Determination of Achievable Information Rates (AIRs) of IM/DD Systems and AIR Loss Due to Chromatic Dispersion and Quantization

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***Abstract*—The achievable information rates (AIRs) of a nonreturn-to-zero optical transmission system operating at 10 Gb/s are determined based on experimentally obtained conditional probability density functions for an amplified spontaneous emission noise-dominated system in the presence of chromatic dispersion (with and without optical dispersion compensation). Specifically, we concentrate on the AIR loss due to the accumulation of chromatic dispersion, as well as the practical constraint of the analog-to-digital conversion and quantization of log-likelihood ratios.**

***Index Terms*—Achievable information rates (AIRs), Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm, finite-sate machine, optical communications, quantization.**

I. Introduction

THERE have been numerous attempts to determine the Shannon's capacity of a fiber-optics communication channel [1]–[3]. The most common approach is to consider the amplified spontaneous emission (ASE) from inline amplifiers as the predominant noise source [1], [2], [4], [5], and include the effect of nonlinearities in an approximate fashion. The fiber nonlinearities were considered either as the perturbation of a linear case [1], or as multiplicative noise [2]. Unfortunately, these approximate methods may yield reliable results only in the regime of relatively small nonlinearities; and, as indicated in [3], they need to be carefully justified for every particular transmission system under study. In addition, authors in [1]–[3] did not provide experimental verification of probability density functions (pdfs) employed in calculation of channel capacity.

Another quantity of high practical interest is the achievable information rate (AIR), which represents the mutual information rate for a predefined stationary input distribution. In this letter, following the approach from [6]–[10], and using a method based on forward step of the Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm [11], we for the first time 1) determine the AIR of a single-channel nonreturn-to-zero (NRZ) optical transmission system operating at 10 Gb/s using experimentally determined conditional pdfs, and 2) investigate the AIRs loss due to analog-to-digital (A/D) conversion and quantization of intrinsic log-likelihood ratios (LLRs). The method consists of two steps: 1) estimating pdfs for different states (bit configurations) experimentally; and 2) calculating the AIRs as explained in [6]–[10]. The numerical results are reported for the ASE noise-dominated channel for a transmission system with 100 km of single-mode fiber (SMF). In particular, we investigated the information rate loss due to chromatic dispersion by calculating AIR with and without optical dispersion compensation. Moreover, we consider the influence of the channel memory assumption on the AIR. We note that the ultimate capacity of the ON–OFF-keyed communication system is known in advance and equals unity. Therefore, this letter is concerned with determination of information rates (or equivalently, the coding overheads) of systems with intensity modulation and direct detection (IM/DD).

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II. Determination of AIRs

The calculation of the AIRs of IM/DD systems is based on a discrete dynamical description of the channel using the finite state machine (FSM) approach [6]–[10]. The optical transmission system (channel) is modeled as an intersymbol interference (ISI) channel, where m previous and m next bits influence the observed bit. The channel output (receiver output) vector is denoted by $\boldsymbol{y} = (y_1, \ldots, y_n)$, with $y_i \in \boldsymbol{Y}$, and the source vector by $\boldsymbol{x} = (x_1, \ldots, x_n)$, with $x_i \in \boldsymbol{X} = \{0, 1\}$. The channel is completely defined by $\mathbf{X}$, $\mathbf{Y}$ and the conditional probability function $P(\boldsymbol{Y}|\boldsymbol{X})$. The state (bit configuration) $s = (x_{j-m}, x_{j-m+1}, \ldots, x_j, x_{j+1}, \ldots, x_{j+m})$ is determined by the sequence of $2m+1$ input bits $x_i \in \mathbf{X}$ that influence the observed bit x_j.

The information rate is defined as

$$I(\mathbf{Y};\mathbf{X}) = H(\mathbf{Y}) - H(\mathbf{Y}|\mathbf{X}). \quad (1)$$

$H(\boldsymbol{U})$ is entropy defined as $H(\boldsymbol{U}) = E(\log_2 P(\boldsymbol{U}))$ for a random variable $\boldsymbol{U}$ with $E(\cdot)$ being the expectation operator. In the rest of the letter, we address a problem often encountered in practice: determination of information rate in the case of independent and uniformly distributed (i.u.d.) input source to the IM/DD channel. Using this source for the calculation results in a lower bound on the channel capacity, also known as the AIR, and is related to the information rate of an IM/DD system that can be achieved in practice.

The Shannon-McMillan-Breiman theorem [12] states that

$$\left(\frac{1}{n}\right) \log_2 P(Y_1^n) \rightarrow E(\log_2 P(\mathbf{Y})), \qquad \text{as } n \rightarrow \infty \quad (2)$$

where we use Y_i^j to denote $(Y_i, \ldots, Y_j)$. Therefore, the problem of estimating the information rate can be reduced to generating

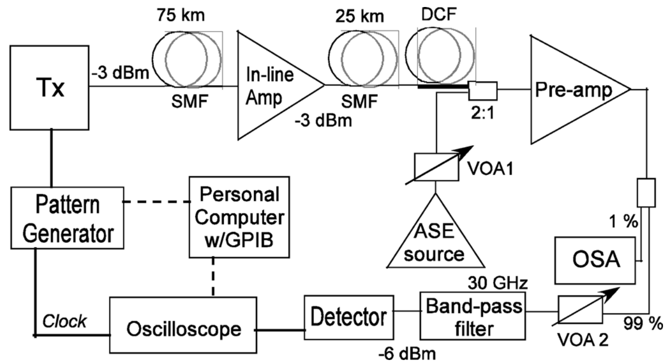

Fig. 1. Experimental setup.

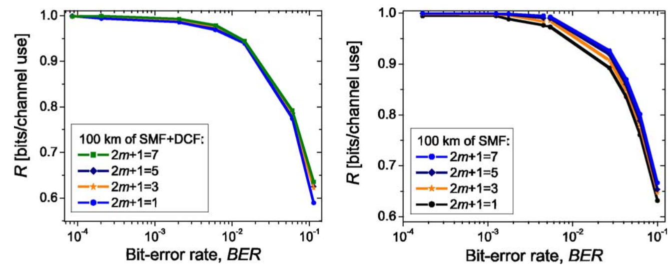

Fig. 2. AIRs versus BER after 100 km of SMF-28 with optical dispersion compensation (left), and without optical dispersion compensation (right).

a long sequence y_1^n and calculating $\log_2 P(y_1^n)$. A fiber-optics communication channel is essentially a discrete nonlinear ISI channel with memory, with memory length determined by the particular dispersion map being employed. In some dispersion maps designed for the pseudolinear transmission (not considered in this letter), the memory of the channel can be in the order of hundreds of bits, but it is finite, suggesting that (2) is applicable as long as the observed sequence is of a sufficient length. The $\log P(y_1^n)$ for an i.u.d. source is estimated by using the forward step of BCJR algorithm [6], [11]

$$\log P\left(y_1^n\right) = \sum_{i=1}^{n} \log P\left(y_i | y_1^{i-1}\right)$$
$$P\left(y_i | y_1^{i-1}\right) = \sum_{s',s} \alpha_{i-1}\left(s'\right) P\left(y_i | s\right) P_{s's}. \tag{3}$$

The probability of the state (bit configuration) s at instance i, denoted as $\alpha_i(s)$, is calculated by

$$\alpha_i\left(s\right) = \frac{\sum\limits_{s'} \alpha_{i-1}\left(s'\right) P\left(y_i | s\right) P_{s's}}{\sum\limits_{s',s} \alpha_{i-1}\left(s'\right) P\left(y_i | s\right) P_{s's}} \tag{4}$$

where $P_{s's}$ is the probability of transition from state s' to state s, which is $1/2$ for two possible transitions, and 0 otherwise.

The BCJR algorithm, proposed by Bahl *et al.* [11], is a maximum *a posteriori* probability decoding algorithm that can be used for any kind of FSM-driven sequence in general. As opposed to the Viterbi algorithm, which is a maximum likelihood sequence detection method, the BCJR algorithm is an optimal detection method that minimizes symbol error probability. Many experiments have been carried out in applying this algorithm not only to decode codes that can be described as a trellis, but also in the detection of data sent over channels with memory. A significant benefit of using the BCJR algorithm is that it provides soft LLRs at the output that can be further used to perform iterative decoding (LDPC/turbo decoding). The method described above is applicable to any modulation format, the NRZ is employed here as an example, while CSRZ was used in [6].

The conditional probability functions $P(y_i|s)$ depend on the physical properties of the channel and are determined experimentally using the setup shown in Fig. 1. A zero-chirp modulator was used to produce an OC-192 NRZ data stream. In-line optical amplification was performed after a 75-km span of SMF span with average dispersion of 17 ps/km · nm. The launch power was maintained at −3 dBm with variable optical attenuators [(VOAs) not shown in the diagram]. Total link length was 100 km, and a dispersion compensating module could be inserted at the end of the link to achieve full dispersion compensation. The end-of-link signal was combined with an ASE source immediately prior to the preamplifier. The ASE power was controlled by VOA1 in order to guarantee an independent optical signal-to-noise ratio adjustment at the receiver. A standard amplified PIN receiver was used for direct detection and was preceded by VOA2 that guaranteed a constant total received power of −6 dBm. De-Brujin pseudorandom bit sequences (PRBS) of 128 bits were synthesized using a 10-Gb/s pattern generator. The sampling oscilloscope, triggered by the data pattern, was used to acquire the received sequences, which were based on a PRBS sequence divided into subsequences of 28 bits overlapping by 3 bits with previous and subsequent subsequences to ensure statistics collection for all of the channel responses, assuming the maximal channel memory of 7 bits. 8192 realizations were collected for each channel response. Notice that the PRBS sequence was used to determine the conditional pdfs only (to ensure that all bit-configurations are present), while i.u.d. sequence is used to determine the AIRs.

III. Experimental Results and Quantization Effects

The AIRs versus the Q-factor of the uncoded signal for the back-to-back thermal noise-dominated channel are presented in our recent paper [7]. To determine the AIRs, we applied the forward step of BCJR algorithm for different numbers of slots (memory) m on each side of a center bit slot. This is of interest for practical applications, because designing electronics that would perform BCJR algorithm on high number of states (2^{2m+1}) is not a trivial task, especially for systems operating at 10 Gb/s and above. The AIRs versus bit-error rate (BER) after 100 km of SMF, with or without dispersion compensating fiber, are shown in Fig. 2. The AIR of the dispersion compensated channel exhibits weak memory effects. These memory effects are substantially larger in the uncompensated case due to the group velocity dispersion-induced ISI [13], [14]. The BER of the uncoded signal, used in the x-axis, is calculated from the collected histograms.

In experimental results shown in Fig. 2, the double precision of LLRs is assumed. In practice, however, hardware implementations require quantizing the LLRs, and A/D conversion of received samples. A fixed-point (FP) representation of a real-valued LLR λ is an integer λ_z with an n_b–bit precision. Out of the n_b bits, d_b bits are used to represent the integer part (including the sign) of λ and p_b bits are used to represent the decimal part of λ [15]. The range of λ_z is defined by (d_b, p_b),

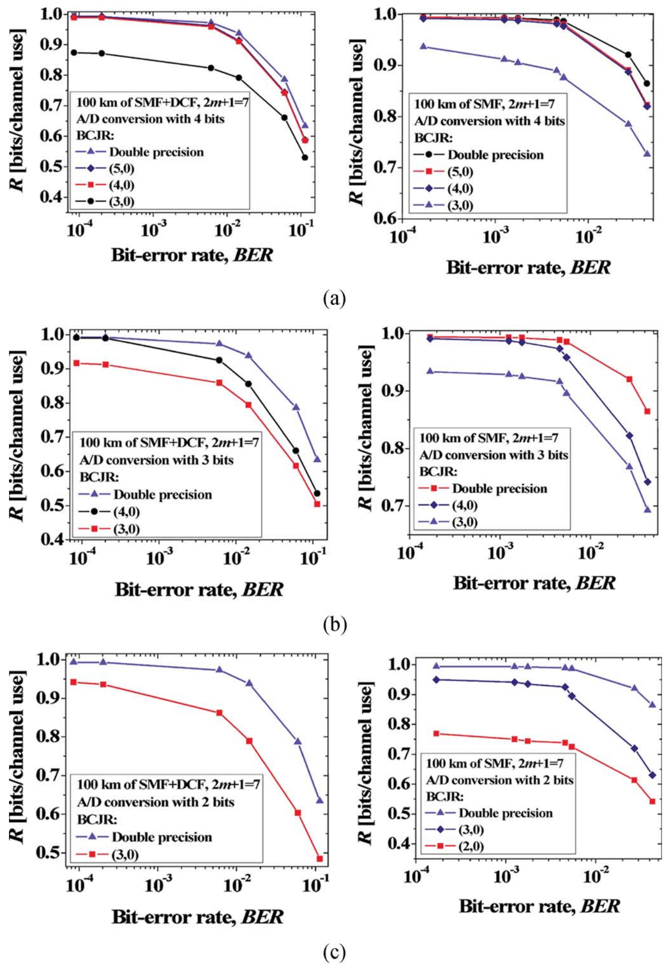

Fig. 3. Estimation of AIR loss due to A/D conversion and quantization after 100 km of SMF-28 for A/D conversion with: (a) 4 bits, (b) 3 bits, and (c) 2 bits. The left and right columns correspond to the cases with and without optical dispersion compensation, respectively.

where $n_b = p_b + d_b$. The FP representation of λ is obtained as follows:

$$\lambda_z = \min\left(\lfloor 2^{p_b}\lambda + 0.5\rfloor, 2^{n_b-1} - 1\right) \quad \text{or } \max\left(\lfloor 2^{p_b}\lambda + 0.5\rfloor, -2^{n_b-1}\right). \quad (5)$$

Hence, the range of λ_z is $[2^{nb-1} - 1, -2^{nb-1}]$, and we refer to λ_z as (d_b, p_b)-quantized. In calculation of AIRs in Fig. 3, the intrinsic information obtained from the channel observations is (d_b, p_b)-quantized and fed to forward log-domain BCJR algorithm. A/D conversion of samples is performed with 4 bits Fig. 3(a), 3 bits Fig. 3(b), and 2 bits Fig. 3(c). The left column in Fig. 3 corresponds to the case with optical dispersion compensation, and the right column to the case without optical dispersion compensation. The BCJR algorithm is implemented with the same number of bits used in A/D conversion or with smaller number of bits. The result of any operation performed within the BCJR algorithm is (d_b, p_b)-quantized. We observe from Fig. 3 that the AIR loss due to quantization is negligible if the number of bits (excluding the sign) is four. Otherwise, significant AIR loss is found.

IV. Conclusion

The AIRs of an NRZ optical transmission system operating at 10 Gb/s are determined based on conditional pdfs obtained experimentally. Two cases of high practical interest are considered: 1) an ASE noise-dominated channel without dispersion, and 2) an ASE noise-dominated system with dispersion. It was found that the dispersion compensated channel exhibits weak memory effects, while memory effects are substantially larger in the uncompensated case due to group velocity dispersion induced ISI. We note that the presented results without optical dispersion compensation can be interpreted as AIR loss due to electronic equalization by means of a Viterbi or BCJR detector, as opposed to all optical approaches. The AIR capacity loss due to quantization of intrinsic LLRs and A/D conversion is found to be negligible for number of bits equal to four. As expected, the AIR loss is becoming more important as the number of bits used in A/D conversion and BCJR algorithm decreases. The AIR loss due to A/D conversion and quantization of LLRs is more important in the case without dispersion compensation.

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