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Scaling laws for evaporative cooling in time-dependent optical traps

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We derive scaling laws for the number of atoms, collision rate, and phase-space density as a function of trap depth for evaporative cooling in an adiabadically lowered optical trap. The results are in excellent agreement with a Boltzmann equation model and show that very large increases in phase-space density can be obtained without excessive slowing of the evaporation rate. Predictions are in reasonable agreement with a recent experiment that achieves Bose-Einstein condensation by evaporation in an optical trap. We also discuss evaporation of fermionic mixtures and explain why Pauli blocking does not strongly inhibit cooling.

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Far-off-resonance optical dipole traps are well known to provide nearly state-independent confining potentials for neutral atoms [1]. This enables study of systems that cannot be stored in magnetic traps, such as stable states of cesium atoms or diamagnetic atoms and molecules. Shallow optical traps have been used to confine multiple spin-state spinor Bose condensates [2]. Optical traps also are likely to play an important role in studies of cold neutral fermions, where specific pairs of states are required to achieve superfluidity [3–5]. A degenerate Fermi gas has been produced by direct evaporation of a two-state mixture of ^{40}K in a magnetic trap, using a dual radio-frequency-knife method [6]. Sympathetic cooling of fermionic ^{6}Li to degeneracy also has been achieved by using mixtures of ^{6}Li with bosonic ^{7}Li in a magnetic trap [7,8]. However, for both ^{6}Li and ^{40}K, exploration of superfluidity will require an optical trap. Generally, the procedure for producing an optically trapped degenerate gas has been to obtain a degenerate gas by optical cooling and evaporation in a magnetic trap, followed by transfer to an optical trap. However, direct production of a degenerate gas in an optical trap would greatly simplify many experiments, and has been explored for many years without success [9,10].

Recently, a Bose-Einstein condensate (BEC) has been produced by Barrett *et al.*, using forced evaporation of ^{87}Rb in a CO_2 laser trap that is continuously lowered [11]. In those experiments, an extremely high initial spatial density is obtained, producing a very large elastic-scattering rate as well as a high initial phase-space density. Hence, the trap depth can be lowered rapidly, producing a BEC in a few seconds.

In this paper, we describe a scaling law model of forced evaporation in a continuously lowered optical trap. The scaling law follows from a simple energy evolution equation for the trapped atoms, which includes the energy loss arising both from evaporation and from adiabatic lowering of the trap potential. Previous derivations of scaling laws for evaporation in magnetic traps have not explicitly included the time-dependent potential [12]. This is appropriate for evaporation in traps with a constant potential where the radio-frequency-knife method is employed to lower the trap threshold. Since the trap strength does not change, the explicit time-dependence of the potential has little effect, as shown by modeling evaporation from a magnetic trap with a time-dependent evaporation threshold [13]. To determine the correct scaling laws for adiabatically lowered optical traps, where the trap strength varies in time, we explicitly include the time-dependent potential and derive scaling laws for the phase-space density, number, and elastic collision rate as a function of the well depth. Results are obtained for a fixed large ratio η of trap depth U to thermal energy kT, to lowest order in the small parameter $\exp(-\eta)$, using a harmonic oscillator approximation. For $\eta = U/kT = 10$, which is typical for evaporation in optical traps, the scaling laws show that by lowering the well depth by a factor of 100, the phase-space density can be increased by a factor of 400. The corresponding elastic collision rate is reduced by a factor of 24. By including the effects of loss arising from background gas collisions, we obtain reasonable agreement with the results of Barrett *et al.* [11].

It is well known that the evaporation rate of a gas from an optical trap of fixed depth stagnates as the temperature drops [14]. At low temperatures, the number of colliding pairs of atoms with enough energy for one to leave the trap is determined by the tail of the Boltzmann distribution. Hence, the evaporation rate is suppressed by a factor $\exp(-U/kT)$. For $U/kT > 10$, the evaporation slows dramatically, and it is necessary to force evaporation by adiabatically lowering the trap depth [15].

The optical trapping potential can be written generally as

$$U(\mathbf{x},t) = -U(t)g(\mathbf{x}), \tag{1}$$

where $g(\mathbf{x})$ describes the trap shape and $g(|\mathbf{x}| \to \infty) \to 0$ with $g(0) = 1$. We assume that evaporation is carried out at low temperatures near stagnation, where the average thermal energy $kT \ll U$.

To determine how the number of trapped atoms N, collision rate γ, and phase-space density ρ scale as the trap depth U is lowered, we first estimate the rate of energy loss from the trap, neglecting atom loss arising from background gas collisions. Taking the zero of energy to be at the bottom of the trap, evaporating atoms will have an average energy $U + \alpha kT$, where $0 \leq \alpha \leq 1$ [14]. From the s-wave Boltzmann equation with $kT \ll U$, we find $\alpha = (\eta - 5)/(\eta - 4)$ for any potential that is harmonic near the minimum [16]. The energy-loss rate arising from evaporation is then $\dot{N}(U + \alpha kT)$, where $\dot{N}$ is the rate at which atoms evaporate from the trap. In addition, as the trap depth is lowered adiabati-

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cally at a rate $\dot{U}$, an energy change arises from the change in potential energy. Since $kT \ll U$, the atoms vibrate near the trap bottom in an approximately harmonic potential, where $E/2$ is the average potential energy. The potential energy then changes at a rate $(\dot{U}/U)E/2$ and the total energy E obeys the approximate evolution equation

$$\dot{E} = \dot{N}(U + \alpha kT) + \frac{\dot{U}}{U}\frac{E}{2}. \tag{2}$$

In the classical limit, $E = 3NkT$ is the total energy of the trapped gas so that $\dot{E} = 3Nk\dot{T} + 3\dot{N}kT$. Then, the contribution to $\dot{T}$ from evaporation is proportional to $\dot{N}(U + \alpha kT - 3kT)$. Hence, the cooling rate is proportional to the difference between the average energy carried away per particle $(\simeq U + \alpha kT)$ and the average thermal energy $3kT$, as it should be.

Solving Eq. (2) with a fixed value of $U/kT = \eta$, the number of trapped atoms is found to vary with trap depth as

$$\frac{N}{N_i} = \left(\frac{U}{U_i}\right)^{3/[2(\eta' - 3)]}, \tag{3}$$

where i denotes the initial condition at $t = 0$, $N = N(t)$, and $U = U(t)$. Here, $\eta' = \eta + \alpha = \eta + (\eta - 5)/(\eta - 4)$. The corresponding phase-space density in the classical regime is $\rho = N(h\nu)^3/(kT)^3$, where $\nu = \nu(t) \propto \sqrt{U}$ is the geometric mean of the trap oscillation frequencies. Using Eq. (3), it is easy to show that $\rho = \rho(t)$ scales with trap depth and number as

$$\frac{\rho}{\rho_i} = \left(\frac{U_i}{U}\right)^{3(\eta' - 4)/[2(\eta' - 3)]} = \left(\frac{N_i}{N}\right)^{\eta' - 4}. \tag{4}$$

Equation (4) shows that for $\eta = 10$, $\rho/\rho_i = (U_i/U)^{1.3}$. For an initial phase-space density of $\rho_i = 3 \times 10^{-3}$, lowering the well depth by a factor of 84 yields $\rho = 1$. For an energy-independent scattering cross section, the elastic collision rate $\gamma = \gamma(t) \propto N\nu^3/(kT)$ scales with trap depth as

$$\frac{\gamma}{\gamma_i} = \left(\frac{U}{U_i}\right)^{\eta'/[2(\eta' - 3)]}, \tag{5}$$

and is reduced by a factor of 21 for a factor of 84 reduction in well depth when $\eta = 10$.

An important feature of Eq. (4) is that the increase in phase-space density with decreasing number is identical to that obtained using a radio-frequency-knife method with a trap depth to thermal energy ratio of η. This is a consequence of the adiabatic energy loss, which ensures that the phase-space density does not change as the trap depth is lowered when $\dot{N} = 0$. Hence, the phase-space change arises only from evaporation as in the radio-frequency-knife method. For an optical trap with $\eta = 10$, $\rho/\rho_i = (N_i/N)^{6.8}$, and a modest decrease in number leads to a great increase in phase-space density. Unlike evaporation from a fixed well, however, the collision rate for an energy-independent elastic cross section decreases as $(U/U_i)^{0.69}$ for $\eta = 10$, and runaway evaporation is not achieved [17]. Nevertheless, for atoms such as ^{6}Li, where the scattering length is anomalously large, evaporation is still rapid and the background collision induced loss can be minimized despite the reduced collision rate.

The scaling laws are derived neglecting background gas collisions. To include background gas collisions, a loss rate term $-\Gamma_{bg}E$ is included in Eq. (2) and $\dot{N}$ is replaced by $\dot{N}_{evap}$, where the evaporation rate $\dot{N}_{evap} = \dot{N} + \Gamma_{bg}N$. Then, one can show that the scaling laws for the number, collision rate, and phase-space density versus trap depth are reduced by a factor $\exp(-\Gamma_{bg}t)$, where t is the time over which the trap depth is lowered. Different exponential factors can be derived for the scaling laws versus number. Equation (2) is also readily modified to include effects of residual heating.

Although the derivation of the scaling laws does not explicitly include the time dependence of the trap depth, maintaining a constant value of η specifies the time dependence $U(t)$, which follows from the evaporation rate. To lowest order in $\exp(-\eta)$, and neglecting background gas collisions, we obtain from the s-wave Boltzmann equation [14],

$$\dot{N} = -2(\eta - 4)\exp(-\eta)\gamma N. \tag{6}$$

Differentiating Eq. (3) and writing γ/γ_i in terms of U/U_i, we obtain

$$\frac{U(t)}{U_i} = \left(1 + \frac{t}{\tau}\right)^{-2(\eta' - 3)/\eta'}, \tag{7}$$

where the time constant τ is given by

$$\frac{1}{\tau} = \frac{2}{3}\eta'(\eta - 4)\exp(-\eta)\gamma_i. \tag{8}$$

Including background gas collisions changes $t \to [1 - \exp(-\Gamma_{bg}t)]/\Gamma_{bg}$ in Eq. (7). Equation (8) shows that the lowering rate scales with the initial collision rate γ_i. According to Eq. (7), the rate of decrease of the well depth decreases with time as the collision rate, and hence, the evaporation rate declines. The initial elastic collision rate for a single state Bose gas in a harmonic potential is $\gamma_i(\mathrm{sec}^{-1}) = 4\pi N_i M \sigma \nu_i^3/(kT_i)$, where N_i is the total number of atoms initially in the trap, T_i the initial temperature, $\sigma = 8\pi a^2$, and ν_i is the initial mean trap oscillation frequency in Hz. For a two-state 50-50 mixture of fermions with the same mass, scattering length, total number, and trap frequencies as in the Bose case, the rate is reduced by a net factor of 4.

The scaling law predictions are valid for both bosons and fermions in the classical regime, where the effects of quantum statistics can be neglected. We have compared the scaling law predictions to a detailed Boltzmann equation model for evaporation of a two-state mixture of fermionic ^{6}Li in a single gaussian beam optical trap. Assuming sufficient ergodicity, the evolution of a low-temperature trapped gas in a time-dependent potential is described by the s-wave Boltzmann equation [13,14], which we write in the form [18],

$$\frac{\partial f(\epsilon,t)}{\partial t} + \langle \dot{U}(\epsilon,t)\rangle \frac{\partial f(\epsilon,t)}{\partial \epsilon} = \left(\frac{df}{dt}\right)_{coll}. \tag{9}$$

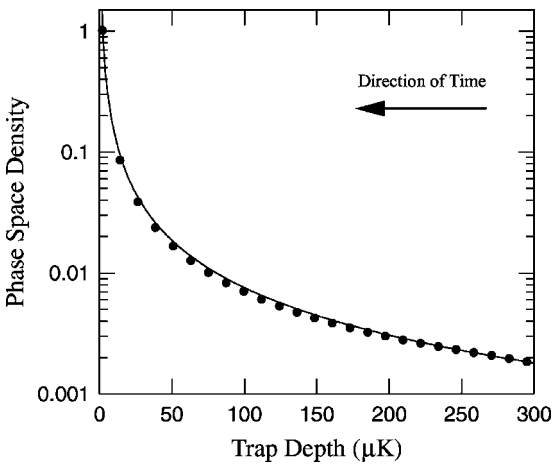

FIG. 1. Phase-space density (each state) vs trap depth. S-wave Boltzmann equation model (dots); scaling law predictions (solid line). The time varies from 0 to 36 sec.

The left-hand side of Eq. (9) describes the adiabatic evolution of the gas in the time-dependent potential. Physically, in a time Δt, the occupation number $f(\epsilon,t)$ changes adiabatically according to $f(\epsilon,t+\Delta t)=f(\epsilon-\Delta t(\partial H/\partial t),t)$. Hamilton's equations require $\langle \partial H/\partial t\rangle=\langle \dot{U}(\epsilon,t)\rangle$, where the angular brackets denote the ergodic average of the time rate of change of the potential [18]. The right-hand side of Eq. (9) is a Boltzmann collision integral, which redistributes the level occupation numbers $f(\epsilon,t)$ including Fermi statistics [18].

A comparison of the results of the scaling laws and the Boltzmann equation predictions for a two-state mixture of ^{6}Li fermions is shown in Figs. 1 and 2. The scattering length is taken to be $a=-300\,a_0$, which can be obtained by applying a magnetic field of 300 G. The trap parameters are as follows: $N_i=4\times10^5$ (total), $\nu_i=1300$ Hz, $T_i=30$ μK, $U_i=300$ μK, i.e., $\eta=10$ and $\Gamma_{bg}=0.003$ s^{-1}. The well depth is lowered according to Eq. (7) with $\eta=10$ using a time constant τ of 1.2 seconds, as predicted by Eq. (8). With these parameters, the Boltzmann model yields U/kT between 10 and 9.6. Using $\eta=10$ in the scaling laws, we obtain nearly perfect fits to the Boltzmann model up to a phase-space density of 1, where the effects of Fermi statistics become important. For exponential lowering of the trap depth, where η is only approximately constant, the scaling law fits are in reasonable agreement with the Boltzmann model, although the fits are not quite as good as for constant η.

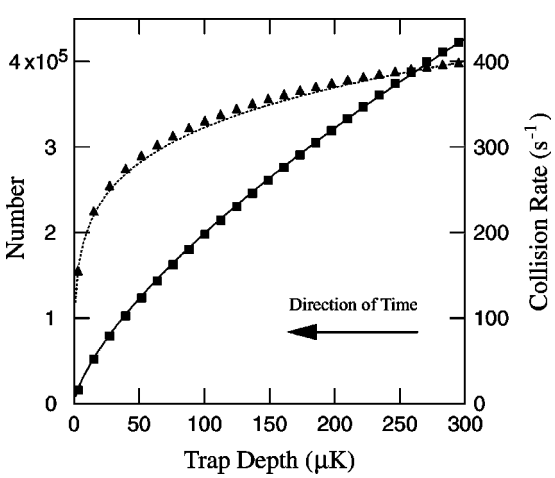

FIG. 2. Number and collision rate vs trap depth. S-wave Boltzmann equation model: Number (triangles); collision rate (squares); scaling law predictions (lines). The time varies from 0 to 36 sec.

For completeness, we describe briefly why Fermi statistics does not severely suppress the efficiency of evaporative cooling. When $T\ll T_F$ the collision rate within the trap is reduced to $\Gamma\propto\gamma_{cl}(T/T_F)^2\ll\gamma_{cl}$, where γ_{cl} is the classical collision rate evaluated at the Fermi surface, T is the temperature, and T_F is the Fermi temperature [19]. The factor $(T/T_F)^2$ is a consequence of Pauli blocking, which forbids collisions into occupied energy states, as observed recently [20]. However, in evaporation, one of the final states is essentially unoccupied, since it is outside the trap. Hence, the evaporation rate is suppressed by only T/T_F, i.e.,

$$\Gamma_{evap}\propto\gamma_{cl}\frac{T}{T_F}\exp\left(-\frac{U-kT_F}{kT}\right).$$

The exponential factor describes the high-energy tail of the Fermi distribution, which is responsible for evaporation when the trap depth $U\gg kT$. This is essentially the same factor that appears in the evaporation of a classical gas. Since the heat capacity also scales as T/T_F [21], the efficiency of evaporative cooling in lowering the temperature is not seriously compromised in a two-component Fermi gas, although the sensitivity to residual heating is increased. Further, the collision rate within the trap is always fast compared to the evaporation rate when $U-kT_F\gg kT$, since $T/T_F\gg\exp[-(U-kT_F)/kT]$ so that rethermalization is faster than evaporation. This picture explains why Pauli blocking does not appear to strongly affect the rate of decrease of T/T_F in a recent theoretical model of evaporation for a two-component Fermi gas [22].

It is interesting to compare the scaling law predictions with the experimental BEC results obtained by Barrett *et al.* [11]. We take N_i to be the stagnation value after 1 second [11], about 1/3 of the maximum number loaded. In the actual experiments, the gas is not given time to stagnate before the trap lowering begins, but one expects that most atoms are lost in a small fraction of a second before the well depth changes appreciably, since the evaporation slows exponentially as the temperature drops to less than 1/10 of the well depth in the first second. For the final conditions, we use the data for a laser power of $P=350$ mW, near the transition between the classical and degenerate regimes, where the scaling law is approximately valid. We use the trap lifetime of 6 sec and a lowering time of 2.5 sec to obtain a background loss factor of $\exp(-\Gamma_{bg}t)=\exp(-2.5/6)=0.66$. The well is assumed to be lowered by a factor $U_f/U_i=(\nu_f/\nu_i)^2=1/84$, based on the measured trap oscillation frequencies. We take $\eta=U/kT=10$ and give results for the case where all atoms are in a single hyperfine state. Table I shows that the predictions are in good agreement with the experiments for these reasonable assumptions about the trap parameters.

TABLE I. Comparison of scaling law predictions with the BEC experiment of Ref. [11].

Initial conditions	Final conditions	Predictions
$\nu_i = 1500$ Hz	$\nu_f = 164$ Hz	
$N_i = 6.7\times 10^5$	$N_f = 1.8\times 10^5$	$N_f = 1.9\times 10^5$
$T_i = 38$ μK	$T_f = 375$ nK	$T_f = 450$ nK
$\gamma_i = 12\times 10^3$ sec^{-1}	$\gamma_f = 300$ sec^{-1}	$\gamma_f = 372$ sec^{-1}
$\rho_i = 1/200$	$\rho_f = 1.4$	$\rho_f = 1.1$

In conclusion, we have derived simple scaling laws for the number, collision rate, and phase-space density as a function of trap depth for atoms in a time-dependent, adiabatically lowered optical trap operating near stagnation. The inclusion of the time-dependent potential in the Boltzmann evolution equation and in the scaling law model is essential, as it accounts for the adiabatic energy loss in the trap, which is needed to preserve the phase-space density in the absence of evaporation. Our results show that the phase-space density in optical traps increases rapidly as the trap depth is lowered, when the ratio of trap depth to temperature is large. The reduction of the elastic collision rate with well depth is mitigated by the large initial spatial [11,23] and phase-space densities obtainable with optical traps, as well as the large scattering lengths obtainable in some systems. Hence, evaporation in optical traps appears quite promising as a means to achieve degeneracy in a variety of atomic and molecular systems. Finally, we have presented a physical picture to explain why Pauli blocking does not strongly inhibit cooling by evaporation in fermionic mixtures.

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