Iteratively Decodable Codes on m Flats for WDM High-Speed Long-Haul Transmission

Sundararajan Sankaranarayanan, *Student Member, IEEE,* Ivan B. Djordjevic, *Member, IEEE,* and Bane Vasic, *Senior Member, IEEE*

***Abstract*—In an earlier paper, we reported that the low-density parity-check (LDPC) codes from finite planes outperform any other known forward error-correction (FEC) scheme for optical communications. However, the number of different LDPC codes of code rate above 0.8 is rather small. As a natural extension of the prior work, in this paper, we consider LDPC codes on m flats derived from projective and affine geometries, which outperform codes from finite planes. The codes on m flats also provide a greater selection of structured LDPC codes of rate 0.8 or higher. The performance of the codes in a long-haul optical-communication system was assessed using an advanced simulator able to capture all important transmission impairments. Specifically, they achieve a coding gain of 10 dB at a bit error rate (BER) of 10^{-9}, outperforming, therefore, the best turbo product codes proposed for optical communications. In addition, the simulator implements a fixed-point (FP) iterative decoder that allows control of the precision of the soft information used in the decoder. Such quantization is required to facilitate hardware implementations of the iterative decoder, and the high-speed operations for long-haul optical transmission systems. The loss in performance due to reduced precision of the soft information can be as low as 0.2 dB.**

***Index Terms*—Finite geometries, forward error correction (FEC), low-density parity-check (LDPC) codes, m flats, optical communications.**

I. Introduction

In RECENT years, low-density parity-check (LDPC) codes have generated great interests in the coding community [6]–[10], and this has resulted in a great deal of understanding of the different aspects of the code and the decoding process. An iterative LDPC decoder based on the sum–product algorithm (SPA) has been shown to achieve a performance of as close as 0.0045 dB to the Shannon limit [6]. The inherent low complexity [11]–[14] of this decoder opens up avenues for its use in different high-speed applications, such as optical communications.

In a series of recent articles [1]–[5], we showed that error performance and decoder hardware complexity offered by turbo codes [15]–[17] can be greatly improved by using iteratively decodable LDPC codes. Although several methods have been proposed to construct "good" LDPC codes, the complexity of LDPC encoders and decoders can be considerably reduced by allowing cyclic or quasi-cyclic structures in the parity-check matrices of the codes. These symmetries are especially critical to enable high-speed forward error-correction (FEC) architectures for optical communications. The cyclic or quasi-cyclic structures of these codes support simple encoders, realized using shift registers, and low-complexity iterative decoders. In [18], we presented a construction method based on the incidence of points and lines of a finite projective plane, and in [1], we extended the above construction method to include affine planes and some secondary structures in projective planes like ovals and unitals. In [3]–[5] and [19]–[21], we proposed several LDPC designs based on integer lattice, mutually orthogonal Latin squares and rectangles, and block-circulant parity-check matrices. These codes have shown encouraging performances, with a coding gain of larger than 10 dB at a bit error rate (BER) of 10^{-9}, outperforming the best turbo codes [17] proposed for optical-communication systems.

The next step of generalization of these codes can be obtained by considering incidence properties of higher dimensional flats over finite (projective and affine) geometries. In [1], LDPC codes are constructed based on the incidence of points (zero-dimensional flat) and lines (one-dimensional flat) in a plane (two-dimensional flat) of projective and affine geometries. In this paper, we construct LDPC codes based on the incidence of m-dimensional flats, where m is not necessarily 0, and $(m+1)$-dimensional flats in an n-dimensional flat, where $n > m+1$, of projective and affine geometries. Apart from excellent coding, these LDPC designs offer a larger number of codes of rate above 0.8. This approach was first suggested by Tang *et al.* [22], and the BER performances of these codes over an additive white Gaussian noise (AWGN) channel presented in the paper were impressive.

In order to make a realistic assessment of the code's performance in long-haul optical communication system, an advanced simulator was developed [1], [23], which successfully captures the effects of important transmission impairments such as fiber nonlinearites [interchannel (four-wave mixing, cross-phase modulation) and intrachannel (intrachannel four-wave mixing, intrachannel cross-phase modulation, self-phase modulation) nonlinearities, stimulated Raman scattering], chromatic-dispersion effects [group-velocity dispersion (GVD), second-order GVD], linear crosstalk effects, (optical and electrical) filtering effects [intersymbol interference (ISI)], amplified spontaneous emission (ASE) noise, and others.

The LDPC decoder iteratively uses soft information, referred to as log-likelihood ratio (LLR), in order to decode the output of the channel. This soft information used in the iterative decoder has to be quantized in order to facilitate digital hardware implementations [30]. The performance loss using a

Manuscript received November 15, 2004; revised June 20, 2005.

The authors are with the Department of Electrical and Computer Engineering, University of Arizona, Tucson, AZ 85721 USA (e-mail: ssundar@ece.arizona.edu; ivan@ece.arizona.edu; vasic@ece.arizona.edu).

Digital Object Identifier 10.1109/JLT.2005.857736

fixed-point (FP) representation for quantized LLRs is assessed as well in this paper. Another approach, which is the "bio-inspired network-decoding" approach [31], has received greater attention in recent years. This approach is promising from the system-level-accuracy, speed, and power-consumption standpoints [31].

In Section II, we present a brief introduction to finite geometries, and outline the algorithm to construct LDPC codes on m flats from these geometries. In Section III, we briefly describe the channel model implemented, and in Section IV, we present the BER performance of the proposed codes. In Section V, the BER performance is assessed by varying the precision setting for LLRs. The final section summarizes the major results of this paper.

II. Iteratively Decodable Codes on m Flats

In this section, we present algorithms to construct two classes of LDPC codes based on the projective and affine geometries of finite-dimensional vector spaces over finite fields. A detailed discussion on finite geometries can be found in [26]–[28]. The following notation will be used in the paper:

1) p is a prime integer, and q is the power of p.
2) GF(q) is a Galois field of order q.
3) m and l are integers less than n, where $l < m$.

The projective geometry of dimension n over GF(q), represented as PG(n, q), is a set of all proper subspaces of a vector space of dimension $n + 1$ over GF(q). In projective-geometric terminology, an $(m + 1)$-dimensional element of the set is referred to as an m flat. For example, points are 0 flats, lines are 1 flats, planes are 2 flats, and m-dimensional spaces are $(m - 1)$ flats. An l-dimension flat is said to be contained in an m-dimension flat if it satisfies a set-theoretic containment. Since any m flat is a finite vector subspace, it is justified to talk about a basis set composed of basis elements independent in GF(p). In the following discussion, the basis element of a 0 flat is referred to as a point and, generally, this should cause no confusion because this basis element is representative of the 0 flat.

Using the above definitions and linear algebraic concepts, it is straightforward to count the number of m flats, defined by a basis with $m + 1$ points, in PG(n, q). The number of such flats, say $N_{\mathrm{PG}}(m, n, q)$, is the number of ways of choosing $m + 1$ independent points in PG(n, q) divided by the number of ways of choosing $m + 1$ independent points in any m flat.

$$\begin{aligned} N_{\mathrm{PG}}(m, n, q) &= \frac{(q^{n+1}-1)(q^{n+1}-q)(q^{n+1}-q^2)\dots(q^{n+1}-q^m)}{(q^{m+1}-1)(q^{m+1}-q)(q^{m+1}-q^2)\dots(q^{m+1}-q^m)} \\ &= \prod_{i=0}^{m} \frac{(q^{n+1-i}-1)}{(q^{m+1-i}-1)}. \end{aligned}$$

Hence, the number of 0 flats in PG(n, q) is $N_{\mathrm{PG}}(0, n, q) = (q^{n+1} - 1)/(q - 1)$, and the number of 1 flats is $N_{\mathrm{PG}}(1, n, q) = [(q^{n+1} - 1)(q^n - 1)] / [(q^2 - 1)(q - 1)]$. When $n = 2$, the number of points is equal to the number of lines, and this agrees with dimensions of point–line incidence matrices of projective plane codes, introduced in [1]. In [1], we considered point–line incidence matrices to construct cyclic LDPC codes, while in this paper, we consider the incidence of m flats and l flats in PG(n, q).

An algorithm to construct an m flat in PG(n, q) recognizes that the elements of GF(q^{n+1}) can be used to represent points of PG(n, q) [24]. If α is the primitive element of GF(q^{n+1}), then α^v, where $v = (q^{n+1} - 1)/(q - 1)$, is a primitive element of GF(q). Each one of the first v powers of α can be taken as the basis of one of the 0 flats in PG(n, q). In other words, if α^i is the basis of a 0 flat in PG(n, q), then every α^k, such that $i \equiv k \bmod v$, is contained in the subspace. In a similar fashion, a set of $m + 1$ powers of α that are independent in GF(q) forms a basis of an m flat. If $\alpha^{s_1}, \alpha^{s_2}, \dots, \alpha^{s_{m+1}}$ is a set of $m + 1$ basis elements of an m flat, then any point in the flat can be written as

$$\zeta_i = \sum_{j=1}^{m+1} \varepsilon_{ij} \alpha^{s_j}$$

where the ε_{ij} are chosen such that no two vectors $\langle \varepsilon_{i1}, \varepsilon_{i2}, \dots, \varepsilon_{i(m+1)} \rangle$ are linear multiples over GF(q). Now, every ζ_i can be equivalently written as a power of the primitive element α.

To construct an LDPC code, we generate all m flats and l flats in PG(n, q) using the method described above. An incidence matrix of m flats and l flats in PG(n, q) is a binary matrix with $N_{\mathrm{PG}}(m, n, q)$ rows and $N_{\mathrm{PG}}(l, n, q)$ columns. The (i, j)th element of the incidence matrix $\mathbf{A}_{\mathrm{PG}}(m, l, n, q)$ is a one if and only if the jth l flat is contained in the ith m flat of the geometry. An LDPC code is constructed by considering the incidence matrix or its transpose as the parity-check matrix of the code. It is widely accepted that the performance of an LDPC code under an iterative decoder is deteriorated by the presence of four cycles in the Tanner graph [25] of the code. In order to avoid four cycles, we impose an additional constraint that l should be one less than m [22]. If l is one less than m, then no two distinct m flats have more than one l-dimensional subspace in common. This guarantees that the girth (length of the shortest cycle) of the graph is 6.

Example: Let us construct the incidence matrix of 2 flats and 1 flats in PG(3,2). For some primitive element α of GF(2^3), each element of the set $P = \{\alpha^0, \alpha^2, \alpha^1, \dots, \alpha^{14}\}$ is a generator of a unique 0 flat in the geometry, and this agrees with the fact that there are $N_{\mathrm{PG}}(0, 3, 2) = 15$ points in the geometry. There are $N_{\mathrm{PG}}(0, 2, 2) = 7$ points in each 2 flat of the geometry, and all points in one of those flats can be generated as linear combinations of $\{\alpha^0, \alpha^1, \alpha^3\}$ in GF(2). By working out all possible linear combinations, one arrives at $\{\alpha^0, \alpha^1, \alpha^3, \alpha^4, \alpha^7, \alpha^9, \alpha^{14}\}$ as the set of all points in the 2 flat. Using the above procedure, all m flats and l flats in the geometry can be generated. The incidence matrix of the m flats and l flats in PG(3, 2) is shown at the bottom of the next page. As mentioned before, there are no four cycles in the Tanner graph of the incidence matrix. This matrix or its transpose will be used as the parity-check matrix of a projective-geometry code.

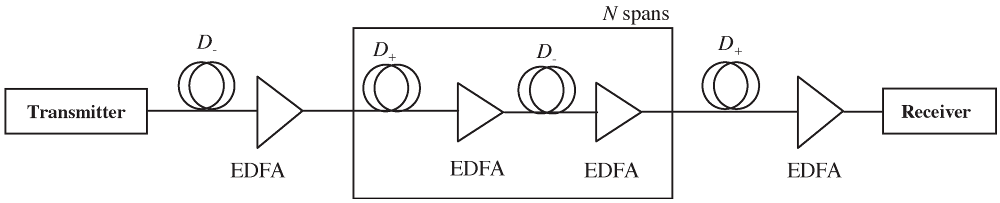

Fig. 1. Dispersion map under study.

The affine geometry of dimension n over GF(q), represented as AG(n, q), is a set of all cosets of the proper subspaces of a vector space of dimension n over GF(q). A coset of a subspace U is defined as the set $\mathbf{x} + U$, where $\mathbf{x}$ is any vector in the space of dimension n over GF(q), and its dimension is equal to the dimension of U. Unlike the projective geometry, an m-dimensional element of the set is referred to as an m flat. For example, points are 0 flats, lines are 1 flats, and planes are 2 flats. Note that points are, in fact, cosets of the unique zero-dimension subspace. Similar to the projective geometry, an l-dimension flat is said to be contained in an m-dimension flat if it satisfies a set-theoretic containment.

One of the interesting properties of this geometry is the concept of parallelism. It can be shown that cosets of the same dimension are parallel if and only if they are cosets of the same subspace [26]–[28]. The construction of AG(n, q) is simplified by the fundamental embedding theorem. The affine geometry of dimension n is obtained from a projective geometry of dimension n by removing a hyperplane and all subspaces contained in it. In other words, to create an m flat in AG(n, q), follow this procedure.

1) Construct a hyperplane in PG(n, q).
2) Construct an m flat in PG(n, q).
3) Remove all points in the hyperplane contained in the m flat.

In order to construct all flats in AG(n, q), fix the hyperplane in PG(n, q) and remove all points in this hyperplane from all flats in PG(n, q). An incidence matrix of m flats and l flats in AG(n, q) is defined similar to the projective-geometry case. Also, the incidence matrix or its transpose can be used as a parity-check matrix that defines an LDPC code. Also, m is constrained to be one more than l in order to avoid four cycles in the resultant LDPC code.

III. Simulation Model Description

The system of interest in this paper is a wavelength division multiplexing (WDM) system. The continuous-wave laser signals at different wavelengths are modulated using independently encoded electrical streams and a Mach–Zehnder (MZ) modulator, WDM multiplexed, and transmitted over the same fiber. The carrier-suppressed return-to-zero (CSRZ) modulator employed is composed of a laser diode, two MZ intensity modulators [the first serving as modulator, the second as a non-RZ (NRZ)-to-RZ converter], a pseudorandom bit sequence (PRBS) generator, and an encoder. The optical signal at the receiver side is split into separate channels by using an optical demultiplexer. Erbium-doped fiber amplifiers (EDFAs) and dispersion-compensating fibers (DCFs) are deployed periodically to compensate the loss and accumulated dispersion of the standard single-mode fiber (SMF). The direct-detection receiver observed is composed of a WDM demultiplexer, a p-i-n photodiode, an electrical filter, and a sampler followed by a decoder. An EDFA is used as a preamplifier.

The propagation of a signal through the transmission media is modeled by a nonlinear Schrödinger equation [29] and solved using the split-step Fourier method. For more details on the transmission-system model implemented, the reader is referred to our previous papers [1], [23].

$$\mathbf{A}_{\mathrm{PG}}(2,1,3,2) = \begin{bmatrix}
1&0&0&0&1&1&0&0&1&0&0&1&0&1&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
1&1&0&1&0&0&0&1&0&0&1&0&0&0&0&0&0&0&1&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&1&1&0&1&0&0&0&0&0&0&0&0&0&1&0&0&0&0&1&0&0&0&0&0&0&0&0&0&1&0&0&0&0&1\\
0&1&0&0&0&1&1&0&0&0&0&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&1&0&0&0&0&0&1&0\\
1&0&1&0&0&0&1&0&0&1&0&0&1&0&0&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&1&0&1&0&0&0&0&0&0&0&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&0&0&0\\
0&0&0&0&0&0&0&0&1&0&1&0&1&0&0&1&0&0&0&0&0&0&0&0&0&1&0&1&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&1&1&1&0&0&0&0&1&0&1&1&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&1&1&0&0&0&0&0&0&1&1&0&0&0&0&0&0&0&0&0&0&1&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&0&0&1&0&1&1&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&1&1&0&0&0&0&0&0&0&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&0&0&1&1&0&1&0&1&0&0&0\\
0&0&1&1&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&0&1&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&0&0&1&0&1&1&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&0&0&1&0
\end{bmatrix}$$

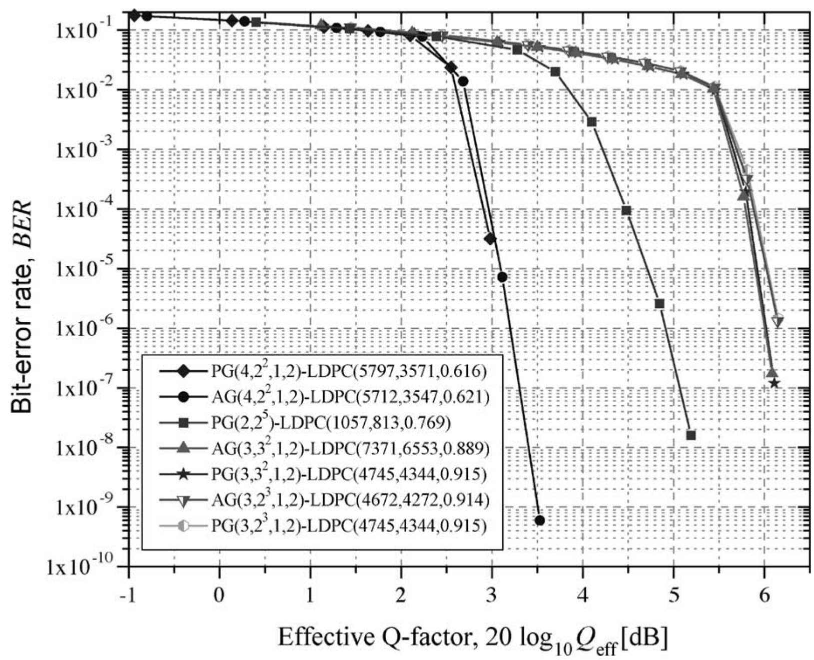

Fig. 2. BER performance of codes on m flats.

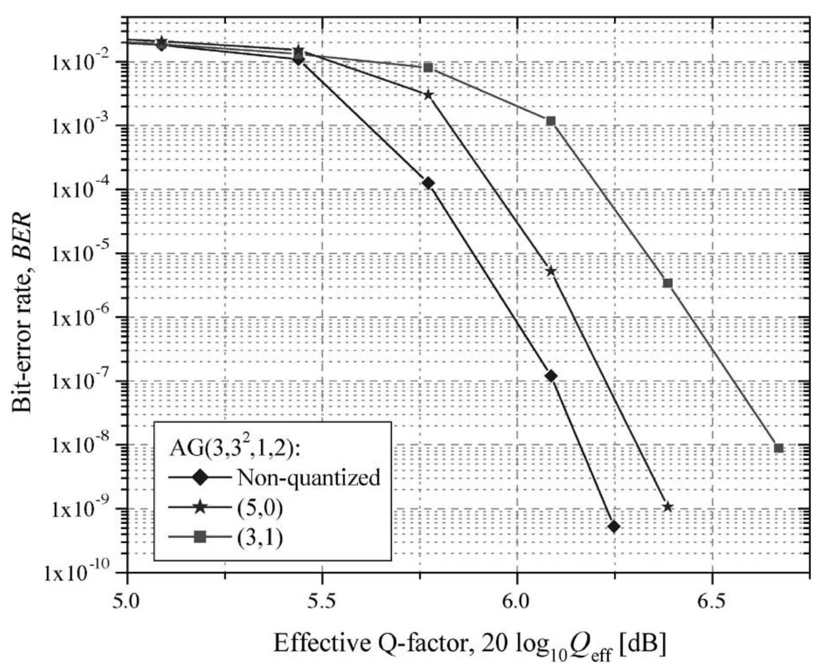

Fig. 3. Performance loss due to FP representation of LLRs.

The dispersion map, shown in Fig. 1, is composed of 25 spans of length $L = 48$ km, each span consisting of $2L/3$ km of D_+ fiber followed by $L/3$ km of D_- fiber. The Q-factor is additionally decreased by noise loading. The fiber parameters are as follows. D_+ fiber: dispersion of 20 ps/(nm · km), dispersion slope of 0.06 ps/(nm^2 · km), effective area equal to 110 μm^2, and loss equal to 0.19 dB/km. D_- fiber: dispersion of −40 ps/(nm · km), dispersion slope of −0.12 ps/(nm^2 · km), effective area equal to 30 μm^2, and loss equal to 0.25 dB/km. The nonlinear Kerr coefficient is 2.6×10^{-20} m^2/W. The precompensation of −320 ps/nm and corresponding post-compensation are also used. The simulations were carried out with an average channel power of 0 dBm, with a central wavelength of 1552.524 nm, and CSRZ modulation format. The influence of six neighboring channels on the observed channel is considered. The optical filter of bandwidth $1.5R_b$ (R_b—the line rate), and electrical filter of bandwidth $0.65R_b$ are observed.

The effective Q-factor is computed using the following expression:

$$Q_{\rm eff} = \sqrt{2}\,{\rm erfc}^{-1}(2 \cdot P_{\rm e,unc})$$

where $P_{\rm e,unc}$ is the BER of the uncoded signal.

IV. m-Flats-Codes Performance

The results of simulation are given in Fig. 2. The code on 1 and 2 flats from $\mathrm{PG}(3, 3^2)$ [denoted as $\mathrm{PG}(3, 3^2, 1, 2)$-LDPC(4745, 4344, 0.915)] of code rate 0.915 (and redundancy of only 9.29%) performs better than the best turbo code with significantly higher redundancy ($\sim$ 24.3%) proposed for optical communications in [17]. At a BER of 10^{-7}, the code from projective geometry achieves a gain of 8.2 dB over the uncoded scheme. Similarly, codes from finite geometries with rate $\sim$ 0.62 achieve a gain of 12 dB at a BER of 10^{-9}. In order to achieve a BER as low as 10^{-9}, we transmit (in simulation) an encoded sequence of length 2^{15} over the whole transmission system only once. Different ASE noise realizations are added to the received sequence until 2000 decoder errors are collected. The efficient realization SPA proposed in [13], employed in simulations, allows an additional 0.5-dB improvement in coding gain compared to the min-sum approximation of SPA implemented in [1] [$\mathrm{PG}(2, 2^5)$ curve].

V. FP Representation of LLRs

The FP representation of a real-valued LLR λ is an integer λ_z with an $n_{\rm b}$-bit precision. Of the $n_{\rm b}$ bits, $d_{\rm b}$ bits are used to represent the integer part (including the sign) of λ and $p_{\rm b}$ bits are used to represent the decimal part of λ. The range of λ_z is defined by $(d_{\rm b}, p_{\rm b})$, where $n_{\rm b} = p_{\rm b} + d_{\rm b}$. The FP representation of λ is obtained as follows:

$$\lambda_z = \begin{cases} \min\left(\lfloor 2^{p_{\rm b}}\lambda + 0.5\rfloor, 2^{n_{\rm b}-1} - 1\right) \\ \max\left(\lfloor 2^{p_{\rm b}}\lambda + 0.5\rfloor, -2^{n_{\rm b}-1}\right) \end{cases}.$$

Hence, the range of λ_z is $[2^{n_{\rm b}-1} - 1, -2^{n_{\rm b}-1}]$, and we refer to λ_z as $(d_{\rm b}, p_{\rm b})$ quantized.

In the decoding stage, the intrinsic information obtained from the channel observations is $(d_{\rm b}, p_{\rm b})$ quantized and fed to the FP iterative decoder. The result of any operation performed within the decoder is $(d_{\rm b}, p_{\rm b})$ quantized. We observe, from Fig. 3, that the performance loss due to 5-bit quantization is within 0.2 dB at a BER of 10^{-9}.

VI. Conclusion

A novel class of error control for long-haul optical-communication systems based on iteratively decodable codes on m flats over affine and projective geometries is presented in this paper. The iterative decoding has been demonstrated to give a coding gain of 9–12 dB, depending on code rate and the minimum distance, at a BER of 10^{-9}. These codes have many unique features, such as high code rate, large minimum distances, and simple decoding algorithms, that may allow for very-high-speed implementations. The BER performance is assessed using the advanced simulator that is able to take into account all major transmission impairments in long-haul optical transmission. The performance loss due to a fixed-point (FP) representation of log-likelihood ratios (LLRs) is found to be insignificant. The decoding complexity of the proposed forward error correction (FEC) scheme is comparable (if not smaller) to that of the turbo product code decoded using the Chase II algorithm [32], [33]. However, the decoder complexity of serial/parallel concatenated turbo codes is significantly higher [34]. The details of a low-density parity-check (LDPC) chip architecture can be found in [35] and details of a turbo-product-code architecture in [17]. On the other hand, the encoding complexity of the LDPC-based scheme is smaller than that of the turbo-code-based scheme [17].

References

[1] I. B. Djordjevic, S. Sankaranarayanan, and B. Vasic, "Projective plane iteratively decodable block codes for WDM high-speed long-haul transmission systems," *J. Lightw. Technol.*, vol. 22, no. 3, pp. 695–702, Mar. 2004.

[2] ——, "Irregular low-density parity-check codes for long haul optical communications," *IEEE Photon. Technol. Lett.*, vol. 16, no. 1, pp. 338–340, Jan. 2004.

[3] I. B. Djordjevic and B. Vasic, "MacNeish-Mann theorem based iteratively decodable codes for optical communication systems," *IEEE Commun. Lett.*, vol. 8, no. 8, pp. 538–540, Aug. 2004.

[4] ——, "High code rate low-density parity-check codes for optical communication systems," *IEEE Photon. Technol. Lett.*, vol. 16, no. 6, pp. 1600–1602, Jun. 2004.

[5] O. Milenkovic, I. B. Djordjevic, and B. Vasic, "Block-circulant low-density parity-check codes for optical communication systems," *IEEE/LEOS J. Sel. Topics Quantum Electron.*, vol. 10, no. 2, pp. 294–299, Mar./Apr. 2004.

[6] T. Richardson, A. Shokrollahi, and R. Urbanke, "Design of capacity-approaching irregular low-density parity-check codes," *IEEE Trans. Inf. Theory*, vol. 47, no. 2, pp. 619–637, Feb. 2001.

[7] D. MacKay and M. Davey, *Evaluation of Gallager Codes for Short Block Length and High Rate Applications*. [Online]. Available: http://www.cs.toronto.edu/~mackay/CodesRegular.html

[8] J. Hagenauer, M. Moerz, and E. Offer, "Analog turbo-networks in VLSI: The next step in turbo decoding and equalization," in *Proc. 2nd Int. Symp. Turbo Codes*, Brest, France, Sep. 4–7, 2000, pp. 209–218.

[9] Y. Kou, S. Lin, and M. P. C. Fossorier, "Low-density parity-check codes based on finite geometries: A rediscovery and new results," *IEEE Trans. Inf. Theory*, vol. 47, no. 7, pp. 2711–2736, Nov. 2001.

[10] S. R. Weller and S. J. Johnson, "Iterative decoding of codes from oval designs," presented at the Defense Applications of Signal Processing, Workshop (DASP), Adelaide, S.A., Australia, Sep. 16–20, 2001.

[11] J. Hagenauer, E. Offer, and L. Papke, "Iterative decoding of binary block and convolutional codes," *IEEE Trans. Inf. Theory*, vol. 42, no. 2, pp. 429–445, Mar. 1996.

[12] E. Yeo, B. Nikolic, and V. Anantharam, "Iterative decoder architectures," *IEEE Commun. Mag.*, vol. 41, no. 8, pp. 132–140, Aug. 2003.

[13] H. Xiao-Yu, E. Eleftheriou, D.-M. Arnold, and A. Dholakia, "Efficient implementations of the sum-product algorithm for decoding LDPC codes," in *Proc. Global Telecommunications (GLOBECOM)*, San Antonio, TX, Nov. 2001, vol. 2, pp. 1036–1036E.

[14] K. Azadet, E. F. Haratsch, H. Kim, F. Saibi, J. H. Saunders, M. Shaffer, L. Song, and Y. Meng-Lin, "Equalization and FEC techniques for optical transceivers," *IEEE J. Solid-State Circuits*, vol. 37, no. 3, pp. 317–327, Mar. 2002.

[15] O. A. Sab, "FEC techniques in submarine transmission systems," in *Proc. Optical Fiber Communication Conf.*, Anaheim, CA, 2001, vol. 2, pp. TuF1-1–TuF1-3.

[16] O. A. Sab and V. Lemarie, "Block turbo code performances for long-haul DWDM optical transmission systems," in *Proc. Optical Fiber Communication Conf.*, Anaheim, CA, 2001, vol. 3, pp. 280–282.

[17] T. Mizuochi *et al.*, “Forward error correction based on block turbo code with 3-bit soft decision for 10 Gb/s optical communication systems,” *IEEE J. Sel. Topics Quantum Electron.*, vol. 10, no. 2, pp. 376–386, Mar./Apr. 2004.
[18] I. B. Djordjevic and B. Vasic, “Projective geometry low-density parity-check codes for ultra-long haul WDM high-speed transmission,” *IEEE Photon. Technol. Lett.*, vol. 15, no. 5, pp. 784–786, May 2003.
[19] B. Vasic, I. B. Djordjevic, and R. Kostuk, “Low-density parity check codes and iterative decoding for long haul optical communication systems,” *J. Lightw. Technol.*, vol. 21, no. 2, pp. 438–446, Feb. 2003.
[20] B. Vasic and I. B. Djordjevic, “Low-density parity check codes for long haul optical communications systems,” *IEEE Photon. Technol. Lett.*, vol. 14, no. 8, pp. 1208–1210, Aug. 2002.
[21] B. Vasic and O. Milenkovic, “Combinatorial constructions of structured low-density parity check codes for iterative decoding,” *IEEE Trans. Inf. Theory*, vol. 50, no. 6, pp. 1156–1176, Jun. 2004.
[22] H. Tang, J. Xu, S. Lin, and K. Abdel-Ghaffar, “Codes on finite geometries,” *IEEE Trans. Inf. Theory*, vol. 51, no. 2, pp. 572–596, Feb. 2005.
[23] I. B. Djordjevic and B. Vasic, “An advanced direct detection receiver model,” *J. Opt. Commun.*, vol. 25, no. 1, pp. 6–9, Feb. 2004.
[24] I. F. Blake and R. C. Mullin, *The Mathematical Theory of Coding*. New York: Academic, 1975.
[25] W. E. Ryan, “An introduction to LDPC codes,” in *CRC Handbook for Coding and Signal Processing for Recording Systems*, B. Vasic, Ed. Boca Raton, FL: CRC, 2004. to be published.
[26] E. F. Assmus, Jr. and J. D. Key, “Baer subplanes, ovals and unitals,” in *Coding Theory and Design Theory, Part I*, vol. 20, *The IMA Volumes in Mathematics and Its Applications*, D. Ray-Chaudhuri, Ed. New York: Springer-Verlag, 1990, pp. 1–8.
[27] ——, *Designs and Their Codes*. Cambridge, U.K.: Cambridge Univ. Press, 1992.
[28] ——, “Designs and codes: An update,” *Designs, Codes Cryptogr.*, vol. 9, no. 1, pp. 7–27, Aug. 1996.
[29] G. P. Agrawal, *Nonlinear Fiber Optics*. San Diego, CA: Academic, 2001.
[30] G. Montorsi and S. Benedetto, “Design of fixed-point iterative decoders for concatenated codes with interleavers,” *IEEE J. Sel. Areas Commun.*, vol. 19, no. 5, pp. 871–882, May 2001.
[31] F. Lustenberger, “On the design of analog VLSI iterative decoders,” Ph.D. thesis, Swiss Federal Inst. Technol., Zurich, Switzerland, Nov. 2000.
[32] D. Chase, “A class of algorithms for decoding of block codes with channel measurements information,” *IEEE Trans. Inf. Theory*, vol. IT-18, no. 1, pp. 170–179, Jan. 1972.
[33] S. A. Hirst, B. Honary, and G. Markarian, “Fast chase algorithm with an application in turbo decoding,” *IEEE Trans. Commun.*, vol. 49, no. 10, pp. 1693–1699, Oct. 2001.
[34] W. E. Ryan, “Concatenated convolutional codes and iterative decoding,” in *Wiley Encyclopedia in Telecommunications*, J. G. Proakis, Ed. New York: Wiley, 2003.
[35] M. Mansour, “Implementation of LDPC decoders,” presented at the IEEE Communications Theory Workshop, Park City, UT, Jun. 13–15, 2005.

Sundararajan Sankaranarayanan (S’05) received the B.E. degree in electronics and communications engineering from the University of Madras, India in 1998, and the M.S. degree in electrical engineering from the University of Arizona, Tucson in 2003. He is currently pursuing the doctoral degree in the area of coding theory from the Electrical and Computer Engineering Department of the University of Arizona.

He was with Philips Research, NY, and Seagate Research, PA, in the summers of 2000 and 2002, respectively. In the fall of 2004, he was with the Systems Group of International Business Machines (IBM) Research Laboratory, Zurich, Switzerland. His research interests include iterative decoders, error floors of low-density parity-check (LDPC) codes, theory of error-control codes and its applications in magnetic recording systems, and optical-communication systems.

Ivan B. Djordjevic (M’04) received the B.Sc., M.Sc., and Ph.D. degrees, all in electrical engineering, from the University of Nis, Serbia, in 1994, 1997, and 1999, respectively.

From 1994 to 1996, he was with the University of Nis, and from 1996 to 2000, he was with the State Telecommunications Company (Serbia Telecom), District Office for Networks-Nis, Serbia. He was involved in digital transmission systems commissioning and acceptance, design, maintenance, installation, and connection. From 2000 to 2001, he was with the National Technical University of Athens, Greece, and with TyCom US Inc. (now TyCo Telecommunications). He was involved in modelling and simulation of WMD systems and networks. During 2002 and 2003, he was with the University of Arizona, Tucson; University of Bristol, UK; and University of the West of England, Bristol, working on forward error correction and iterative decoding for optical transmission, optical CDMA, high-speed transmission and optical switches. He is now with the University of Arizona, Tucson, on leave from the University of the West of England. He is the author of more than 100 international journal articles and international conference papers. His research interests include DWDM fiber-optic communication systems and networks, error control coding, OFDM, free space optics, CDMA, optical packet switching, coherent communications, statistical communication theory, and satellite and wireless communications.

Bane Vasic (SM’02) received the B.Sc., M.Sc., and Ph.D., all in electrical engineering, from the University of Nis, Serbia, in 1990, 1991 and 1994, respectively.

From 1996 to 1997, he was a Visiting Scientist at the Rochester Institute of Technology and Kodak Research, Rochester, NY, where he was involved in research in optical storage channels. From 1998 to 2000, he was with Lucent Technologies, Bell Laboratories. He was involved in research coding schemes and architectures for high-speed applications. He was involved in research in iterative decoding and low-density parity check codes, as well as development of codes and detectors implemented in Lucent (now Agere) chips. Presently, he is a Faculty Member of the Electrical and Computer Engineering Department, University of Arizona, Tucson. His research interests include coding theory, information theory, communication theory, and digital communications and recording.

Dr. Vasic is a Member of the Editorial Board of the IEEE TRANSACTIONS ON MAGNETICS. He served as a Technical Program Chair, IEEE Communication Theory Workshop, 2003, and as Co-organizer of the Center for Discrete Mathematics and Theoretical Computer Science (DIMACS) Workshops on Optical/Magnetic Recording and Optical Transmission and Theoretical Advances in Information Recording, 2004. He is a co-organizer of the Communication Theory Symposium within the International Conference on Communications (ICC 2006).