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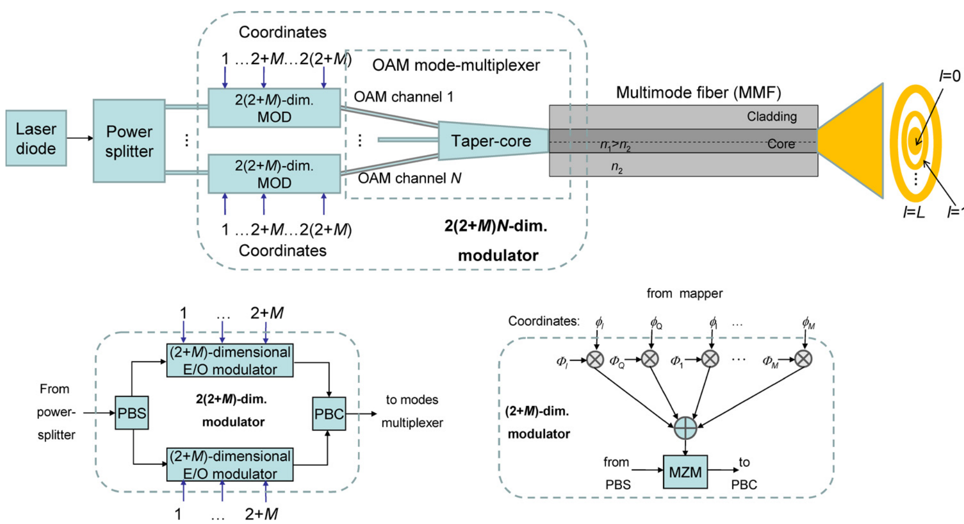
Ivan B. Djordjevic, Senior Member, IEEE
Lei Xu
Ting Wang



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Multidimensional Hybrid Modulations for Ultrahigh-Speed Optical Transport

Ivan B. Djordjevic,[1] *Senior Member, IEEE*, Lei Xu,[2] and Ting Wang[2]

[1]Department of Electrical and Computer Engineering, University of Arizona, Tucson, AZ 85721 USA
[2]NEC Laboratories America, Inc., Princeton, NJ 08540 USA

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Abstract: From Shanon's theory, we know that information capacity is a logarithmic function of signal-to-noise ratio (SNR) but a linear function of the number of dimensions. By increasing the number of dimensions D, we can dramatically improve the spectral efficiency. At the same time, in D-dimensional space $(D > 2)$, for the same average symbol energy, we can increase the Euclidean distance between signal constellation points compared with the conventional in-phase (I)/quadrature (Q) 2-D signal space. The 4-D space, with two phase coordinates per polarization, has already been intensively studied. To satisfy the ever-increasing bandwidth demands, in this paper, we propose the D-dimensional signaling $(D > 4)$ by employing all available degrees of freedom for transmission over a single carrier including amplitude, phase, polarization, and orbital angular momentum (OAM). The proposed modulation scheme can be called *hybrid* D-dimensional modulation as it employs all available degrees of freedom. The proposed hybrid 8-D coded-modulation scheme outperforms its 4-D counterpart by 3.97 dB at a bit error rate (BER) of 10^{-8} while outperforming its corresponding polarization-division-multiplexed (PDM) iterative polar quantization (IPQ)-based counterpart by even a larger margin of 6.41 dB (at the same BER). The improvement of the proposed scheme for two amplitude levels per dimension and $D = 8$ over conventional PDM 64-quadrature amplitude modulation (QAM) is indeed a striking 8.28 dB at a BER of 2×10^{-8}.

Index Terms: Multimode fibers (MMFs), hybrid modulations, low-density parity-check (LDPC) codes, coded-modulation.

1. Introduction

The invention of Internet has fundamentally changed the underlying information communication infrastructure and has led to the worldwide telecom boom in the late 1990s and early 2000s [1], [2]. The volume of Internet traffic continues to grow rapidly fueled by the emergence of new applications, thus increasing the demand for higher bandwidths. The exponential Internet traffic growth projections (for example by CISCO [2]) place enormous transmission rate demand on the underlying information infrastructure at every level, from the core to access networks. Because of this exponential traffic growth, some industry experts believe that the 100-Gb/s Ethernet (100 GbE) standard adopted recently (IEEE 802.3ba) [3] came too late and that 1-Tb/s Ethernet (1 TbE) standard should be available by 2013 [4].

From Shanon's theory, we know that information capacity is a logarithmic function of signal-to-noise ratio (SNR), but a linear function of the number of dimensions. By increasing the number of dimensions D, we can dramatically improve the spectral efficiency. At the same time, in D-dimensional space $(D > 2)$ for the same average symbol energy, we can increase the Euclidean distance between signal constellation points compared with the conventional in-phase (I)/quadrature (Q) 2-D signal space. The 4-D space, with two coordinates being in-phase/quadrature components in x-polarization (I_x and Q_x) and two coordinates being in-phase/quadrature components in y-polarization (I_y and Q_y), has already been intensively studied [5]–[7].

To satisfy the ever-increasing bandwidth demands, in this paper, we propose to use D-dimensional signaling $(D > 4)$ by employing all available degrees of freedom for conveyance of the information over a single carrier including amplitude, phase, polarization, and orbital angular momentum (OAM). The proposed scheme is called the D-dimensional *hybrid* modulation scheme because all available degrees of freedoms are used. The D-dimensional signal-constellation needed for D-dimensional signaling is obtained as the D-dimensional Cartesian product of a pulse-amplitude modulation (PAM) signal constellation. For each polarization, we employ $(2 + M)$-bases functions, which are defined in Section 2. When N orthogonal OAM modes are used in multimode fibers (MMFs) [9]–[12] or multicore fibers, the overall signal space is a $D = 2(2 + M)N$-dimensional, and spectral efficiency can be dramatically improved. To keep the receiver complexity reasonable low, it is possible to use $2(2 + M)$-dimensional signal space and then spatially multiplexed N signal streams. To demonstrate the capabilities of the proposed schemes, we show via Monte Carlo simulations that a hybrid 8-D coded modulation scheme outperforms its corresponding 4-D counterpart [5], [7] by 3.97 dB at the bit error rate (BER) of 10^{-8} while outperforming its corresponding conventional polarization-multiplexed iterative polar quantization (IPQ)-based (this is an optimum 2-D signal constellation in an amplified spontaneous emission noise dominated scenario, as shown in [8]) counterpart by even a larger margin of 6.41 dB (also at the BER of 10^{-8}). Notice that the recent proposal due to Liu *et al.* [13], which can be called pulse-position-multiplexed polarization-division-multiplexed (PDM) quadrature amplitude modulation (QAM), is essentially a multiplexing scheme. In this scheme, different PDM-QPSK streams are multiplexed together by using different time-slots. Our scheme, on the other hand, is a multidimensional *hybrid modulation* scheme, and orthogonal basis functions are used as coordinates for multidimensional signaling. Finally, the proposed scheme is applicable to both single-mode fiber (SMF) and multimode fiber (MMF) links.

The remainder of this paper is organized as follows. In Section 2, we describe the proposed scheme. We present our numerical results and discuss their importance in Section 3. Section 4 is devoted to concluding remarks.

2. Multidimensional Hybrid Coded Modulation

In this section, we provide a detailed description of our proposed multidimensional hybrid coded-modulation approach to enable ultrahigh-speed optical transport. As indicated introduction, the proposed coded-modulation employs all available degrees of freedom, namely, amplitude, phase polarization, and OAM. For each polarization, we employ the following $M + 2$ basis functions:

$$\phi_j(t) = \frac{1}{\sqrt{T_s/M}} \mathrm{rect}\left[\frac{t-(j-1)T_s/M}{T_s/M}\right];\ j = 1, \ldots, M \quad \mathrm{rect}(t) = \begin{cases} 1, & 0 \le t < 1 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

$$\Phi_I(t) = \sqrt{\frac{2}{T_s}} \cos(2\pi t/(T_s/M)) \quad \Phi_Q(t) = \sqrt{\frac{2}{T_s}} \sin(2\pi t/(T_s/M)) \tag{2}$$

where T_s is the symbol duration. In addition, we use N orthogonal OAM states for modulation. Therefore, the corresponding signal space is $2(2 + M)N$-dimensional. By increasing the number of dimensions, i.e., the number of orthogonal OAM basis functions and/or number of pulse-position basis functions, we can increase the aggregate data rate of the system while ensuring reliable transmission at these ultrahigh speeds using capacity-approaching low-density parity-check (LDPC) codes. Apart from increasing the aggregate data rate, an $2(2 + M)N$-dimensional space when compared with the conventional 2-D space can provide larger Euclidean distances between signal constellation points, resulting in improved BER performance.

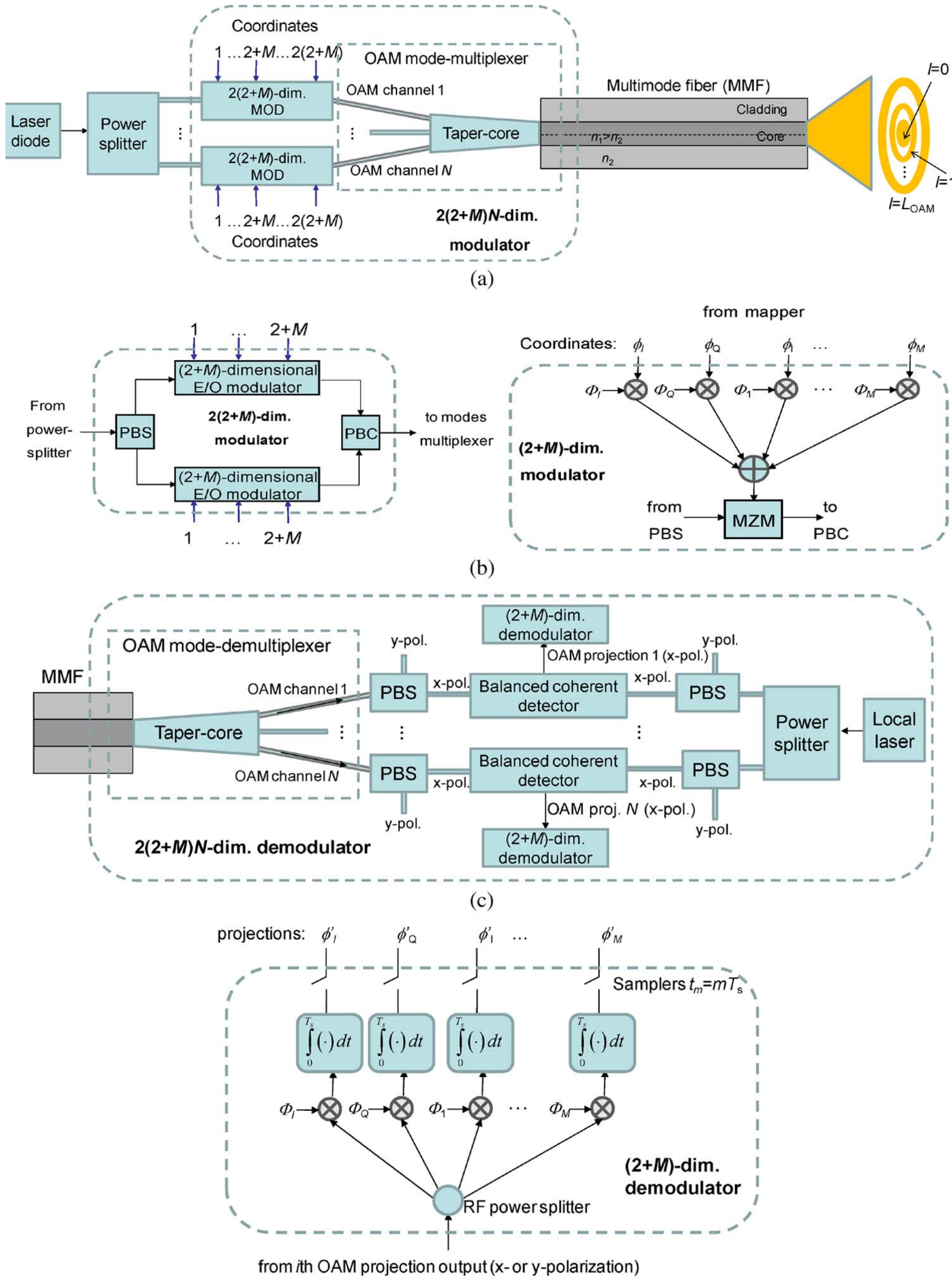

Fig. 1. Proposed multidimensional modulation scheme. (a) Transmitter configuration. (b) $2(2+M)$-dimensional modulator configuration. (c) $(2+M)N$-dimensional demodulator configuration. (d) $(2+M)$-dimensional demodulator configuration. [The excited OAM modes $l = 0, 1, \ldots, L_{\mathrm{OAM}}$ are shown in Fig. 1(a).]

The overall system configuration is depicted in Fig. 1. In Fig. 1(a), we show the transmitter architecture. A continuous wave laser diode signal is split into N branches by using a power splitter (such as $1:N$ star coupler) to feed $2(2+M)$-dimensional electrooptical modulators, each corresponding to one out of N OAM modes. The $2(2+M)$-dimensional electrooptical modulator is implemented as shown in Fig. 1(b). The OAM mode multiplexer is composed of N waveguides, taper-core fiber, and MMF, properly designed to excite orthogonal OAM modes in MMF (see [9] and

[10]). Namely, the azimuthal modes $u_{l,p}$, where $l = 0, 1, \ldots, L_{\text{OAM}}$ for fixed p, which are illustrated in Fig. 1(a), are mutually orthogonal as

$$(u_{m,p}, u_{n,p}) = \int u^*_{m,p}(r, \phi, z) u_{n,p}(r, \phi, z) r\, dr\, d\phi$$
$$= \begin{cases} \int |u_{m,p}|^2 r\, dr\, d\phi, & n = m \\ 0, & n \neq m \end{cases}; m, n \in \{0, 1, \ldots, L_{OAM}\} \tag{3}$$

and can be used as basis functions for multidimensional signaling. (In (3), r denotes the radial distance, ϕ denotes the azimuthal angle, and z denotes the propagation distance.) In addition, for $p = 0$, the intensity of a Laguerre–Gaussian (LG) mode is a ring of radius proportional to $|l|$, and as such, it can easily be detected by donut-shaped photodetector designed to capture the lth mode. The $2(2+M)$-modulator is composed of two $(2+M)$-dimensional modulators, i.e., one for each polarization, whose configuration is shown in Fig. 1(b) (on the right). The $(2+M)$-coordinates from the mapper are used as inputs to the modulator, properly multiplied with corresponding basis functions [see (1) and (2)], combined and after driver amplifier used as a radio-frequency (RF) input to the Mach–Zehnder modulator (MZM). Notice that the electrical portion of the modulator can be implemented as a DSP module followed by a digital-to-analog converter (DAC). For the first initial results of fabrication OAM mode multiplexer, see [11]. To facilitate this implementation, few mode fibers should be developed, which is an active research topic [12]. The $(2+M)$-dimensional modulator outputs are combined into a single OAM stream by the polarization beam combiner (PBC). The N OAM streams, such obtained, are combined by the OAM mode-multiplexer as described above. The $2(2+M)N$-dimensional demodulator architecture is shown in Fig. 1(c). We first perform OAM mode-demultiplexing in the OAM-demux block [see Fig. 1(c)], whose outputs are $2(2+M)$-dimensional projections along N OAM states.

The nth OAM projection is used as input to the polarization-beam splitter (PBS). The x- (y-) polarization output is used as input to the balanced coherent detector [the second input is the corresponding output that originates from the local laser, as shown in Fig. 1(c)]. The balanced coherent detector provides the $(2+M)$-dimensional signal used as input to the corresponding demodulator, shown in Fig. 1(d). The outputs of correlators, from Fig. 1(d), provide projections along corresponding basis functions [given by (1) and (2)]. Instead of the bank of correlators, we can use a bank of matched filters. This particular version is suitable for DSP implementation. Namely, we can use analog-to-digital conversion blocks and perform the match filtering in digital domain followed by dot-product calculation among the received vector and candidate constellation point vectors. Such obtained projections are used as inputs of the corresponding *a posteriori* probability demapper (APP), as shown in Fig. 2(d). The $(2+M)$-dimensional modulator [see Fig. 1(b)] should also be implemented in discrete-time (DT) domain by multiplying the coordinates with the DT version of basis functions, summing up the results of multiplications, performing oversampling, and, after digital-to-analog conversion of real and imaginary parts, use them as I- and Q-inputs of an I/Q modulator. The analog version shown in Fig. 1 is provided from pedagogical point of view to facilitate the explanation.

After this description of transmitter and receiver architectures, we turn our attention to the description of the corresponding multidimensional coded-modulation scheme, which is shown in Fig. 2. As shown in Fig. 2(a), K independent bit streams coming from different information sources are first encoded using binary (n, k) LDPC codes. The outputs of the encoders are then interleaved by a $K \times n$ block interleaver. The block interleaver accepts data from the encoders row-wise and outputs the data column-wise to the mapper that accepts K bits at the time instance i.

The multidimensional mapper determines the corresponding $L^{2(2+M)N}$-ary signal constellation point, where L is the number of amplitude levels per dimension, using

$$s_i = C_{2(2+M)N} \sum_{j=1}^{2(2+M)N} \phi_{i,j} \Phi_j. \tag{4}$$

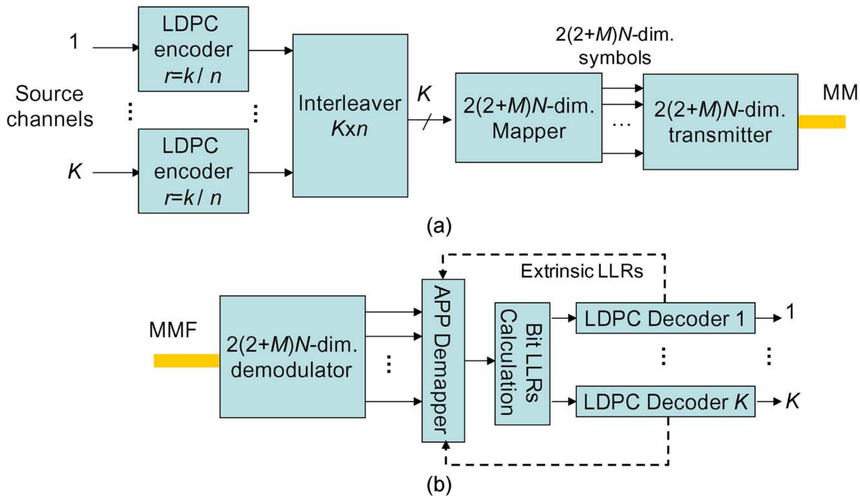

Fig. 2. Multidimensional LDPC-coded modulation scheme. (a) Transmitter and (b) receiver configurations.

In (4), the set $\{\Phi_1, \Phi_2, \ldots, \Phi_{2(2+M)N}\}$ represents the set of $2(2+M)N$ orthonormal basis functions, $C_{2(2+M)N}$ is a normalization factor, and $\phi_{i,j}$ are signal-constellation coordinates obtained as elements of the $2(2+M)N$-dimensional Cartesian product of PAM signal constellation

$$X^{2(2+M)N} = \underbrace{X \times X \times \cdots \times X}_{2(2+M)N \text{ times}} = \left\{\left(x_1, x_2, \ldots, x_{2(2+M)N}\right) | x_d \in X, \quad 1 \leq d \leq 2(2+M)N\right\} \tag{5}$$

where $X = \{2l - 1 - L | l = 1, 2, \ldots, L\}$. Each coordinate of the D-dimensional mapper output, where $D = 2(2+M)N$, is used as the corresponding RF input to the D-dimensional modulator [see Fig. 1(a)]. Finally, the modulated signals are sent over the MMF/multicore fiber system of interest after being combined into an optical wave via a mode-multiplexer, which was discussed above [see Fig. 1(a)]. Notice that the D-dimensional mapper is just a lookup table (LUT), which can be implemented using a single field-programmable gate array. In PDM-QAM, on the other hand, two LUTs are needed: one in each polarization. If somebody wants to use the OAM multiplexing, the number of required LUTs will be $2N$. Therefore, the complexity of D-dimensional mapper is lower than that that of conventional OAM-multiplexed PDM-QAM.

The number of bits per signal constellation point is determined by $\log_2(L^D) = K$. The dimensionality D, can be adjusted according to the desired final rate by varying the parameters M, N, and L. The aggregate data rate (per single wavelength) is determined by

$$\log_2\left(L^{2(2+M)N}\right) \frac{\text{ch.bits}}{\text{ch.sym.}} \times R_s \frac{\text{ch.sym.}}{\text{s}} \times r \frac{\text{info.bits}}{\text{ch.bits}} \tag{6}$$

where r is the code rate, which is assumed to be equal for LDPC codes at each level, and R_s is the symbol rate. For example, by setting the symbol rate to 12 GS/s, and the parameters L, M, and N all to 4, the aggregate data rate is 1152 Gb/s (for $r = 1$), which is compatible with 1-Tb/s Ethernet. Notice that the overall signal space is 96-dimensional so that the complexity of the receiver will be too high if Gaussian approximation for conditional probability density functions (PDFs) cannot be used. In this case, we can use a spatially polarization-multiplexed $(2+M)$-dimensional system instead.

The aggregate data rate of this system is given by

$$2N \times \log_2(L^{2+M}) \frac{\text{ch.bits}}{\text{ch.sym.}} \times R_s \frac{\text{ch.sym.}}{\text{s}} \times r \frac{\text{info.bits}}{\text{ch.bits}} \tag{7}$$

which is essentially the same as that given by (6). For the same parameters as in the previous example, the spatially polarization-multiplexed system achieves the same aggregate rate, while the corresponding signal space is 6-D, and the receiver complexity is reasonable low even in the presence of nonlinearities (when Gaussian approximation is not valid).

The receiver configuration for coherent detection is depicted as in Fig. 2(b). After D-dimensional demodulation, the corresponding projections are forwarded to the D-dimensional APP demapper. Notice that the proposed hybrid D-dimensional coded-modulation scheme requires only one D-dimensional demapper, while conventional coded OAM-multiplexed PDM-QAM requires $2N$ 2-D APP demappers. In the quasi-linear regime, when Gaussian approximation is sufficiently good, the complexity of the D-dimensional receiver is significantly simpler than that of coded OAM-multiplexed PDM-QAM. The D-dimensional demapper provides the symbol log-likelihood ratios (LLRs), which are used by the bit LLR calculation block to compute bit LLRs required for iterative decoding in binary LDPC decoders. To improve the overall system performance, we iterate extrinsic information between LDPC decoders and the APP demapper until convergence or until a predetermined number of iterations has been reached. Finally, the outputs of the K binary LDPC decoders are provided to the user as the estimates of the K information streams sent by the transmitter. Another difficulty for coded OAM-multiplexed PDM-QAM is that extrinsic information needs to be iterated between LDPC decoders and $2N$ APP demappers, which further increases the silicon area in application-specific integrated circuit implementation, compared with single D-dimensional APP demapper.

From the description of the transmitter and the receiver setups, it is clear that the system is scalable to any number of dimensions with small penalty in terms of BER performance, as long as the orthonormality among basis functions is preserved. The orthogonality among OAM modes in realistic multimode/multicore fibers can be reestablished by considering fibers as $N \times N$ MIMO systems. (For example, in [14], the 6×6 MIMO MMF system has been studied.) Because OAM modes are orthogonal to each other, the problem of OAM coupling can be solved in similar fashion as was done with PMD in PDM-QAM systems with SMF links by considering the problem as a 2×2 MIMO system [2]. Since we are concerned with the fundamental limits of the proposed scheme, we assume perfect compensation of OAM crosstalk. Notice that a similar approach was used by Karlsson and Agrell [15], which is related to SMF transmission. Given the fact that the MMF-EDFA can be implemented by OAM mode demultiplexing, independently amplifying each of OAM mode, and multiplexing them back into a single optical signal, the same assumptions applied in [15] are applicable here. Moreover, the proposed scheme is applicable in both SMF, in which case, we set $N = 1$ and MMF links when $N > 1$. As already mentioned above, the increase in the number of dimensions leads to an increase in complexity, and hence, a compromise between the desired aggregate rate and the receiver complexity should be made in practice. Notice that some industry experts [16] believe that SMF channel capacity will be reached by 2025. Therefore, it is of utmost importance to study alternative approaches to improve the total capacity in the fiber, and our paper represents a step forward in that direction.

The spectral efficiency of the proposed D-dimensional scheme, where $D = 2(2 + M)N$, is

$$\frac{S_E^{\mathrm{PDM}-D-\mathrm{dim.}}}{S_E^{\mathrm{PDM-QAM}}} = \frac{\log_2 L^D}{2\log_2 M_{\mathrm{QAM}}} = \frac{D\log_2 L}{2\log_2 M_{\mathrm{QAM}}} \tag{8}$$

times better than that of PDM-QAM. In (8), with M_{QAM}, we denoted the QAM signal constellation size. Therefore, for the same number of amplitude levels per dimension ($M_{\mathrm{QAM}} = L^2$), the spectral efficiency of the proposed scheme is $(D/4)$-times better than that of QAM. For example, for $M = N = L = 2$ ($D = 16$) and $M_{\mathrm{QAM}} = L^2 = 4$, the spectral efficiency of proposed scheme is 4 times better. (The comparison is, therefore, performed for the same number of amplitude levels per dimension.)

3. Performance Evaluation

In this Section, we evaluate the BER performance of the proposed hybrid multidimensional coded-modulation scheme by performing Monte Carlo simulations. The results of simulations are

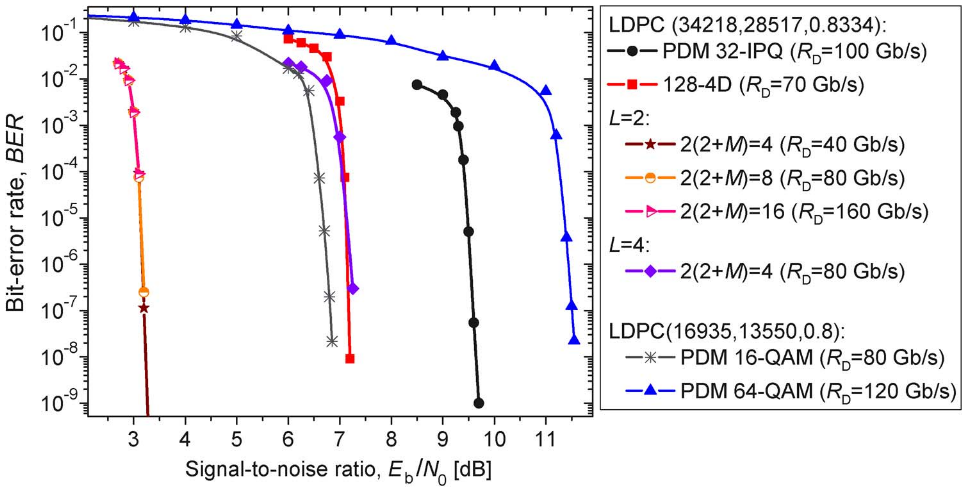

Fig. 3. BER performance of the $2(2+M)$-dimensional LDPC-coded modulation schemes per *single* OAM mode. The symbol rate used in simulations was $R_s = 12.5$ GS/s, and the signal bandwidth was 12.5 GHz. R_D denotes the aggregate data rate per OAM dimension.

summarized in Fig. 3, which are obtained for radial mode $p = 0$ and either two or four azimuthal modes. In simulations, the OAM multiplexer (demultiplexer) shown in Fig. 1(a) [see Fig. 1(c)] is used and designed as explained in [9]–[11]. The results of simulations are obtained in a back-to-back configuration, similarly as in [19], as we are concerned here with fundamental limits of proposed hybrid D-dimensional coded-modulation scheme. For transmission over MMFs, we will need to employ the MIMO approaches as discussed in [18] to restore the orthogonality of OAM modes. The residual, uncompensated, OAM crosstalk can be modeled by invoking the central limit theorem, and the bit SNR from Fig. 3, in this situation, will instead represent the signal-to-noise-and-OAM crosstalk ratio (SNCR). The SNCR can be related to SNR by

$$\text{SNCR [dB]} = 10\log_{10}\left(\frac{E_b}{N_0 + N_x}\right) = \text{SNR [dB]} - 10\log_{10}\left(1 + \frac{N_x}{N_0}\right),\ \text{SNR [dB]} = 10\log_{10}\left(\frac{E_b}{N_0}\right) \quad (9)$$

where E_b is the bit energy and N_0 is the power spectral density (PSD) of noise, while N_x is the PSD of OAM crosstalk.

We see that a spatially multiplexed 8-D coded modulation scheme, which is obtained by setting $L = M = 2$ and $R_s = 12.5$ GS/s for $N = 4$ OAM modes, outperforms its corresponding 4-D counterpart [5], [7] by 3.97 dB at the BER of 10^{-8}. The improvement of the proposed scheme obtained by setting $L = 2$, $M = 2$ over conventional PDM 64-QAM of lower aggregate data rate is indeed striking 8.275 dB at BER of 2×10^{-8}! The aggregate data rate of this scheme is 400 Gb/s, which is compatible with next-generation Ethernet. Notice that the aggregate data rate of 4-D scheme is only 4×12.5 Gb/s. The same scheme outperforms its corresponding optimum PDM IPQ-based counterpart [8] by a margin of 6.41 dB (also at the BER of 10^{-8}). The scheme with parameters $L = 2$, $M = 0$ outperforms the conventional PDM 16-QAM scheme (having the same number of constellation points) by 3.589 dB at a BER of 2×10^{-8}. By increasing the number of dimensions to 16, from Fig. 3, it is clear that the BER performance loss compared with the $D = 8$ case is negligible, provided that orthogonality among basis functions is preserved.

By setting $L = 4$, $M = 2$, and $R_s = 12.5$ for $N = 5$ OAM modes, the aggregated data rate of the spatially multiplexed 16-dimensional coded-modulation scheme is 1 Tb/s. Therefore, the proposed D-dimensional hybrid coded-modulation scheme is both 400-Gb/s and 1-Tb/s Ethernet enabling technology, while employing mature 10-Gb/s technology to perform encoding, decoding, and DSP. Because the operating symbol rate is 12.5 GS/s, this scheme is much more suitable than previous proposals, including the recent 100 G standard (IEEE 802.3ba) [3], which is based on 25-GS/s effective information symbol rate.

4. Conclusion

Inspired by the high potential of multidimensional signal constellations and recent demonstrations [9], [10] in which OAM modes are successfully excited in MMFs, we proposed the use of $2(2+M)N$-dimensional modulation as the next-generation Ethernet enabling technology. The 8-D coded-modulation scheme outperforms its corresponding 4-D counterpart by 3.97 dB at the BER of 10^{-8}, while outperforming its corresponding conventional PDM IPQ-based counterpart by even a larger margin of 6.41 dB (at the same BER). The improvement of the proposed scheme obtained for $L=2$, $M=2$ over conventional PDM 64-QAM, of lower aggregate data rate, is even 8.275 dB at BER of 2×10^{-8}! The spectral efficiency of the proposed scheme, for $M=N=L=2$ $(D=16)$ and $M_{\text{QAM}}=L^2=4$, with the same number of amplitude levels per dimension, is 4 times better. The recent demonstration [17] in which a 21.4-Gb/s signal was transmitted over 200 km of MMF, a recent demonstration where four independent data streams were successfully transmitted in four OAM modes [10], and a recent experiment in which 12.8-bit/s/Hz spectral efficiency is achieved by employing four LG modes [19], are leading to optimism that the proposed coded modulation scheme can be used for various applications ranging from short-haul to long-haul.

In terms of complexity, the proposed D-dimensional scheme requires one LUT to implement a D-dimensional mapper, while the corresponding OAM-multiplexed PDM-QAM counterpart requires $2N$ 2-D LUTs. On the receiver side, the proposed scheme requires one D-dimensional APP demapper, while the OAM-multiplexed PDM-QAM counterpart requires N 2-D APP demappers. Therefore, in the quasi-linear regime when Gaussian approximation is sufficiently good, the complexity of the proposed scheme is significantly lower than that of the OAM-multiplexed PDM-QAM.

As the final remark, notice that the proposed scheme is very flexible; it can be used in both i) SMF links, in which case, we set $N=1$, meaning that the corresponding signal space is $2(2+M)$-dimensional; and ii) MMF links, in which case, we set $N>1$, meaning that the signal space is $2N(2+M)$-dimensional. The proposed scheme can be used as a $2N(2+M)$-dimensional *modulation* scheme or as a multiplexing scheme in which N OAM streams, each carrying $2(2+M)$-dimensional signal streams, are multiplexed together. The particular choice of parameters depends on the application of interest.

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