow for opportunistic (on demand) access to the spectrum. Cognitive radio (CR) is a technology that promises to offer such an opportunistic capability without noticeably affecting *primary radio* (PR) users. CRs are mainly characterized by their cognitive capability and reconfigurability. The cognitive capability provides spectrum awareness, whereas reconfigurability enables a CR user to dynamically adapt its operating parameters to the surrounding RF environment. In an environment where several licensed PR networks (PRNs) are operating, CR users that co-exist with PR users should frequently sense their operating channels for active PR signals to discover spectrum opportunities, and should vacate these channels if a PR signal is detected. Given the available spectrum opportunities at different CR users, a crucial challenge in this domain is how nodes in a CRN can cooperate to access the spectrum in order to efficiently utilize those opportunities while improving the overall network throughput.



(a) Ideal filter (b) Actual filter

Figure 1: Ideal vs. actual transmission filters.

1.1 Motivation

Various channel assignment algorithms for CRNs have been proposed in the literature (e.g., [3–10]). Most of them were designed assuming no adjacent-channel interference (ACI) (i.e., ideal filtering)[1]. Such ACI-free schemes require ideal (rectangular) transmission filters (see Figure 1(a)). In practice, however, spectrum spill-over is common during the filtering of signals. To mitigate ACI and protect neighboring PR/CR reception, frequency separation (unused portion of spectrum) between adjacent channels are needed[2]. Such frequency separation is referred to as a guard band. The imposition of guard bands adds a constraint on the effective use of the spectrum. Therefore, when assigning transmission channels/powers to CR transmissions, it is necessary to consider the guard-band issue to improve spectrum utilization. Note that guard bands are not needed between contiguous channels assigned to the same transmission (we refer to *contiguously* assigned channels as a *frequency block*). For every frequency block (which may comprise one or more channels), one guard channel on each side of the block is needed (e.g., in Figure 1(b), one frequency block of three

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[1] ACI is caused by extraneous power received from a transmission operating on an adjacent channel.

[2] Without loss of generality, we assume that the spectrum is grouped into equal bandwidth frequency channels.

channels is assigned to transmission 1, which requires two guard channels).

Another aspect of previously proposed channel assignment mechanisms is that they are typically based on selecting the "best" channel, or set of channels, for a given transmission (e.g., [9,11]). In here, the *best* channel is the one that has the highest received SINR. We refer to this approach as the *greedy approach.* When the greedy approach is employed in a CRN, the number of required guard channels may significantly increase. This results in a higher blocking probability for CR transmissions, leading to a significant reduction in network throughput. To illustrate, consider a transmission that requires m data channels. Assume that the best m channels are non-contiguous and one guard channel on each side of each channel is available. According to the greedy approach, the total number of required channels (data-plus-guard channels) is $m + 2m = 3m$. In general, if the m selected data channels are obtained from k non-contiguous frequency *blocks*, then the required number of channels is $m + 2k$ (e.g., if $k = 1$, then only $m + 2$ channels are required).

As a numerical example, assume that a given CR transmission requires $m = 8$ data channels. Assume that the best 8 channels are non-contiguous and one guard channel on each side of a data channel is available. According to the greedy scheme, this CR transmission requires 16 guard channels, which results in spectrum efficiency of 33%. Here, spectrum efficiency is defined as the ratio of the number of data channels and the total number of required data-plus-guard channels. On the other hand, if 8 adjacent data channels are available (i.e., one frequency block), the transmission will require only 2 guard channels. This results in spectrum efficiency of 80%. Figure 2 shows the total number of required channels as a function of k for $m = 8$. This figure and the above example reveal that an efficient channel assignment algorithm should try to minimize k (ideally, selecting $k = 1$), which would minimize the number of guard channels per data channel.

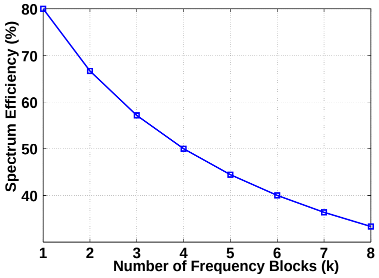
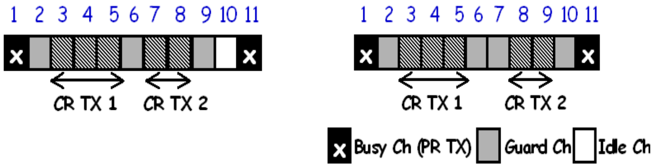


Figure 2: Number of required channels vs. number of blocks ($m = 8$).

It is worth mentioning that in a given neighborhood, the *optimal* channel/power assignment that maximizes the number of simultaneous CR transmissions requires perfect knowledge of the SINR of each link and the rate demands of all contending CR users in that locality. Therefore, determining this optimal assignment incurs high control overhead and delay. Even if perfect knowledge is available, this problem without even accounting for the guard-band issue is NP-hard [12,13]. Since computing the optimal assignment grows exponentially with the size of the network and the number of available channels [12], suboptimal algorithms are needed. Such algorithms should attempt to compute the channel assignment that improves spectrum utilization in a purely distributed manner while relying only on local information.

1.2 Contributions

In this work, we consider the joint power control and channel assignment problem in multi-channel CRNs under the realistic assumption of non-ideal filters (i.e., the need for guard bands). Our goal is to improve network throughput by attempting to maximize spectrum efficiency. This is equivalent to minimizing the number of required guard channels for a given transmission, which can be achieved through a proper guard-band-aware channel assignment scheme. Our scheme exploits the benefits of synchronized contiguous multi-channel transmission while considering the local channel availability, the already introduced guard channels, and the non-adjacency of channels assigned to neighboring CR users. According to this scheme, a CR user that intends to transmit has to account for *potential* future transmissions in its neighborhood. It does that by assigning to its transmission the set of channels that requires the minimum number of guard bands and that satisfies the rate demand. We propose two variants of the guard-band-aware channel assignment mechanism. The first variant is suitable for CRNs with a transmission technology that does not allow two neighboring CR transmissions to share the same guard channel (no guard-band reuse), while the other variant is for CRNs with a transmission technology that allows for guard-band reuse. Figures 3 illustrates channel assignment with and without guard-band reuse.



(a) With guard-band reuse

(b) Without guard-band reuse

Figure 3: Example that illustrates channel assignment with/without guard-band reuse. In part (a)/(b), transmission 2 can/cannot reuse a guard channel assigned to transmission 1 (i.e., channel 6 in this example).

1.3 Organization

The rest of the paper is organized as follows. In Section 2, we describe the system model, state the main design constraints, and formulate the channel/power assignment optimization problem. Section 3 introduces our proposed guard-band-aware channel assignment scheme. Simulation results and discussion are presented in Section 4. Finally, Section 5 gives concluding remarks.

2 Models and Problem Formulation

2.1 Network Model

We consider an ad hoc CRN that coexists geographically with L different PRNs. PR users are legacy radios that cannot be controlled by the CRN. The PRNs are licensed to operate over non-overlapping channels. For the ith PRN, its available bandwidth (B_i) is divided into C_i adjacent but non-overlapping frequency channels each of Fourier bandwidth W (in Hz). Let M denote the total number of channels in the network; $M = \sum_{i=1}^{L} C_i$. CR users continuously scan the spectrum, identifying potential spectrum holes (idle PR channels) and opportunistically exploiting them for their transmissions. For a given physical-layer encoding scheme, we assume that the data rate of an idle channel is proportional to the channel bandwidth [14]. Accordingly, a bandwidth model that delivers 1 bit per 1 Hz is considered if the received SINR is greater than a given threshold (μ^*) [14]. Formally, for an idle channel $i \in M$, its transmission rate (R_i) is obtained according to the following rate-SINR relationship:

$$R_i = \begin{cases} W \text{ Mbps}, & \text{if SINR}^{(i)} \geq \mu^* \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

where $\text{SINR}^{(i)}$ denotes the received SINR over channel i.

Depending on PR/CR activities, a CR transmission may proceed over multiple contiguous or non-contiguous idle channels to avoid harmful interference to PR/CR users. This synchronized multi-channel transmission capability can be realized by the existing frequency division multiplexing (FDM) technology, or using discontinuous orthogonal frequency division multiplexing (D-OFDM) technology [5, 15, 16].

2.1.1 FDM-based CRNs

In this case, each CR user is equipped with n_t half-duplex radio transceivers, $1 \leq n_t \leq M$, that can be used simultaneously. A CR user can transmit over an arbitrary segment of the available bandwidth by using tunable raised-cosine pulse filters, such that each frequency block is transmitted using one of the available transceivers. When a raised-cosine filter is used, the required number of guard channels depends on the number of channels in a frequency block and the rolloff factor of the raised-cosine filter (β). This β is a measure of the excess bandwidth of the filter. Formally, for a CR transmission that uses a frequency block of m adjacent channels, the excess bandwidth on each side of the frequency block is $\Delta f = mW\frac{\beta}{2}$. Thus, a necessary and sufficient condition to mitigate ACI using only one guard channel of bandwidth W on each side of a frequency block is $\Delta f \leq W \Rightarrow mW\frac{\beta}{2} \leq W \Rightarrow m \leq \frac{2}{\beta}$. For practical values of m, β, and W, the above condition often holds. For example, for $\beta = 0.1$ and $W = 3$ MHz, $m \leq 20$ channels (i.e., a data rate of up to 60 Mbps). Note that the IEEE 802.11a WLAN can provide data rates ranging from 6 Mbps to 54 Mbps [17]. Accordingly, it is reasonable to assume that a guard-band of bandwidth W on each side of a frequency block is sufficient to protect the reception over that block and avoid causing harmful interference to neighboring transmissions. This means that two guard channels are needed to separate any two distinct frequency blocks assigned to neighboring transmissions. This represents the case where a guard channel that is reserved for a CR transmission cannot be reused (shared) by another CR transmission (Figure 3(b)).

2.1.2 D-OFDM-based CRNs

According to D-OFDM, a CR transmission can simultaneously proceed over multiple channels (contiguous or non-contagious) using a single half-duplex radio, where each channel consists of a distinct block of the same number of contiguous sub-carriers [5, 15]. In essence, this capability can be achieved through power allocation by assigning 0 powers to all sub-carriers of non-assigned/busy channels. For a given CR transmission and a set of assigned channels, all sub-carries belonging to the selected channels will be used for that transmission [5, 15]. It has been shown that only the nearest sub-carriers in a neighboring frequency block that is assigned to a different CR transmission can be considered as a major source of interference to any demodulated sub-carrier [18, 19]. Therefore, to prevent ACI, it is sufficient to assign one guard channel between any two frequency blocks that are allocated to two neighboring CR transmissions, irrespective of the size of the frequency blocks [18, 19]. This represents the case where a reserved guard channel for a given CR transmission can be reused by another CR transmission (Figure 3(a)).

It is worth mentioning that the available channel set for CR transmissions depends on whether a guard-band reuse is possible or not. To illustrate, consider the channel status table in Figure 4. Channels $\{2, 6, 15, 16, 17, 18, 19\}$ are idle (i.e., unallocated). In the no guard-band reuse (guard-band reuse) case, only channels $\{16, 17, 18\}$ ($\{2, 6, 15, 16, 17, 18\}$) are available for data transmissions by CR users. As explained in Section 3.2.2, this difference in guard band reservation makes channel assignment under both cases different. In this paper, we investigate the problem of channel/power allocation for both cases.

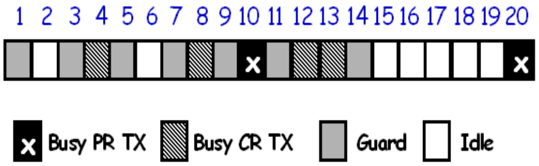


Figure 4: Example that illustrates the impact of guard-band reuse.

2.2 Design Constraints

For a given CR transmission, both the transmitter and receiver need to cooperatively select appropriate frequency channels and the transmission powers over these channels while meeting the following constraints:

1. *Half-duplex operation*: While transmitting, a CR user cannot receive/listen.
2. *Fixed rate per channel*: Each channel i can support a rate W (in bps) if its received SINR is $\geq \mu^*$.
3. *Exclusive channel occupancy policy*: A selected channel cannot be assigned to more than one data transmission in the same neighborhood.
4. *Rate demand requirement*: A CR transmission j requires a rate demand $R_D(j) = m_j W$, where $m_j \leq M$ is the number of required data channels.
5. *Maximum transmission power*: For a given CR transmission, the total transmission power (P_{tot}) over the selected channels is limited to $P_{\max}$.
6. *Guard-band reservation*: A guard channel (to protect a CR reception) cannot be used for CR transmissions. As an example, in Figure 4, guard channels $\{1, 3, 5, 7, 9, 11, 14\}$ cannot be used for data transmission.
7. *PR protection*: To protect a PR reception, an adjacent idle channel to a busy channel occupied by a PR user cannot be used for CR data transmissions [20]. In Figure 4, channels $\{9, 11, 19\}$ cannot be used for data transmissions. Note, however, these channels can be used as guard bands for CR transmissions.
8. *Guard-band reuse*: First, we consider the case where guard-band reuse in not allowed. In Section 3.2.2, we relax this constraint by considering the case of guard-band reuse.

2.3 Optimal Channel/Power Allocation

It has been shown that the joint power control/channel assignment problem that aims at maximizing the overall network throughput in a multi-channel CRNs is a challenging optimization problem. In fact, even without considering the guard-band constraint, this problem is known to be NP-hard [12, 13]. Worse yet, it requires perfect knowledge of the SINR at each CR receiver and the rate demands of all contending CR users. Hence, in this paper, we develop a channel assignment scheme that provides a suboptimal solution with low complexity and good spectrum utilization. Our scheme exploits guard-band awareness. The key idea behind it is to minimize the number of required guard channels for a given transmission while relying only on information provided by the two communicating users.

2.4 Guard-band-aware Channel Assignment: Problem Statement and Formulation

For a given CR transmission j, the sender and receiver need to cooperatively select an appropriate set of channels and the transmit powers over the selected channels such that spectrum efficiency is maximized by minimizing the number of assigned frequency blocks k (equivalently, minimizing the number of required guard channels) while meeting the aforementioned constraints. If multiple solutions exist for our optimization problem, we seek the one that requires the least amount of total transmission power. Let $\mathcal{I}_j$, $\mathcal{G}_j$, and $\mathcal{B}_j$ respectively denote the sets of idle, guard, and busy channels, as presently seen by the jth transmitter-receiver pair. Because our focus is on computing a feasible channel assignment $\Omega_j \subseteq \mathcal{I}_j$ for a given transmission j, the subscript j (i.e., the transmission index) is dropped in the rest of this paper to simplify the notation.

Given the current status of all channels (i.e., $\mathcal{I}$, $\mathcal{G}$, and $\mathcal{B}$), the channel gain and measured interference over every channel $i \in \mathcal{I}$ along link j, the rate demand (m channels), and the SINR threshold μ^*, the receiver of the jth CR transmission can compute the minimum required power (P_i) for every idle channel $i \in \mathcal{I}$ such that the received SINR is $\geq \mu^*$. Using this fact, the channel assignment problem can be stated as follows:

$$\begin{aligned} &\min_{\Omega} \left[k(\Omega) + \frac{P_{tot}(\Omega)}{P_{\max}} \right] \\ \text{s.t.} \quad & P_{tot}(\Omega) \overset{\text{def}}{=} \sum_{i \in \Omega} P_i \leq P_{\max} \\ & |\Omega| = m. \end{aligned} \tag{2}$$

The second term in the objective function ensures that if multiple solutions exist for the optimization problem, the one with the least amount of total transmission power will be selected. Note that the first constraint in (2) ensures that $\frac{P_{tot}(\Omega)}{P_{\max}} \leq 1 \leq k(\Omega)$ for any feasible assignment Ω. So, for any two feasible assignment Ω_1 and Ω_2 with $k(\Omega_1) < k(\Omega_2)$, the above formulation will also selects Ω_1 over Ω_2, irrespective of P_{tot}.

For $i = 0, \dots, M+1$, let α_i be a binary variable that is defined as follows:

$$\alpha_i = \begin{cases} 1, & \text{if channel } i \in \Omega \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

We let $\alpha_0 \overset{\text{def}}{=} \alpha_{M+1} \overset{\text{def}}{=} 0$. By introducing the binary variables α_i, the number of non-adjacent frequency blocks for a given assignment Ω (i.e., $k(\Omega)$) can be written as:

$$k(\Omega) = \frac{1}{2} \sum_{i=1}^{M+1} (\alpha_i - \alpha_{i-1})^2. \tag{4}$$

Substituting (4) into (2), the optimization problem becomes:

$$\begin{aligned} \min_{\alpha_i} & \left[\frac{1}{2} \sum_{i=1}^{M+1} (\alpha_i - \alpha_{i-1})^2 + \sum_{i=1}^{M} \frac{P_i}{P_{\max}} \alpha_i \right] \\ \text{s.t.} \quad & \sum_{i=1}^{M} \alpha_i = m \\ & \sum_{i=1}^{M} \alpha_i P_i \le P_{\max} \\ & \alpha_i \in \{0,1\}, i \in \{1, \ldots, M\}. \end{aligned} \tag{5}$$

For any channel $i \notin \mathcal{I}$, α_i is set to 0 a priori. An unavailable channel can be either an already reserved channel (for a PR or another CR transmission) or a channel that is adjacent to a busy channel (i.e., a guard channel). Note that the optimization problem in (5) is a binary quadratic program (BQP).

Proposition 1 *The optimization problem in (5) can be transformed into a binary linear programming (BLP) with a linear objective and linear constraints.*

Proof: The BQP Formulation in (5) can be easily transformed into BLP by introducing a new auxiliary variable z_i, where $i = 1, \ldots, M+1$, and adding the following constraints on z_i:

$$\begin{cases} z_i \ge \alpha_i - \alpha_{i-1}, \\ z_i \ge \alpha_{i-1} - \alpha_i. \end{cases} \tag{6}$$

According to (6), if channels i and $i-1$ have the same status, then $z_i = 0$. Otherwise, z_i must be at the same time greater than or equal -1 and 1. Thus, it will be 1. Formally,

$$z_i \overset{\text{def}}{=} \begin{cases} 0, & \text{if channels } i \text{ and } i-1 \text{ have the same status,} \\ 1, & \text{otherwise.} \end{cases} \tag{7}$$

With the introduction of z_i, the quadratic term of the objective function in (5) can be consequently changed to $\frac{1}{2} \sum_{i=1}^{M+1} z_i$.

This results in the following (equivalent) formulation to the original BQP problem in (5):

$$
\begin{aligned}
\min_{\alpha_i, z_i} & \left[\frac{1}{2} \sum_{i=1}^{M+1} z_i + \sum_{i=1}^{M} \frac{P_i}{P_{\max}} \alpha_i \right] \\
\text{s.t.} \quad & \sum_{i=1}^{M} \alpha_i = m \\
& \sum_{i=1}^{M} \alpha_i P_i \leq P_{\max} \\
\alpha_i - \alpha_{i-1} - z_i \leq 0, & \; i \in \{1, \dots, M+1\} \\
-\alpha_i + \alpha_{i-1} - z_i \leq 0, & \; i \in \{1, \dots, M+1\} \\
\alpha_i \in \{0, 1\}, & \; i \in \{1, \dots, M\} \\
z_i \in \{0, 1\}, & \; i \in \{1, \dots, M+1\}.
\end{aligned}
\tag{8}
$$

It is clear that the optimization problem in (8) is a BLP, which can be expressed in standard matrix form as follows:

$$
\begin{aligned}
\min_{\mathbf{X} \in \{0,1\}} & \; f(\mathbf{X}) = \mathbf{c}^T \mathbf{X} \\
\text{s.t.} \; & \; \mathbf{A}_{\text{eq}} \mathbf{X} = m \\
& \; \mathbf{A} \mathbf{X} \leq \mathbf{b}
\end{aligned}
\tag{9}
$$

where $\mathbf{c}^T = \begin{pmatrix} \frac{\mathbf{P}^T}{P_{\max}} & \frac{1}{2}\mathbf{e}^T \end{pmatrix}$ is an $(2M{+}1)$-dimensional vector describing the linear objective function, $\mathbf{P} \overset{\text{def}}{=} \begin{pmatrix} P_1 & P_2 & \dots & P_M \end{pmatrix}$ $\mathbf{e}$ is an $(M+1)$-dimensional column vector of all 1's, $\mathbf{X} \overset{\text{def}}{=} \begin{pmatrix} \alpha_1 & \alpha_2 & \dots & \alpha_M & z_1 & z_2 & \dots & z_{M+1} \end{pmatrix}^T$ is an $(2M+1)$-dimensional column vector of all decision variables, $\mathbf{A}_{\text{eq}} = \begin{pmatrix} \mathbf{e}^T & \mathbf{0}^T \end{pmatrix}$ is an $(2M{+}1)$-dimensional row vector describing the linear equality constraint, $\mathbf{0}$ is an $(M{+}1)$-dimensional column vector of all 0's, $\mathbf{b} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix}$ is an $(2M{+}2)$-dimensional column vector of all 0's, $\mathbf{A} = \begin{pmatrix} -\mathbf{B} & -\mathbf{I} \\ \mathbf{B} & -\mathbf{I} \end{pmatrix}$ is an $(2M + 2 \times 2M + 1)$ matrix describing linear inequality constraints, $\mathbf{I}$

is an $(M+1 \times M+1)$ identity matrix, and $\mathbf{B}$ is an $(M+1 \times M)$ matrix that is given by:

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ -1 & 1 & 0 & 0 & \dots & 0 \\ 0 & -1 & 1 & 0 & \dots & 0 \\ 0 & 0 & -1 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & -1 & 1 \\ 0 & 0 & 0 & \dots & 0 & -1 \end{bmatrix}_{M+1 \times M} .$$

3 Channel Assignment Schemes

In this section, we present two channel assignment algorithms: a greedy guard-band-unaware assignment and a near-optimal guard-band-aware assignment. Because of its simplicity and low processing overhead, the greedy approach is often employed in multi-channel systems [9]. However, it results in higher blocking probability for data transmissions, leading to a reduction in network throughput. Hence, we propose a novel guard-band-aware spectrum sharing algorithm to improve the throughput performance of CRNs.

3.1 Greedy Algorithm

The greedy approach proceeds in three steps:

1. Given $\mathcal{I}, \mathcal{G}, \mathcal{B}$, channel gains and measured interference over every channel $i \in \mathcal{I}$ along the given CR link, and μ^*, the algorithm calculates the required power $P_i, \forall i \in \mathcal{I}$.
2. The algorithm sorts the idle channels in an increasing order of their P_i.
3. The algorithm picks the first m channels from the top of the sorted list. If the total transmission power over the best m channels exceeds $P_{\max}$, then there is no feasible channel assignment.

Lemma 1 *For a given CR transmission with a rate demand, if the greedy solution is infeasible, then there is no feasible channel assignment that can support the given rate demand.*

3.2 Suboptimal Algorithm Based on Sequential Fixing

A BLP is a combinatorial problem. Its solution, in general, is NP-hard [21]. There exist several methods for approximately solving BLP problems, including cutting plane methods, decomposition methods, and branch-and-bound methods [22]. It has been shown [23] that the branch-and-bound algorithm is the superior method for solving BLP problems and is often sufficient to approximately solve such problems. However, the worst-case time complexity of this approximation is still exponential. Instead of employing a branch-and-bound algorithm, we develop polynomial-time near-optimal algorithm by exploiting the special structure of the problem. The aforementioned observations indicate that if we relax the binary constraints $\alpha_i \in \{0,1\}$ and $z_i \in \{0,1\}$ to the continuous interval $[0,1]$, then the resulting *linear relaxation* (LR) is solvable in polynomial time [24]. The main idea behind our fast solution is to fix the values of α_i sequentially through solving a series of relaxed LP problems, with at least one α_i finalized to a binary value at each iteration. Our suboptimal algorithm is called sequential fixing LP (SFLP). Two variants of the SFLP algorithm are proposed. The first variant is suitable for CRNs with a transmission technology that does not allow for guard-band sharing, whereas the second one is for CRNs with a transmission technology that allows for guard-band sharing.

3.2.1 SFLP-based Channel Assignment with No Guard-band Reuse

In this section, we present the SFLP algorithm for the no guard-band reuse case. In the first iteration, we relax the binary constraints by allowing α_i's and z_i's to take real values in $[0,1]$. For an unavailable (guard or busy) channel $i \notin \mathcal{I}$, we set $\alpha_i = 0$ (i.e., cannot be assigned to CR transmissions). We also set $\alpha_i = 0$ for any idle channel that is adjacent to a busy channel occupied by a PR user or to an already allocated guard channel. We refer to the resulting formulation as $\text{LR}^{(1)}$, which must have a feasible solution if the original BLP has a feasible solution (i.e., if $\text{LR}^{(1)}$ problem is infeasible, then there is no feasible channel assignment). The solution to $\text{LR}^{(1)}$ provides a lower bound on the optimal solution to (9), because the feasibility region of the BLP is a subset of that of $\text{LR}^{(1)}$. However, the solution of $\text{LR}^{(1)}$ is, in general, not a feasible solution to the original BLP problem, because α_i's and z_i's can now take values between 0 and 1. Among all newly obtained real-valued α_i's, we then set the one that has the largest value to 1. Then, at iteration $i, i = 2, \dots m$, the algorithm proceeds as follows:

i. The algorithm relaxes all unfixed α_i's and all z_i's to real values in $[0,1]$.

ii. The algorithm checks the feasibility region of the new LR, called $\text{LR}^{(i)}$. If this region is empty, this means the fixing in the $(i-1)$th iteration was not correct. Thus, we flip the value of the last fixed variable to 0 and update $\text{LR}^{(i)}$. Note that the revised $\text{LR}^{(i)}$ problem must be feasible (see Lemma 3).

iii. The algorithm solves the resulting LR program ($\text{LR}^{(i)}$), whose variables do not include those that have been fixed after the execution of $\text{LR}^{(i-1)}$.

iv. The algorithm chooses the largest α_i and fix it to 1.

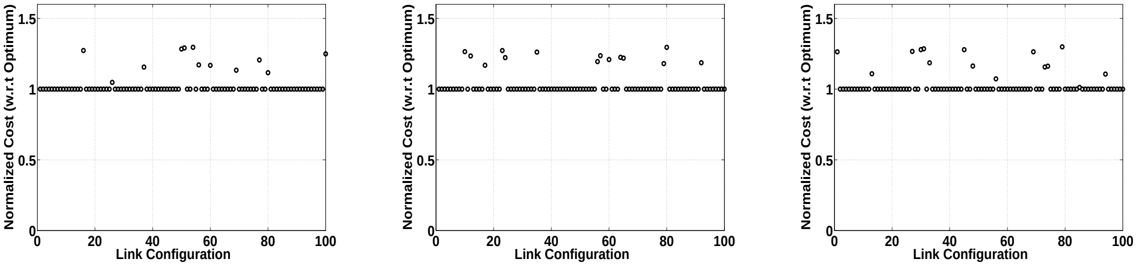
v. The process is repeated until a total of m α_i's are set to 1 (feasible assignment) or all α_i's are fixed and no feasible channel assignment can be found.

3.2.2 SFLP-based Channel Assignment with Guard-band Reuse

Now, we consider the case in which guard-band reuse is allowed. Recall that, to improve spectrum efficiency, the number of introduced guard channels should be minimized. When guard-band sharing is not allowed, minimizing the number of frequency blocks is equivalent to minimizing the number of newly introduced guard channels. However, when guard-band reuse is allowed, the number of introduced guard channels is minimized by attempting to reuse existing guard channels (introduce no new guard channels) and at the same time minimize the number of frequency blocks required for a given transmission, which may result in 100% spectrum efficiency. This can be achieved by selecting frequency blocks that do not introduce additional guard channels (i.e., already has a guard channel on each side and can reuse it). To illustrate, consider the channel status table in Figure 4. Suppose that a prospective CR transmission requires 2 data channels. Assume that any possible combination of two idle channels is power-feasible (i.e., $P_{tot} \leq P_{\max}$). Also assume that channels 16 and 17 require the minimum P_{tot} among all possible combinations of two adjacent channels. According to the SFLP algorithm proposed in Section 3.2.1, channels 16 and 17 will be selected. This assignment introduces 2 additional guard channels (50% spectrum efficiency). However, if channels 2 and 6 are selected, no additional guard channels will be introduced, leading to 100% spectrum efficiency. Thus, for efficient spectrum utilization, the SFLP algorithm cannot be directly applied to CRNs under the possibility of guard-band reuse. We now modify the SFLP algorithm to be suitable for such networks.

In the first iteration, we relax the binary constraints by allowing α_i's and z_i's to take real values in $[0, 1]$. For a busy channel $i \in \mathcal{B}$ (occupied by PR or CR user), we set $\alpha_i = 0$. We also set $\alpha_i = 0$ for all channels that are adjacent to a busy channel occupied by a PR user. For a guard channel $i \in \mathcal{G}$, we set $\alpha_i = 1$. By setting $\alpha_i = 1, \forall i \in \mathcal{G}$, our algorithm will prefer frequency blocks that already have guard channels reserved by neighboring transmissions. Note that because $\alpha_i, \forall i \in \mathcal{G}$ is set to 1, the constraint on the number of selected channels in the original BLP (i.e., $\mathbf{e}^T\boldsymbol{\alpha} = m$) should be updated as follow: $\mathbf{A}_{\text{eq}}\mathbf{X} = m + |\mathcal{G}|$. We refer to the resulting formulation as $\text{LR}^{(1)}$, which must have a feasible solution if the modified BLP has a feasible solution. Among all α_i's of the optimal solution of $\text{LR}^{(1)}$, we set the one that has the largest value to 1. Then, for the subsequent iterations ($i = 2, \dots m$), the same algorithm used for SFLP with no guard-band reuse is used to compute a feasible channel/power assignment. In the rest of this paper, we refer to the channel assignment mechanism that uses the modified SFLP algorithm as SFLP-GR, and the one that uses the original

SFLP algorithm as SFLP.



(a) Low activity ($P_B = 0.1$, average = 1.0287, variance = 0.0059)

(b) Moderate activity ($P_B = 0.4$, average = 1.0425, variance = 0.0074)

(c) High activity ($P_B = 0.7$, average = 1.0291, variance = 0.0052)

Figure 5: Normalized cost function for the SFLP solution w.r.t. the optimal solution ($m = 4$).

Lemma 2 *If the greedy solution in Section 3.1 is feasible, then the original BLP and the corresponding LR (i.e., LR$^{(1)}$) have feasible solutions.*

Proof: The feasibility regions of the BLP and LR$^{(1)}$ include the greedy solution (i.e., the greedy solution is one of the possible combinations of the binary variables that need to be exhausted to solve the BLP).

Lemma 3 *The updated LR$^{(i)}$ problem in Step (ii) must be feasible.*

Proof: (By induction). In the second iteration, if LR$^{(2)}$ has no feasible solution, it will be updated by switching the value of the last fixed variable to 0. The total transmission power will therefore decrease. Consequently, the total transmission power constraint will not be violated. Thus, the updated LR$^{(2)}$ must have at least one feasible solution. In the second iteration, LR$^{(2)}$ comes from either a feasible LR$^{(2)}$ or an updated feasible LR$^{(2)}$ of the first iteration. Thus, LR$^{(2)}$ must be feasible in the second iteration. Given that LR$^{(2)}$ is feasible in the second iteration, the rationale used in proving the feasibility of the first iteration also applies here to prove the feasibility of LR$^{(3)}$ in the third iteration. This induction is repeated for all iterations. Noting that all variables are bounded in $[0, 1]$, Lemma 3 holds.

Theorem 1 *The SFLP algorithm can determine a feasible solution or no feasible solution in no more than* $\max\{m, |\mathcal{I}|\}$ *iterations.*

Proof: The proof follows from Lemma 2 and 3. It is guaranteed that in each iteration, one new α_i variable will be fixed to either 0 or 1 and a new feasible LR will be generated for the next iteration. If all the generated LR$^{(i)}$ are feasible, then m iterations are required. Otherwise, a maximum of $|\mathcal{I}|$ iterations are required to determine whether m feasible channels can be found.

Based on Theorem 1, it is easy to show that the time complexity of the proposed SFLP algorithm is bounded by the complexity of the LR solver times $\max\{m, |\mathcal{I}|\}$. Because a LR solver (LP solver) has a polynomial complexity, the complexity of our sequential fixing algorithm is also polynomial. Our simulations show that in most cases our algorithm requires m iterations to find a feasible assignment. In addition, the performance gap between the SFLP solution and the optimal solution (obtained through an exhausted search) is shown to be very small (below 5%), and in most cases it is zero. We also provide a lower bound on the optimal BLP solution, which is the solution to $\mathrm{LR}^{(1)}$ in the first iteration. Our simulations show that this bound is typically loose.

4 Performance Evaluation

4.1 Simulation Setup

We consider N CR links in a 100 meter × 100 meter area. We assume that there are $M = 21$ channels, each licensed to one PRN. CR users can opportunistically access the 21 channels. Each channel has 1 MHz of bandwidth. The carrier frequency of the ith PRN is $f_i = 900 + i$ MHz, for $i = 1, \dots M$. We set μ^* to 0.63 for all channels. The status of a PR signal is modeled as a 2-state Markov model that alternates between two states: IDLE and BUSY. A BUSY (IDLE) state indicates that some (no) PR user is transmitting over the given channel. For channel i, denote the average IDLE and BUSY durations of the PR signal by λ_i and μ_i, respectively. In any given slot, the ith PRN is active with probability $P_B^{(i)} = \frac{\lambda_i}{\lambda_i + \mu_i}$. We set $\mu_i = 100$ ms and $\lambda_i = \lambda, \forall i \in \{1, \dots, M\}$. Accordingly, $P_B^{(i)} = P_B, \forall i$. We consider a Rayleigh fading model to describe the channel gain between any two users. Specifically, for a transmitter-receiver separation d, the received power over the ith channel is given by:

$$P_r{}^{(i)} = P_o^{(i)} \left(\frac{d}{d_o{}^{(i)}} \right)^{-n} \xi^{(i)}, \qquad d \geq d_o{}^{(i)} \tag{10}$$

where $P_o^{(i)} = \frac{P_t{}^{(i)} G_t^{(i)} G_r^{(i)} l_i^2}{(4\pi d_o^{(i)})^2}$ is the path loss of the close-in distance $d_o{}^{(i)} = \max\{\frac{2D^2}{l_i}, D, l_i\}$, D is the antenna length, $P_t{}^{(i)}$ is the transmission power, $G_t^{(i)}$ is the antenna gain at the transmitter, $G_r^{(i)}$ is the antenna gain at the receiver, l_i is the wavelength of f_i, n is the path loss exponent, and $\xi^{(i)}$ is a normalized random variable that represents the power gain of the fading process. For Rayleigh fading, $\xi^{(i)}$ is exponentially distributed; $\Pr(\xi^{(i)} \leq y) = 1 - e^{-y}$ [25]. We set the maximum transmission power of a CR user to $P_{\max} = 1$ W, the thermal noise power density to 10^{-21} W/Hz for all channels, the path loss exponent to $n = 4$, and the antenna length to $D = 5$ cm.

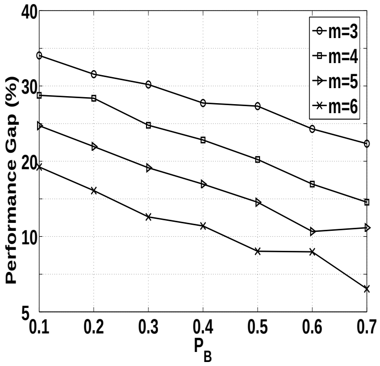


Figure 6: Relative performance gap between SFLP solution and lower bound (%).

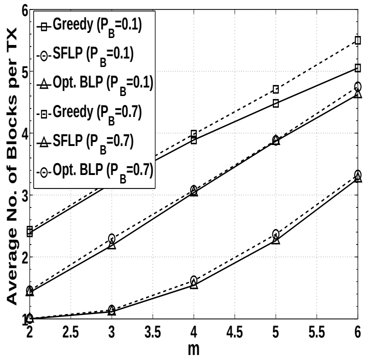


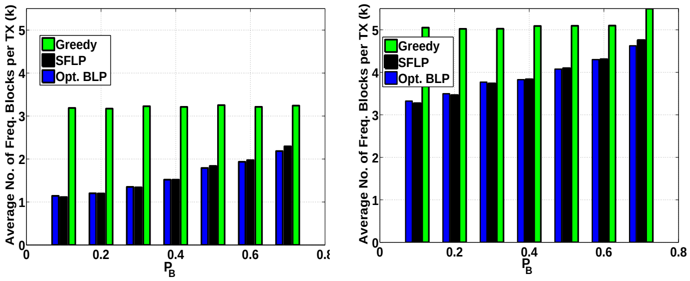
Figure 7: Average number of assigned blocks vs. m.

4.2 Results

4.2.1 Link-level Simulations

First, we use MATLAB simulations to empirically verify the validity of our SFLP algorithm and highlight its advantages. We consider a single CR link, and investigate the performance of the SFLP algorithm as a function of various system parameters. The simulation results are presented for 100 "link configuration" (i.e., optimization instances) that can produce feasible solutions. For each configuration, the source-destination distance is randomly generated, the power gain of the fading process $\xi^{(i)}$, $\forall i$ is exponentially distributed, and the status of a PR channel is determined according to the 2-state Markov model described before. The SFLP algorithm is used to determine the channel assignment and the cost function (number of non-contiguous frequency blocks plus the normalized total transmission power). We compare these results with the lower bound (the solution for $\mathrm{LR}^{(1)}$), the optimal solution, and the greedy solution.

For $m = 4$ and for three different values of P_B (0.1, 0.4, and 0.7), Figure 5 shows the normalized cost obtained by the SFLP algorithm w.r.t. the optimal cost obtained through exhaustive search for 100 link configurations. In most cases, the SFLP solution is identical to the optimal solution. Other results (not shown here) indicate that for various settings



(a) $m = 3$ (b) $m = 6$

Figure 8: Average number of assigned blocks vs. P_B for different values of m.

of the design parameters, the mean and variance of the normalized cost are ≤ 1.04 and ≤ 0.007, respectively. Hence, the SFLP algorithm achieves a near-optimal solution.

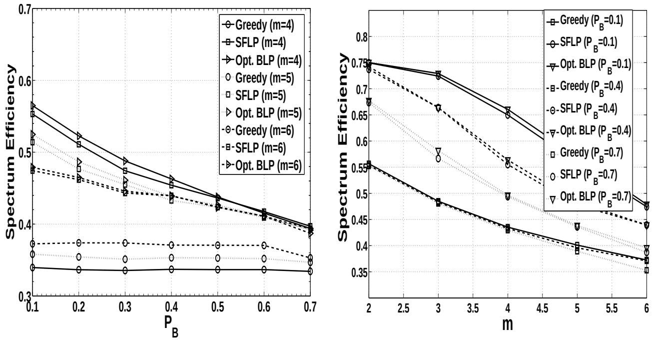
Figure 6 depicts the gap (as a percentage) between the SFLP solution and the lower bound generated after the first iteration of the SFLP algorithm. The gap is plotted as a function of P_B for different values of m. Two observations can be drawn from this figure. First, the lower bound is often loose. Second, the performance gap is smaller at larger m.

Figure 7 illustrates the average number of non-contiguous frequency blocks (k) as a function m (equivalently, the rate demand) under various assignment schemes and for two values of P_B: 0.1 and 0.7. It is clear that SFLP and the optimal BLP solutions significantly outperform the greedy approach. At the same time, the figure reveals that SFLP and the optimal solutions have comparable performance in terms of k. The greedy approach requires roughly the same k, irrespective to P_B. This is because the greedy approach always selects the best available m channels to support the rate demand. For both SFLP and optimal BLP, k is smaller at smaller P_B (see Figure 8). This is because a smaller P_B increases the number of idle channels, and consequently increases the chances of finding contiguous channels to support the rate demand. Figure 8 indicates that, for a given P_B, a larger value of m results in a larger k.

Figure 9 depicts the spectrum efficiency (i.e., ratio of the number of data channels to the number of assigned data-plus-guard channels) as a function of m and P_B. These figures reveal that both SFLP and the optimal BLP achieve comparable performance, which significantly outperforms the greedy scheme. As demonstrated, spectrum efficiency decreases as m increases. This is expected since the larger m, the lower is the likelihood of finding contiguous channels. This increases the number of required guard channels, which consequently reduces spectrum efficiency.

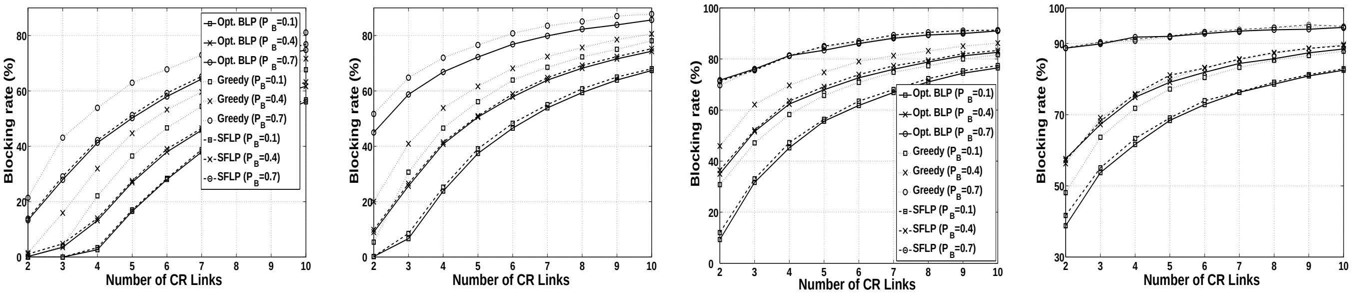
4.2.2 Network-level Simulations

In order to study the performance in a multi-user environment, we use the same simulation setup described in Section 4.1, but we consider $N = 2, 3, \ldots, 10$ CR transmitter-receiver pairs. To resolve channel contention between CR pairs, in our



(a) Spectrum efficiency vs. P_B (b) Spectrum efficiency vs. m

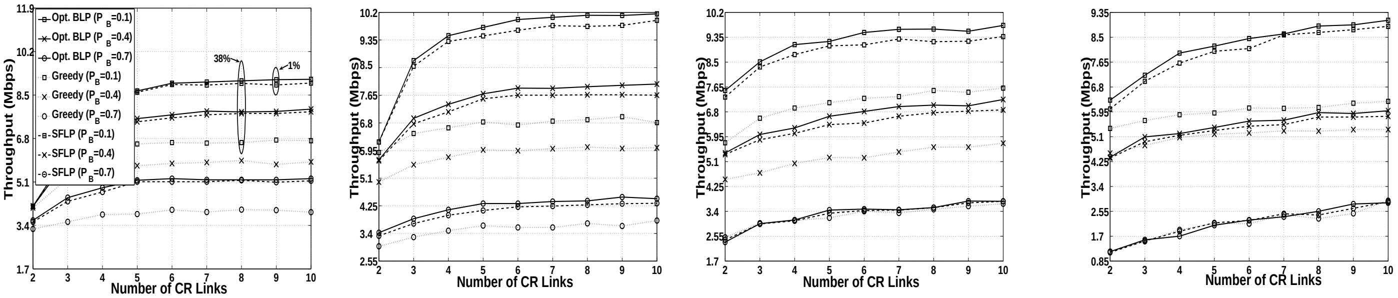
Figure 9: Spectrum efficiency under various channel assignment schemes.



(a) $m = 2$ (b) $m = 3$ (c) $m = 4$ (d) $m = 5$

Figure 10: Blocking rate vs. N for different values of P_B (without guard-band reuse).

simulations, we adopt the cognitive channel access mechanism proposed in [9]. This mechanism is a CSMA/CA-based random access scheme that uses contention-based handshaking for exchanging control information. The main objectives for the use of the control packet exchanges: (1) conducting and announcing the channel assignment, (2) prompting both the transmitter and the receiver to tune to the agreed on channels before transmission commences, (3) ensuring non-overlapping local channel occupancy between CR users (i.e., exclusive channel occupancy). Our results are based on simulation experiments conducted using CSIM (a C-based, process-oriented, discrete-event simulation package [26]).



(a) $m = 2$ (b) $m = 3$ (c) $m = 4$ (d) $m = 5$

Figure 11: Network throughput vs. N for different values of P_B and m.

Each CR sender generates fixed-size (2 KB) data packets and requires m data channels. The time is divided into slots, each corresponding to the transmission of one packet at a rate of m Mbps. We assume that there is always a packet to transmit for each CR user. The locations of the CR transmitters and receivers are randomly assigned within the simulation region. In any given slot, the PR activity over a given channel is determined according to the 2-states Markov model described in Section 4.1. Our performance metrics include: (1) network throughput, (2) CR blocking rate, and (3) average energy consumption for successfully transmitting one data packet (E_p). The CR blocking rate is defined as the percentage of transmission requests (packets in this case) that are blocked due to the unavailability of a feasible channel assignment. The results presented below are based on the average of 25 randomly generated topologies, with a simulation time of 10000 time slots for each topology.

Channel Assignment with No Guard-band Reuse: We first simulate a CRN, where no guard-band reuse is allowed. Our SFLP scheme is compared with two other channel assignment schemes: an optimal scheme (which performs exhaustive search) and the greedy scheme. We study the throughput performance as a function of N, m, and P_B. Figures 10 and 11 show that the SFLP algorithm significantly reduces the packet blocking rate and improves the overall throughput by up to 38% compared with the greedy approach for various settings of the design parameters. In all cases, the SFLP solution is within 5% of the optimal one. Figure 11 reveals that the throughput gain of SFLP over the greedy approach is smaller at larger P_B. This is expected since the larger the value of P_B, the lower the chances of finding contiguous channels. This increases the number of required guard channels, and consequently reduces the throughput gain. Note that for large values of m and P_B, all schemes achieve comparable throughput performance.

In Figure 12, we investigate the impact of various channel assignment strategies on E_p[3]. It is clear that the greedy approach performs better in terms of energy consumption (because the greedy approach always selects the channels with the highest received SINR). Thus, the throughput advantage of SFLP comes at the expense of additional energy consumption.

Channel Assignment with Guard-band Reuse: We now consider a CRN where guard-band reuse is allowed. Our proposed scheme (SFLP-GR) is compared with two other assignment schemes: the original SFLP (which tries to minimize the number of frequency blocks) and the greedy scheme. We adapt the operation of both schemes such that a guard channel can be reused (i.e., an idle channel that is adjacent to an already assigned guard channel can be used for CR data transmissions). We first study the throughput performance. Figures 13-15 show that SFLP-GR significantly outperforms the other schemes. SFLP-GR reduces the CR blocking rate and improves the overall throughput by up to

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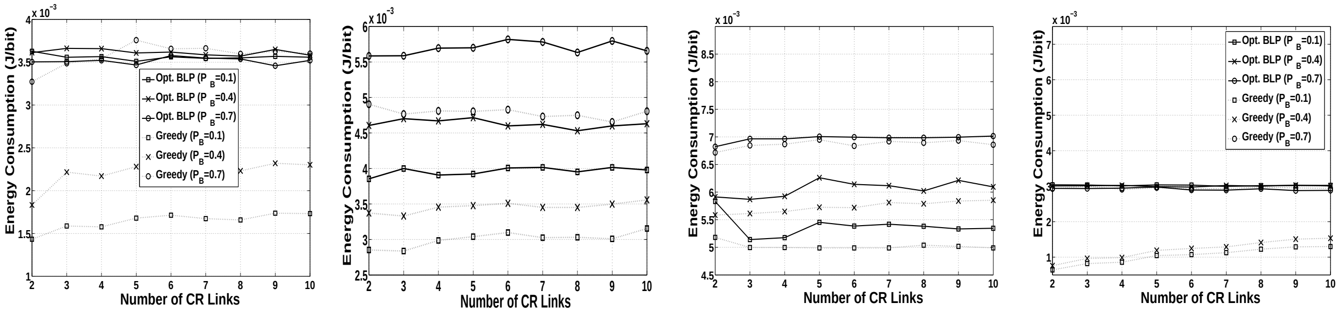
[3] The performance in terms of E_p under SFLP is comparable to the one for the optimal solution. Thus, for clarity, Figure 12 does not show the energy consumption (E_p) of SFLP.

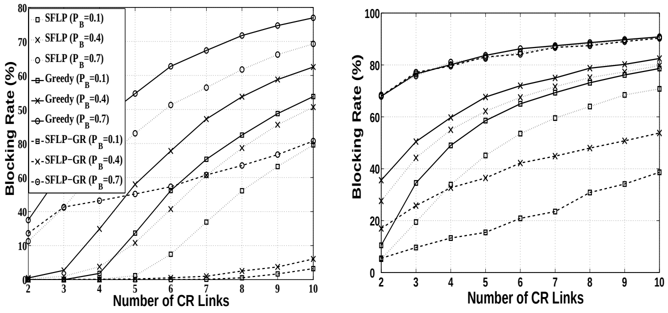


(a) $m = 2$ (b) $m = 3$ (c) $m = 4$ (d) $m = 5$

Figure 12: Energy consumption vs. N for different values of P_B and m.

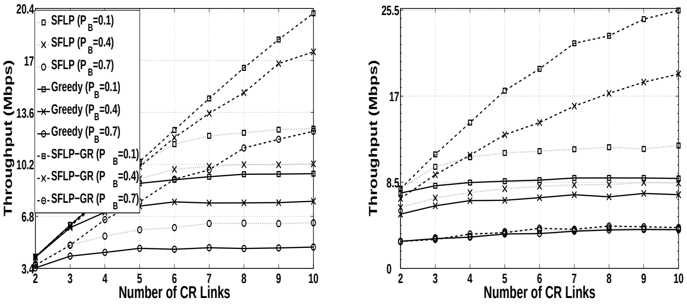
180% compared with the greedy approach and 110% compared with the SFLP algorithm (when $m = 4$ and $N = 10$). This improvement is mostly attributed to the proper channel assignment, which attempts to reuse already allocated guard channels. Consequently, our scheme preserves more channels for future CR transmissions, leading to an increase in the number of simultaneous transmissions. Similar to the case of no guard-band reuse, Figure 15 shows that the achieved throughput is smaller at larger values of P_B and m.

Finally, Figure 16 indicates that similar trends in terms of E_p to the no guard-band reuse case are observed here.



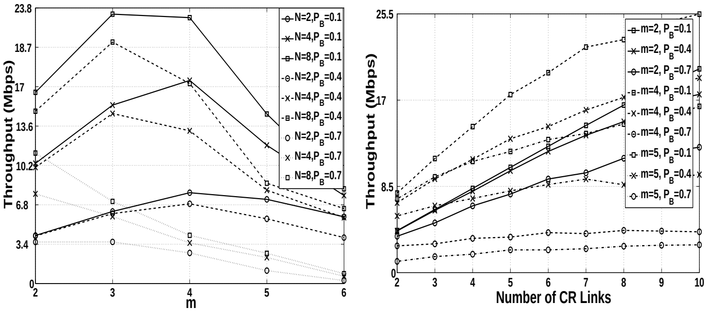
(a) $m = 2$ (b) $m = 4$

Figure 13: Blocking rate vs. N.



(a) $m = 2$ (b) $m = 4$

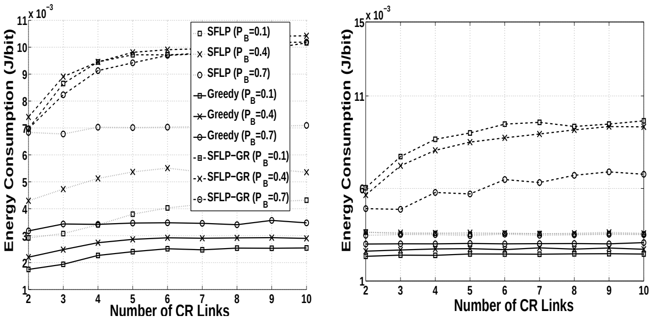
Figure 14: Throughput vs. N (similar behavior for other values of m was observed).



(a) Throughput vs. m ($N = 2, 5, 8$)

(b) Throughput vs. N ($m = 2, 4, 5$)

Figure 15: Throughput comparison for different values of P_B under SFLP-GR assignment.



(a) $m = 2$

(b) $m = 4$

Figure 16: Energy consumption vs. N for different values of P_B.

5 Conclusions

In this paper, we proposed an opportunistic guard-band-aware channel assignment for CRNs. Our scheme improves the CRN throughput through cooperative channel assignment, taking into consideration the guard-band constraint. The proposed channel assignment mechanism reduces the number of required guard channels for a given transmission by assigning adjacent channels as much as possible to that transmission, which significantly improves spectrum efficiency and network throughput. We first formulated the channel access as a joint power control and channel assignment optimization problem, with the objective of minimizing the required spectrum for a given transmission. We showed that this problem can be formulated as a BLP. Because of the exponential worst-case time complexity of BLP, we presented a near-optimal algorithm to solve this problem based on a sequential fixing procedure, where the binary variables are determined iteratively by solving a sequence of LPs. Simulation results verified the accuracy of the proposed algorithm. We compared the performance of our scheme with that of a reference scheme (greedy). We showed that, our proposed scheme achieves up to a 180% increase in throughput over the greedy scheme, with manageable processing overhead. To the best of our knowledge, our proposed channel assignment scheme is the first to account for the guard-band constraint to improve the overall CRN throughput.

References

[1] "FCC, spectrum policy task force report, ET docket no. 02-155," Nov. 2002.

[2] H. Bany Salameh and M. Krunz, "Channel access protocols for multihop opportunistic networks: Challenges and recent developments," *IEEE Network-Special Issue on Networking over Multi-hop Cognitive Networks*, July 2009.

[3] Y. Yuan, P. Bahl, R. Chandra, T. Moscibroda, and Y. Wu., "Allocating dynamic time-spectrum blocks in cognitive radio networks," in *Proceedings of the ACM MobiHoc Conference*, Sept. 2007.

[4] H. Bany Salameh, "Throughput-oriented channel assignment for opportunistic spectrum access networks," *Mathematical and Computer Modelling*, 2010.

[5] J. Jia, J. Zhang, and Q. Zhang, "Cooperative relay for cognitive radio networks," in *Proceedings of the IEEE INFOCOM Conference*, April 2009, pp. 794–802.

[6] S. Sankaranarayanan, P. Papadimitratos, A. Mishra, and S. Hershey, "A bandwidth sharing approach to improve licensed spectrum utilization," in *Proceedings of the IEEE DySPAN Conference*, Nov. 2005, pp. 279–288.

[7] J. Jia, Q. Zhang, and X. Shen, "HC-MAC: A Hardware-Constrained Cognitive MAC for Efficient Spectrum Management," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 1, pp. 106–117, January 2008.

[8] A. Sabharwal, A. Khoshnevis, and E. Knightly, "Opportunistic spectral usage: Bounds and a multi-band CSMA/CA protocol," *IEEE/ACM Transactions on Networking*, vol. 15, no. 3, pp. 533–545, 2007.

[9] H. Bany Salameh, M. Krunz, and O. Younis, "MAC protocol for opportunistic cognitive radio networks with soft guarantees," *IEEE Transactions on Mobile Computing*, vol. 8, no. 6, pp. 1339–1352, 2009.

[10] H. Bany Salameh, M. Krunz, and O. Younis, "Cooperative adaptive spectrum sharing in cognitive radio networks," *IEEE/ACM Transactions on Networking*, 2010.

[11] A. Nasipuri and S. Das, "Performance of multi-channel ad hoc networks," *International Journal of Wireless and Mobile Computing*, vol. 1, no. 3/4, pp. 191–203, 2006.

[12] E. Arikan, "Some complexity results about packet radio networks," *IEEE Transactions on Information Theory*, vol. 30, no. 4, pp. 681–685, 1984.

[13] A. Behzad and I. Rubin, “Multiple access protocol for power-controlled wireless access nets,” *IEEE Transactions on Mobile Computing*, vol. 3, no. 4, pp. 307–316, 2004.

[14] J. Proakis, *Digital Communications*, McGraw Hill, 2001.

[15] J. Poston and W. Horne, “Discontiguous OFDM considerations for dynamic spectrum access in idle TV channels,” in *Proceedings of the IEEE DySPAN Conference*, 2005, pp. 607–610.

[16] K. Nolan, P. Sutton, L. Doyle, T. Rondeau, B. Le, and C. Bostian, “Dynamic spectrum access and coexistence experiences involving two independently developed cognitive radio testbeds,” in *Proceedings of the IEEE DySPAN Conference*, 2007, pp. 270–275.

[17] “IEEE 802.11a, Part 11: wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) specifications,” 1999.

[18] I. Trigui, M. Siala, and H. Boujemaa, “Optimized pulse shaping for OFDM multi-user communications over doubly dispersive channels,” in *The 9th International Symposium on Signal Processing and Its Applications (ISSPA)*, October 2007, pp. 1–4.

[19] A. Tonello, N. Laurenti, and S. Pupolin, “Analysis of the uplink of an asynchronous multi-user DMT OFDMA system impaired by time offsets, frequency offsets, and multi-path fading,” in *Proceedings of IEEE Fall Vehicular Technology Conference (VTC)*, October 2000, pp. 1094–1099.

[20] “Second Report and Order and Memorandum Opinion and Order, ET Docket No. 04-186; FCC 08-260,” 2008.

[21] L. Wolsey., *Integer Programming*, John Wiley & Sons, Inc., 1998.

[22] A. Bemporad and M. Morari, “Control of systems integrating logic, dynamics, and constraints,” *Automatica*, vol. 35, pp. 407–427, 1999.

[23] M. Garey and D. Johnson, *Computers and Intractability: A guide to the Theory of NP-Completeness*, Freeman, 1979.

[24] N. Karmarkar, “A new polynomial-time algorithm for linear programming,” *Combinatorica*, vol. 4, pp. 373–396, 1984.

[25] T. S. Rappaport, *Wireless Communications-Principles and Practice*, Prentice-Hall Press, 2001, 2nd edition.

[26] "Mesquite Software Incorporation," www.mesquite.com.