Low-Density Parity Check Codes for Long-Haul Optical Communication Systems

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***Abstract*—Foward error correction (FEC) scheme based on low density parity check codes (LDPC) codes is presented in this paper. We show that LDPC codes provide a significant system performance enhancement with respect to the state-of-the-art FEC schemes employed in optical communication systems.**

***Index Terms*—Forward error correction, long-haul transmission, low-density parity check codes, optical communications.**

I. Introduction

DURING the last few years a number of high-speed long haul optical communications systems have been demonstrated, commercialized, or implemented [1]. Although data flow through these systems has increased tremendously, it has become widely recognized that a full utilization of the available bandwidth cannot be achieved without powerful error control schemes.

Fortunately, there has been a great deal of research activity in the area of error control coding during the last few years ignited by the excellent bit-error-rate (BER) performance of the turbo decoding algorithm demonstrated by Berrou *et al.* [2]. During the last several years, turbo decoding has been generalized and mathematically formulated through the concept of codes on graphs. The prime examples of codes on graphs are low density parity check codes (LDPC). Extensive simulation results showed that in many channels [such as additive white noise Gaussian (AWGN) channel, binary symmetric channel and erasure channel] LDPC codes perform nearly as well as earlier developed turbo codes. The theory of codes on graphs has not only improved the error performance, but has also opened new research avenues for investigating alternative suboptimal decoding schemes such as belief propagation. The belief propagation algorithms and graphical models have been developed in the expert systems literature by Pearl [3] and in the case of LDPC codes are at least an order of magnitude simpler than the Bahl, Cocke, Jelinek, and Raviv (BCJR) algorithm [4] used in turbo decoding algorithm.

We show that LDPC codes provide a significant system performance enhancement with respect to the state-of-the-art foward error correction (FEC) schemes employed in optical communication systems.

II. A Novel FEC Based on LDPC Codes

A significant effort has been made to apply FEC coding techniques to optical transmission systems starting with Grover's proposal of applying FEC codes to dispersion limited lightwave systems with laser impairments [5]. Recently, particularly in transoceanic submarine systems error-correcting codes, such as the Bose–Chandhuri–Hocquenghem (BCH) code or the Reed–Solomon code have been selected for implementation [6]. Sab and Lemaire's proposed using turbo codes for Alcatel long-haul submarine transmission system [7]. The turbo decoding algorithm is incompatible with optical transmission technology, and its complexity is unacceptably high. Error performance and decoder hardware complexity can be further improved by using another type of iteratively decodable coding schemes, in particular low-density parity check codes. To our knowledge there is no published work on this subject.

We propose a scheme in which presently used Hamming and BCH codes are replaced by LDPC. None of the existing literature on this subject contains codes appropriate for the targeted application, which seeks multigigabit per second operation. We favor finite geometric and combinatorial designs of LDPC codes because they require only simple encoders and decoders and because we can be assured of large minimum codeword distances. Since the spectral efficiency must stay high, we are interested in high rate codes. We have demonstrated [8]–[10] that good codes can be constructed without using random interleavers or random sparse parity check matrices. This is diametrically opposite from the common but not very practical "random code" assumption that has been widely used in recent literature. The structure of our codes is of crucial importance for high-speed implementations, because these codes can lend themselves to extremely simple encoders. The structure can also be used to provide flexible error protection.

III. Code Construction

We propose a construction based on balanced incomplete block designs (BIBD) [10]. More specifically, the codes are based on sub designs of a 2-$(v, k, 1)$ BIBD, where v is a number of parity bits, and k is the column weight of a parity check matrix. The BIBD is given as a set of *blocks*, each of size k, containing v different elements called *points*. We define the point-block incidence matrix of a design as a $v \times b$ matrix $H = (h_{ij})$, in which $h_{ij} = 1$ if the ith element of a point set occurs in the jth block, and $h_{ij} = 0$ otherwise. The row weight is r, column weight is k, and the code rate is $R = (b - \mathrm{rank}(H))/b$. We consider such designs in which no more than one block contains the same pair of points. We developed a novel construction of 2-$(v, k, 1)$ designs with arbitrary block size. The designs are lines connecting points of a rectangular integer lattice. The idea is to trade a code rate and number of blocks for the simplicity of construction and flexibility of choosing design parameters.

To illustrate the code construction we give an example of a small lattice of dimensions $m = 5$ by $k = 3$. This will result

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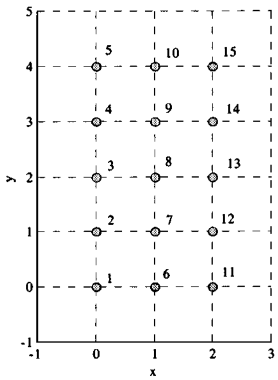

Fig. 1. An example of the rectangular grid for $m = 5$ and $k = 3$.

TABLE I
A LATTICE 2-(15, 3, 1) DESIGN

s=0	s=1	s=2	s=3	s=4
1 6 11	1 7 13	1 8 15	1 9 12	1 10 14
2 7 12	2 8 14	2 9 11	2 10 13	2 6 15
3 8 13	3 9 15	3 10 12	3 6 14	3 7 11
4 9 14	4 10 11	4 6 13	4 7 15	4 8 12
5 10 15	5 6 12	5 7 14	5 8 11	5 9 13

in a 2-(15, 3, 1)-BIBD that gives a parity check matrix with the column weight $k = 3$. The rectangular integer lattice is shown in Fig. 1. The subsets of points (but not all subsets) are referred to as lines (or blocks), and the design is defined as a set of lines of different slopes. A line with slope s, $0 \leq s \leq m-1$, *starting at* the point (x, a), contains the points $\{(x, a + sx \bmod m)\colon 0 \leq x \leq k-1\}$, where $0 \leq a \leq m-1$. The total number of lines is equal to $b = m^2$. The resulting design is given in Table I. The columns correspond to lines of different slopes in Fig. 1.

For example, a line of slope $s = 1$ starting at (0, 0) contains the points with labels $\{1, 7, 13\}$. As mentioned above, each line of a design specifies positions of nonzero elements in a column of parity check matrix H. The parity check matrix corresponding to the Table I is

$$H = \begin{bmatrix}
1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0\\
0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0\\
0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0\\
0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0\\
0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1\\
1&0&0&0&0&0&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0\\
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0&1&0&0&0&0&0&0&0&1&0&0&1&0&0&1&0&0&0&0&0&0&0&1&0\\
0&0&1&0&0&1&0&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&0&1\\
0&0&0&1&0&0&1&0&0&0&0&0&0&0&1&0&0&1&0&0&1&0&0&0&0\\
0&0&0&0&1&0&0&1&0&0&1&0&0&0&0&0&0&0&1&0&0&1&0&0&0
\end{bmatrix}.$$

For example, the sixth column of H, $h_6 = (1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0)^T$ corresponds to a line $\{1, 7, 13\}$.

Generally, the parity check matrix of lattice codes can be written in the form

$$H = \begin{bmatrix} H_{1,1} & H_{1,2} & \cdots & H_{1,m} \\ H_{2,1} & H_{2,2} & \cdots & H_{2,m} \\ \vdots & \vdots & & \vdots \\ H_{k,1} & H_{k,2} & \cdots & H_{k,m} \end{bmatrix}$$

where each submatrix is a permutation matrix. Using the last property, the complexity of the encoders and decoder can be drastically reduced.

Iterative decoding is based on the message-passing algorithm [11] that is much simpler than RS or turbo decoding algorithm.

IV. LDPC CODE PERFORMANCE

The main impairments in long-haul optical transmission come from amplified spontaneous emission noise, pulse distortion due to fiber nonlinearities, chromatic dispersion or polarization dispersion, crosstalk effects, intersymbol-interference (ISI), etc. The models presented so far [6], [7] in considering the FEC are based on AWGN assumption. In contrast, our simulator allows for taking into account in a natural way all major impairments in a long-haul optical transmission such as ASE noise, pulse distortion due to fiber nonlinearities, chromatic dispersion or polarization dispersion, crosstalk effects, intersymbol-interference, etc. In addition it can be used to analyze a variety of FEC schemes, especially those based on iterative decoding.

In this paper we present a FEC scheme based on LDPC codes and study the performance of LDPC codes in the presence of residual dispersion, fiber nonlinearities, ISI and receiver noise resulting from signal–noise and noise–noise interaction on PIN photodiode. The influence of the transfer functions of optical and electrical filters is taken into account as well. A wavelength-division-multiplexing system with 10-Gb/s bit rate per channel and a channel spacing of 50 GHz is considered. It is assumed that the observed channel is located at 1552.524 nm (193.1 THz) and that there exists a non negligible interaction with six neighboring channels. The extinction ratio is set to 13 dB. The transmitter and receiver imperfection is described through a back-to-back Q-factor which is set to 23 dB. A combinatorially constructed LDPC (1369, 1260) code with code rate of $R = 0.92$ (redundancy of 8.7%), constructed as described in Section III, is considered.

Results of a Monte Carlo simulation for the AWGN channel are shown in Fig. 2. For 10^{-6} the LDPC scheme outperforms the RS (255, 239) code by 1.1 dB. It should be noted that the complexity of the RS decoder is much higher than that of a LDPC decoder of equal or comparable length. Another important observation is that coding gain at lower BER (not shown in the figure) will be larger because of a steep waterfall curve of LDPC codes [10].

Fig. 3 shows the BER results of a Monte Carlo simulation for a typical receiver comprised of an erbium-doped fiber amplifier (EDFA) as a pre-amplifier, an optical filter (modeled as super-Gaussian filter of the eight order and bandwidth $2R_b$, R_b-bit rate over code rate), a PIN photodiode, an electrical filter (modeled as a Gaussian filter of bandwidth $0.65R_b$), a sampler and a decision circuit.

In Fig. 4 the same assumptions hold, except that a more realistic transmission is considered with a dispersion map

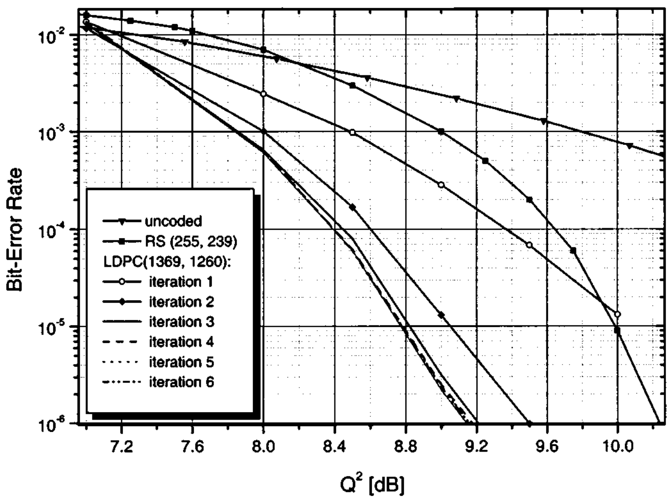

Fig. 2. BER versus Q-factor for LDPC (1369, 1260) code for the AWGN channel.

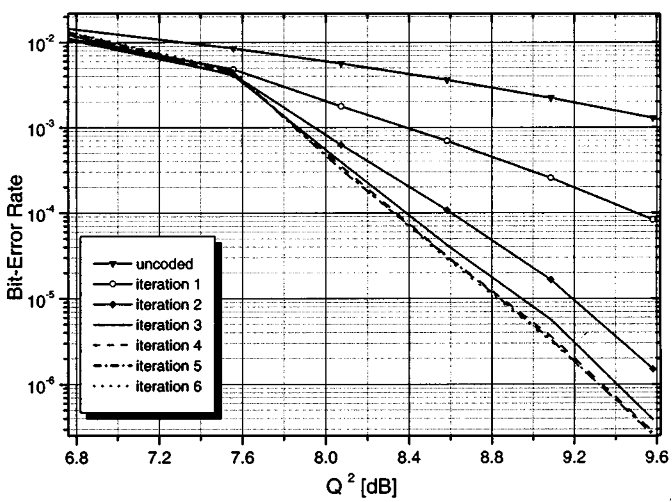

Fig. 3. BER versus Q-factor for LDPC (1369, 1260) code and a realistic receiver model.

composed of standard single-mode fiber (SMF) and dispersion-compensating fiber (DCF) sections giving the residual dispersion of 272 ps/nm. The average power per channel of 0 dBm and a nonreturn-to-zero format are assumed. The SMF attenuation coefficient, dispersion, dispersion slope, nonlinear refractive index and effective cross-sectional area are set to 0.21 dB/km, 17 ps$\cdot$ nm$^{-1}\cdot$ km^{-1}, 0.065 ps$\cdot$ nm$^{-2}\cdot$ km^{-1}, $2.6\cdot10^{-20}$ m^2/W and 80 μm^2, respectively. Corresponding DCF parameters are 0.5 dB/km, -100 ps $\cdot$nm$^{-1}\cdot$ km^{-1}, -0.33 ps $\cdot$nm^{-2} $\cdot$km^{-1}, $2.6\cdot10^{-20}$ m^2/W and 30 μm^2.

V. Conclusion

We have presented a FEC scheme based on LDPC codes. We demonstrate a significant coding gain of LDPC with respect to the state-of-the-art FEC schemes employed in optical communications systems. These codes have many unique features that may allow for very high-speed implementations. For example Hagenaruer *et al.* [12] realized that the sum-product algorithm is well suited for analog very large scale integration implementation. Not only does the code graph specify a natural layout, but also the sum-product operations are well matched to the natural nonlinear physical behavior of transistors. This kind of fast, analog iterative decoder is a very attractive option for optical communications.

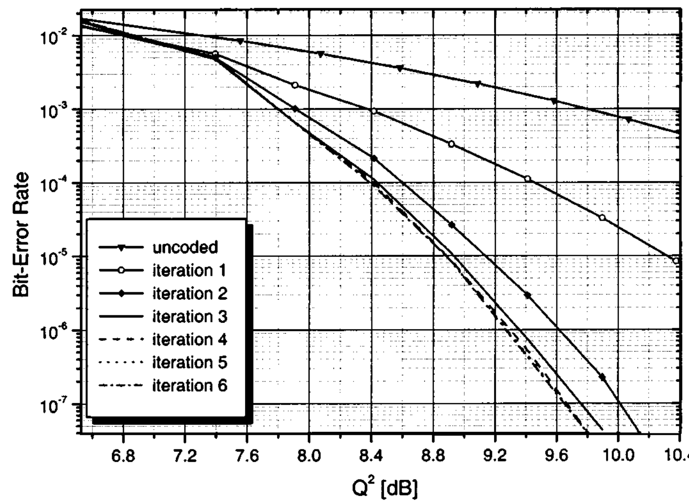

Fig. 4. BER versus Q-factor for LDPC (1369, 1260) code including the transmission medium and a realistic receiver model.

To improve overall system performances these new LDPC schemes can also be considered in conjunction with traditional algebraic decoding schemes such as BCH and RS codes, which may be required to further lower the BER. This approach is valuable not only because most of today's systems use algebraic codes, but also because the error floor of LDPC in the presence of burst of errors due to nonlinearities and signal dependent noise has not yet been assessed. These and the issues of high rate code constructions are left for the future research.

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