On the Channel Capacity of Multilevel Modulation Schemes with Coherent Detection

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Abstract: We describe a method to determine the channel capacity of an arbitrary multilevel modulation scheme by modeling the fiber-optic channel as a dynamical *nonlinear* intersymbol interference (ISI) channel with *memory*. We also propose a multilevel low-density parity-check (LDPC)-coded turbo-equalization scheme that is able closely to approach the channel capacity.

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1. Introduction

As data rates transmitted over the optical fiber increase, a fundamental question about physical limitations of optical fiber arises. The problem of determining capacity of optical transmission systems has been addressed by numerous researchers (see [1],[2] and references therein). The nonlinear nature of the propagation of light in optical fiber systems plays a crucial role in limiting the capacity and makes these limits difficult to calculate. In our recent paper [2] we proposed a method to determine the achievable information rates (lower bounds on channel capacity) for high-speed optical transmission using the finite state machine approach, when the combined effects of different channel impairments and different noise sources are taken into account. In these publications, the channel capacity study has been performed for *binary* modulation formats only. In [1] the authors calculate the channel capacity for nonbinary modulation formats by assuming that the optical channel is discrete and *memoryless*.

In this paper we describe a method to determine the channel capacity of arbitrary *multilevel* modulation scheme by modeling the fiber-optic channel as dynamical nonlinear intersymbol interference (ISI) channel with *memory*. Moreover, we describe a coding scheme that is able is able closely to approach the channel capacity. This scheme is based on multilevel turbo equalization and coded-modulation, and employs the best known low-density parity check (LDPC) codes as channel codes. We show that with this scheme we are able straightforwardly to upgrade currently installed 10 Gb/s optical transmission systems to 100 Gb/s. We show that implementing the proposed scheme we are able to achieve 100 Gb/s per DWDM channel transmission over 9600 km.

2. Channel Capacity of Multilevel Modulation Schemes

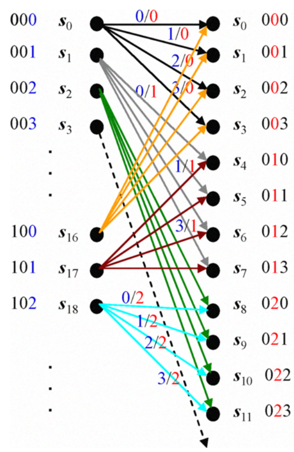
To calculate the channel capacity, we model the whole transmission system as the dynamical nonlinear ISI channel with memory, in which m previous and next m symbols influence the observed symbol. The optical communication system is characterized by the conditional probability density function (PDF) of the output complex vector of samples $\boldsymbol{y}=(y_1,\ldots,y_n,\ldots)$, where $y_i=(y_{i,\mathrm{I}},\ y_{i,\mathrm{Q}})\in \boldsymbol{Y}$ ($y_{i,\mathrm{I}}$ denotes the in-phase channel sample, and $y_{i,\mathrm{Q}}$ denotes the quadrature channel sample) given the source sequence $\boldsymbol{x}=(x_1,\ldots,x_n,\ldots)$, $x_i\in\boldsymbol{X}=\{0,1,\ldots,M\text{-}1\}$. The set $\boldsymbol{X}$ represents the set of indices of constellation points in corresponding M-ary two-dimensional signal constellation diagram (M-ary PSK, QAM, etc.), while $\boldsymbol{Y}$ represents the set of all possible channel outputs. An example of dynamical channel description by means of trellis diagram is shown in Fig. 1(a) for 4-level modulation formats (QPSK). This dynamical trellis is uniquely defined by the following triplet: the previous state, the next state, and the channel output. The state in the trellis is defined as $\boldsymbol{s}_j=(x_{j-m},x_{j-m+1},..,x_j,x_{j+1},\ldots,x_{j+m})=\boldsymbol{x}[j\text{-}m,j+m]$, where x_k denotes the index of the symbol from the following set of possible indices $\boldsymbol{X}=\{0,1,\ldots,M\text{-}1\}$. Every symbol carries $l=\log_2 M$ bits, using the appropriate mapping rule (natural, Gray, anti-Gray, etc.) The memory of the state is equal to $2m+1$, with $2m$ being the number of *symbols* that influence the observed symbol from both sides. The trellis has $M^{2m+1}=64$ states ($\boldsymbol{s}_0$, $\boldsymbol{s}_1,\ldots,\boldsymbol{s}_{63}$), each of which corresponds to a different 3-symbol patterns. The state index is determined by considering ($2m+1$) symbols as digits in numerical system with the base M. The left column in dynamic trellis represents the current states and the right column denotes the terminal states. The branches are labeled by two symbols, the input symbol is the last symbol in initial state (the blue symbol), the output symbol is the central symbol of terminal state (the red symbol). For the complete description of the dynamical trellis, the transition PDFs $p(y_j|x_j)=p(y_j|\boldsymbol{s})$, $\boldsymbol{s}\in\boldsymbol{S}$ are needed; where $\boldsymbol{S}$ is the set of states in the trellis. The conditional PDFs can be determined by using *instanton-Edgeworth expansion* method we proposed in [3]. The information rate can be calculated by [4]:

$$I(\mathbf{Y};\mathbf{X}) = H(\mathbf{Y}) - H(\mathbf{Y}|\mathbf{X}), \tag{1}$$

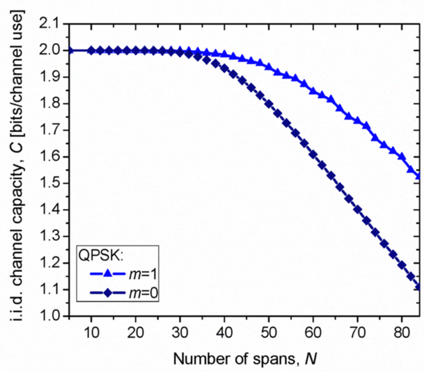
where $H(\boldsymbol{U}) = \mathrm{E}(log_2 P(\boldsymbol{U})$ denotes the entropy of a random variable $\boldsymbol{U}$ and E(·) denotes the mathematical expectation operator. By using the Shannon-McMillan-Brieman theorem [4], which states that the information rate can be determined by calculating $\log_2(P(\boldsymbol{y}[1,n]))$, we obtain the following expression suitable for practical calculation of information rate

$$I(\boldsymbol{Y};\boldsymbol{X}) = \lim_{n\to\infty}\frac{1}{n}\left[\sum_{i=1}^{n}\log_2 P\left(y_i \left|\boldsymbol{y}[1,i-1],\boldsymbol{x}[1,n]\right.\right) - \sum_{i=1}^{n}\log_2 P\left(y_i \left|\boldsymbol{y}[1,i-1]\right.\right)\right]. \tag{2}$$

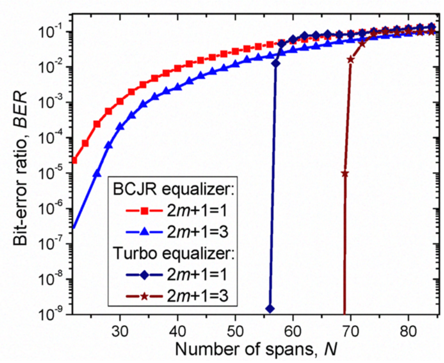
The first term in (2) can be straightforwardly calculated from conditional PDFs because $P(y_i|\mathbf{y}[1,i\text{-}1],\boldsymbol{x}[1,n]) = P(y_i|\boldsymbol{x}[i\text{-}m,i\text{+}m]) = P(y_i|\boldsymbol{s})$. To calculate $\log_2 P(y_i|\boldsymbol{y}[1,i\text{-}1])$ we use the forward recursion of the multilevel BCJR algorithm [5]. In Fig. 1(b) we show the channel capacity against the number of spans, when a PRBS sequence is used as an information source, for two different states memories in trellis description of channel. The QPSK is used as modulation format with aggregate data rate of 100 Gb/s (the symbol rate is 50 Giga symbols/s). The dispersion map is composed of periodically deployed sections of D_+ and D_- fibers, as described in [2]. The span length is set to L=120 km, and each span consists of $2L/3$ km of D_+ fiber followed by $L/3$ km of D_- fiber, with pre-compensation of -1600 ps/nm and corresponding post-compensation. We see that the total transmission distance for state memory m=1 and channel code of code rate R=0.8 is 9600 km, which is 2400 km better than that for state memory m=0.



(a)



(b)



(c)

Fig. 1 (a) Trellis description of channel, (b) channel capacity for QPSK of aggregate rate 100 Gb/s, and (c) turbo equalizer BER performance.

We further describe a channel capacity approaching coding scheme. The proposed scheme is based on *multilevel* (M>2) maximum *a posteriori* probability (MAP) turbo equalization. It is composed of two ingredients: (i) the multilevel BCJR algorithm based equalizer, and (ii) the LDPC decoder. The BCJR equalizer operates on trellis channel description shown in Fig. 1(a), and provides soft *symbol* log-likelihood ratios (LLRs) used in LDPC decoding process. To improve the fiber nonlinearities tolerance, extrinsic LLRs are iterated back and forward between BCJR (MAP) equalizer and LDPC decoder. The results of simulations for a single-channel optical QPSK transmission system operating at 50 Giga symbols/s (with dispersion map described above) are shown in Fig. 1(c). We can see that for 22 spans 4-evel the BCJR equalizer with sate memory m=1 provides more than one order in magnitude improvement in BER over that for state memory m=0. For the multilevel turbo equalization scheme based on 4-level BCJR equalizer of memory m=0 and the LDPC(16935,13550) code of girth-10 and column weight 3, we achieve transmission over 55 spans (6600 km) without any error. On the other hand, for the turbo equalization scheme based on 4-level BCJR equalizer of state memory m=1 and the same LDPC code, we are able to achieve 8160 km of error free transmission at aggregate rate of 100 Gb/s. To achieve the channel capacity (9600 km) with code rate R=0.8, better modulation formats are to be invented. Notice that symbol-by-symbol MAP equalizers cannot be used at all for those transmission distances because they exhibit an early error floor phenomenon.

In conclusion, we described a method to determine the channel capacity of arbitrary *multilevel* modulation scheme by modeling the optical channel as dynamical nonlinear ISI channel with *memory*. We proposed a multilevel turbo equalization scheme that is able closely to approach the channel capacity. By implementing the proposed scheme we will be able to upgrade currently installed 10 Gb/s optical transmission systems to 100 Gb/s.

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