ECE 566

Logic-based Knowledge Representation and Rule Systems

Representations of Knowledge

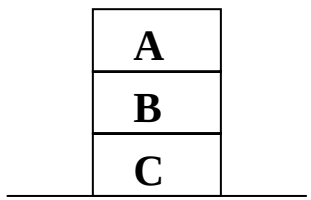
Logic-Based Methods

First Order Predicate Logic: A Language

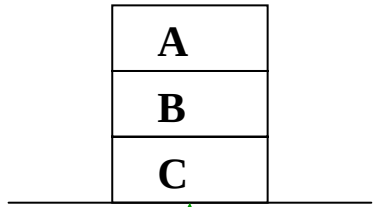
Everything is represented as a sentence: wff

Logical Connectives	Predicates	Terms	Quantifier
$\wedge$	On( )	Constant A	$\forall$
$\vee$	On (A,B)	Variable ?x	$\exists$
$\neg$		Function f(?x)	
$\Rightarrow$			

wff: a well-formed formula



Ontology for representing:



Terms:

A: a constant

B: a constant

C: a constant

On(?x ?y): predicate, ?x is on ?y

Block(?x): predicate, ?x is a block

FOPL representation

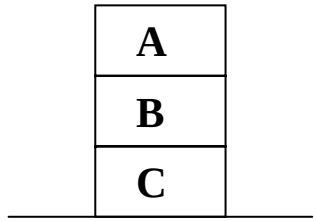
$$\underbrace{\text{Block}(A)\wedge\text{Block}(B)\wedge\text{Block}(C)\wedge\text{On}(A,B)\wedge\text{On}(B,C)}_{\text{wff: W}}$$

Properties of Wffs

If the truth values of two wffs are the same regardless of their interpretation, these are equivalent. The equivalences below can be established by truth tables:

Formula	Equivalent
$\neg (\neg W1)$	$W1$
$W1 \vee W2$	$\neg W1 \Rightarrow W2$
$\neg (W1 \vee W2)$	$\neg W1 \wedge \neg W2$
$\neg (W1 \wedge W2)$	$\neg W1 \vee \neg W2$
$W1 \wedge (W2 \wedge W3)$	$(W1 \wedge W2) \wedge W3$
$W1 \vee (W2 \vee W3)$	$(W1 \vee W2) \vee W3$
$W1 \Rightarrow W2$	$\neg W2 \Rightarrow \neg W1$

Quantification



$\forall$: for all variable values

$\exists$: for some variable value

For all blocks, if one block is above a 2nd block which is itself above a 3rd block, then the first block is above the 3rd one.

$$(\forall ?x)(\forall ?y)(\forall ?z)[[\text{Above}\ (?x, ?y) \wedge \text{Above}(?y, ?z)] \Rightarrow \text{Above}(?x, ?z)]$$

W2

Equivalence Properties for Quantified Formulas

Formula	Equivalent
$\neg (\exists ?x)\, W(?x)$	$(\forall ?x)\, [\neg W(?x)]$
$\neg (\forall ?x)\, W(?x)$	$(\exists ?x)\, [\neg W(?x)]$
$(\forall ?x)[W1?x) \wedge W2(?x)]$	$(\forall ?x) W1(?x) \wedge (\forall ?y) W2(?y)$
$(\exists ?x)[W1(?x) \vee W2(?x)]$	$(\exists ?x) W1(?x) \vee (\exists ?y) W2(?y)$

In order to Interpret Predicate Logic Representations:

1. To each predicate symbol, we must assign a corresponding relationship in the domain.

2. To each constant, we must assign a unique entity.

3. To each function, we must assign a mapping in the domain.

These assignments together define the semantics of the given predicate logic language.

Once semantics is established, we determine the Truth (T) or False (F) of each wff sentence. The value of the wff should be T if the corresponding interpretation is correct in the domain.

In converting problems to symbolic representations, we must come up with an ontology.

Ontology consists of: Predicates
Constants
Variables

This process is referred to as ontologic Eng.

Unification

Finding correct substitutions for terms (variables, functions).

A substitution is an instance of a wff obtained by substituting terms for variables.

Ex: $[\text{Above}(?x, ?y) \wedge \text{Above}(?y, ?z)] \Rightarrow \text{Above}(?x, ?z)$

$$\left.\begin{array}{l}\text{Above}(A, B)\\ \text{Above}(B, C)\end{array}\right\} \Rightarrow \text{Above}(A, C)$$

$s=\{A/?x, B/?y, C/?z\}$

$s=\{t_1/v_1, t_2/v_2 \ldots, t_n/v_n\}$

t_i/v_i means the term t_i is substituted for variable v_i.

Two rules about substitutions:

1. Every occurrence of a variable v_i must be substituted with the same term

2. No variable can be substituted by a term that contains the same variable.

{g(?x)/?x} - incorrect

How do I compose two substitutions together?

If we have S1 and S2, the combined substitution is

obtained by:

1. Apply the substitutions in S2 to the terms in S1.
2. We add all pairs in S2 which do not have corresponding variables in S1.

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<u>Ex1:</u>

S1 = {A/?x} S2 = {B/?y}

S1S2 = {A/?x, B/?y}