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Spectral Amplitude-coding Optical CDMA Systems Based on Steiner Systems

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Summary

In spectral amplitude-coding optical code-division multiple-access (CDMA) the unipolar codes having fixed in-phase cross-correlation are very important as they are able to eliminate multiuser interference and suppress the phase-induced intensity noise. A novel class of unipolar codes, having ideal in-phase cross-correlation, for spectral-amplitude coding optical CDMA systems based on Steiner systems is proposed. It offers a significant improvement of the system performance with respect to previously reported codes.

1 Introduction

Among many multiple-access techniques, the optical code division multiple-access (CDMA) is the most robust as the speed and the scale of information processing elements grow and the number of nodes reaches the order of hundreds to thousands [1–7]. Optical CDMA offers several attractive features like asynchronous access, privacy and security in transmission, ability to support variable bit rate and bursty traffic, scalability of the network, etc.

Since CDMA systems can support asynchronous bursty traffic, transparency to overlaid protocols and decentralized operations they are suitable for local area networks (LAN) and possibly metropolitan area networks (MAN). Discrimination of unwanted signal is achieved by assigning minimally interfering codes (address sequences, signature sequences) to each user, selected from a family of so called optical orthogonal codes.

The main factor of performance degradation is the multiuser interference (MUI). In spectral amplitude-coding (SAC) optical CDMA systems MUI is solely a function of the in-phase cross-correlation values among the address (signature) sequences [2–7]. If in-phase cross-correlation among the address sequences is fixed, then the balanced detection receiver proposed in [5] is able to suppress MUI completely. Hadamard code, m-sequence, modified quadratic congruence code (MQC) and modified frequency hopping (MFH) codes are proposed so far for such applications [2-7]. The paper [7] gives an excellent review of the current constructions of codes for SAC applications.

In this paper a novel ideal in-phase cross-correlation code suitable for spectral amplitude-coding CDMA systems is proposed. It is constructed using Steiner systems. We show that the Steiner system based code families outperform the Hadamard code, MQC and MFH. A novel simplified fiber-Bragg grating (FBG) based SAC scheme capable to suppress MUI is proposed, and a universal formula for performance determination applicable to any class of code having fixed in-phase cross-correlation is derived.

2. System description and problem statement

The simplest OCDMA network consists of N transmitter/receiver pairs connected in a star configuration [1]. To send the information from i^{th} to j^{th} user, the address code for the receiver j is impressed upon data by the encoder at the i^{th} node. The transmitter and receiver structures based on FBGs are shown in Fig. 1. Notice that our encoding/decoding scheme is simpler in implementation than previously proposed in [5]. When bit "1" is sent an optical pulse from a broadband source is launched into encoder, no optical pulse is launched for data bit "0". The optical pulse passes through the linear FBG array in encoder and corresponding spectral components, according to the spectral distribution $A(\lambda)$, are reflected. For the reconfiguration of destination address code all gratings in encoder are tunable. At the receiver, each grating is fixed according to receiver s address. For proper decoding, the peak wavelengths are arranged in opposite order so that round-trip delays of different spectral components are compensated (all reflected components have the same delay and can be merged into a pulse again). The output from the top of a linear FBG array in receiver is used as the complementary-code-correlated output $\bar{A}(\lambda)S(\lambda)$, with $S(\lambda)$ being the received signal spectrum and $\bar{A}(\lambda)$

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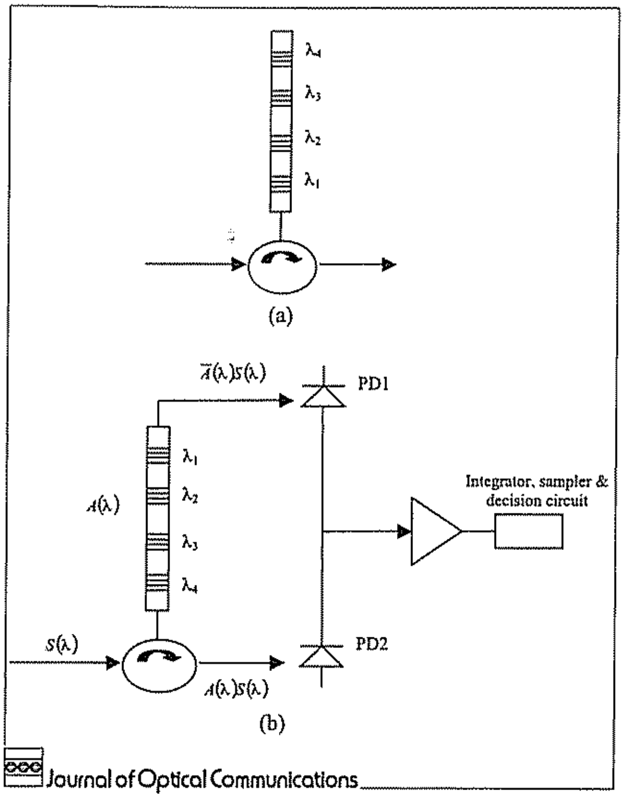

Fig. 1: SAC optical CDMA based on FBGs: (a) encoder, (b) decoder

being the complement of receiver address sequence (non-reflected components passes FBG unchanged). MUI can be canceled using the balanced detector when the in-phase cross-correlation between any two codewords is the same. Therefore, the problem of constructing code sequences having fixed in-phase cross-correlation between any two codewords is of special interest for SAC schemes.

Let $x = (x_1, x_2, \dots, x_v)$ and $y = (y_1, y_2, \dots, y_v)$ be two different code sequences, and let the in-phase cross-correlation between them be defined as $\lambda = \Sigma_{n=1}^{v} x_n y_n$. The code having fixed in-phase cross-correlation between any two codewords (address sequences) is denoted as (v, κ, λ), where v, and κ are codeword length and weights, respectively (λ is the previously defined in-phase cross-correlation). Since $\lambda = 1$ is the minimum value that can be achieved it is called here the ideal in-phase cross-correlation constraint, similarly as in [7].

In spectral amplitude-coding optical CDMA system, although MUI is canceled by balanced reception, another important factor in performance degradation is the phase-induced intensity noise [3–5], which comes from spontaneous emission of a broadband source. In order to suppress it the in-phase cross-correlation constraint should be kept as low as possible, and therefore the codes construction having the ideal in-phase cross-correlation become very attractive for SAC applications [4–7].

In the next section several constructions based on Steiner systems are described.

3. Ideal in-phase cross-correlation codes based on Steiner systems

In general, in combinatorics a design is a pair (V,B), where V is a set of some elements called points, and B is a collection of subsets of V called blocks. The numbers of points and blocks are denoted by $v = |V|$ and $b = |B|$, respectively. If $t \le v$ is an integer parameter, such that any subset of t points from V is contained in exactly λ blocks, we deal with a t-design. A Balanced Incomplete Block Design (BIBD) is a t-design for which each block contains the same number of points k, and every point is contained in the same number of blocks r.

In this paper, we consider only BIBD with $t = 2$. Although such a BIBD still has five integer parameters v, k, λ, b and r, only three of them are independent, since

$$b \cdot k = v \cdot r, \quad (1)$$

$$r(k-1) = \lambda(v-1), \quad (2)$$

(the proof is quite straightforward; see, for example [8]). Therefore, given three parameters we can always find the other two parameters using (1) and (2). The notation (v, k, λ)-BIBD is used for a BIBD with v points, block size k, index λ and $t = 2$. A (v, k, 1)-BIBD with $\lambda = 1$ is called a Steiner system. A Steiner system with $k = 3$ is called a Steiner triple system. A BIBD is resolvable if there exists a nontrivial partition of its blocks set B into parallel classes each of which partitions the point set V. The resolvable Steiner triple system is called Kirkman system. These combinatorial objects originate from Kirkmans famous schoolgirl problem posted in 1850 [9], and have many interesting properties.

For example the collection $B = \{B_1, B_2, \dots, B_7\}$ of blocks $B_1 = \{0, 1, 3\}$, $B_2 = \{1, 2, 4\}$, $B_3 = \{2, 3, 5\}$, $B_4 = \{3, 4, 6\}$, $B_5 = \{0, 4, 5\}$, $B_6 = \{1, 5, 6\}$ and $B_7 = \{0, 2, 6\}$ is a BIBD(7, 3, 1) system or a Kirkman system with $v = 7$, and $b = 7$. Labels within a block defines the positions of ones within a codeword, so that corresponding codewords are: (1 1 0 1 0 0 0), (0 1 1 0 1 0 0), (0 0 1 1 0 1 0), (0 0 0 1 1 0 1), (1 0 0 0 1 1 0), (0 1 0 0 0 1 1), (1 0 1 0 0 0 1). The number of codewords in the Steiner system, determined using the expressions (1) and (2), is $b = v(v-1)/k(k-1)$.

The classes of known Steiner system families, not necessarily resolvable, giving the ideal in-phase cross-correlation sequences are listed in table 1 [9].

Table 1: Known infinite Steiner system families

k	V	Name
q	q^n ($n \ge 2$, q – a prime power)	Affine geometries
q+1	$(q^n - 1)/(q - 1)$ ($n \ge 2$, q – a prime power)	Projective geometries
q+1	$q^3 + 1$ (q – a prime power)	Unitals
2 to m	$2^m(2^s + 1) - 2^s$ ($2 \le m < s$)	Denniston designs

Table 2: The orbit of base block {0, 1, 3, 9} is a (13, 4, 1) BIBD

{0, 1, 3, 9} Orbit			
$b_1 + g$	$b_2 + g$	$b_3 + g$	$b_3 + g$
0	1	3	9
1	2	4	10
2	3	5	11
3	4	6	12
4	5	7	0
5	6	8	1
6	7	9	2
7	8	10	3
8	9	11	4
9	10	12	5
10	11	0	6
11	12	1	7
12	0	2	8

In the following paragraph we present a construction based on difference families (more details on difference families can be found in [8–9]).

Let V be an additive Abelian group of order v. Then k-element subset of G, $B_1 = \{b_{1,1}, \ldots, b_{1,k}\}$ forms a (v, k, 1) difference family (DF) if every nonzero element of G can be represented as a difference of two elements from B_1, i.e. occurs ones among the differences $b_{1,m} - b_{1,n}$, $1 \le m, n \le k$. The set B_1 is called the base block. If V is isomorphic with Z_v,a group of integers modulo v, then a (v, k, 1) DF is called here a cyclic difference family (CDF) or orbit. For example, the block $B_1 = \{0, 1, 3\}$ is a base block of a (7, 3, 1) CDF. To illustrate this, we create an array $\Delta = (\Delta_{i,j})$, of differences $\Delta_{i,j} = b_{1,i} - b_{1,j}$

$$\Delta = \begin{bmatrix} 0 & 1 & 3 \\ 6 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}$$

As it can be seen, each nonzero element of Z_7 occurs only once in Δ. Given the base block, the blocks of orbit B containing the base block B_1 can be calculated as $B_g = \{b_{1,1} + g, \ldots, b_{1,k} + g\}$ where $g \in V$. A construction of a code is completed by creating all possible blocks for the base block. For example, it can be easily verified (by creating the array Δ) that the block $B_1 = \{0, 1, 3, 9\}$ is the base block of a (13, 4, 1) CDF of a group $V = Z_{13}$. The orbit is given in table 2. The labels in table 2 denotes, as mentioned earlier, the positions of ones in corresponding codewords. The number of codewords is determined by the codeword length, $b = v = k^2 - k + 1$, $k = q + 1$ (q is a prime power).

4 Performance analysis

The signal-to-noise ratio (SNR) of the spectral-amplitude-coding system employing any of the BIBD class of fixed in-phase cross-correlation is calculated by using the method described in [3, 5, 7], and is given by the following formula

$$\mathrm{SNR} = \frac{R^2 P_{sr}^2 k^2 / v^2}{eBRP_{sr}\dfrac{k + (\lambda+1)(N-1)}{v} + \dfrac{BR^2P_{sr}^2 kN}{2v^2\Delta f}\left(\dfrac{N-1}{k-\lambda} + k + \lambda(N-1)\right) + \dfrac{4k_B T_r B}{R_L}} \qquad (3)$$

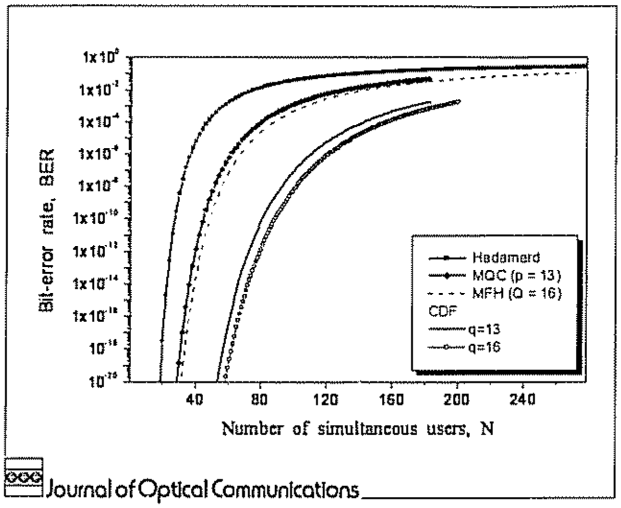

Fig. 2: Bit-error rate versus number of simultaneous users for different code families

where R is the photodiode responsivity, P_{sr} is the effective power of a broad-band source at the receiver, e is an electron charge, B – the electrical equivalent noise bandwidth of the receiver, k_B – the Boltzmanns constant, T_r- the absolute temperature of receiver noise, R_L – the load resistance, Δf – the optical source bandwidth, N is the number of simultaneously active users, and v, k and λ are the parameters of the BIBD. The connection between BIBD parameters and proposed ideal in-phase cross-correlation families is given in corresponding section III. The phase induced intensity noise, the photodiode shot noise and the thermal noise are taken into account. The balanced detection scheme capable of suppressing the multiuser interference, shown in Fig. 1, is considered.

Bit-error rate can be calculated using the Gaussian approximation [3, 5, 7]

$$P_e = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\mathrm{SNR}/8}\right) \qquad (4)$$

where erfc(.) is the complementary error function $\mathrm{erfc}(x) = \left(2/\sqrt{\pi}\right)\int_x^{\infty} e^{-u^2} du$.

The performance of the proposed DCF family in the presence of the phase induced intensity noise, the photodiode shot noise and the thermal noise is illustrated in Fig. 2. The system parameters are: Δv = 3.75 THz, B = 80 MHz, bit-rate 155 Mb/s, central wavelength 1550 nm, T_r = 300 K, R_L = 1030 Ω, photodiode quantum efficiency 0.6. The performance comparison of the Steiner system based CDF SAC optical CDMA system, Hadamard code [3], modified quadratic congruence code [5] and modified frequency hopping code [7] can be made from the same Figure. Simple DCF code significantly outperforms so far proposed codes. Notice that DCF codes can support smaller number of users than MQC or MFH code, although neither one code family can be used for number of user greater than 180 due

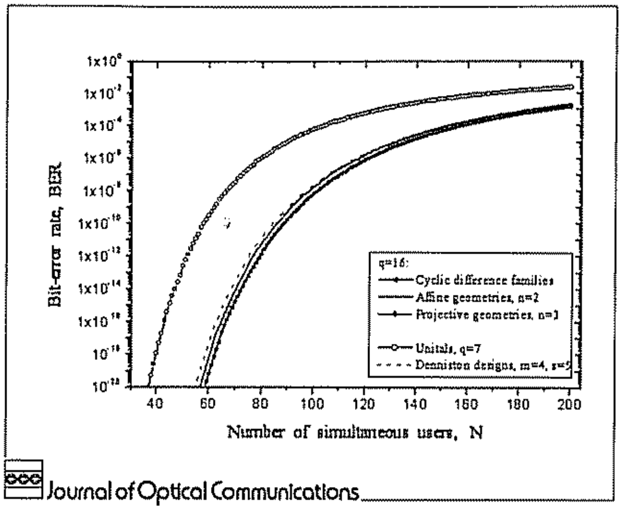

Fig. 3: Bit-error rate versus number of simultaneous users for different Steiner system families

to too poor performance (for given parameters). The comparison among different Steiner system families is shown in Fig. 3. The comparison is made for comparable codewords weights and lengths, except for unitals whose weight was set lower in order to have comparable lengths. Excluding unitals, different Steiner families perform comparable.

5 Conclusion

A novel construction of unipolar codes, having ideal in-phase cross-correlation, for spectral-amplitude coding optical CDMA systems based on Steiner systems is proposed. The system performance is significantly improved by using the Steiner system based codes instead of previously reported codes such as Hadamard code, MQC and MFH. A novel simplified fiber-Bragg grating based spectral-amplitude coding scheme capable to suppress MUI is proposed, and a universal formula for the required SNR as a function of number of users and code parameters is derived. The SNR formula is applicable to any class of codes having fixed in-phase cross-correlation.

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