Homework #7
Assigned November 14, 2008

1. Show that in an isotropic, nonmagnetic, and inhomogeneous dielectric medium, Maxwell’s equations can be manipulated to yield the inhomogeneous wave equation:

\[ \nabla^2 E + 2 \nabla (E \cdot \nabla \ln (n)) - \frac{n^2}{c^2} \frac{\partial^2 E}{\partial t^2} = 0 \]

2. Assuming unit-amplitude normally incident plane wave illumination, find the angular spectrum of:
   (a) A circular aperture of diameter d.
   (b) A circular opaque disk of diameter d.

3. Consider a spherical wave expanding about the point (0,0,-z_0) in a Cartesian coordinate system. The wavelength of the light is \( \lambda \) and \( z_0 > 0 \).
   (a) Express the phase distribution of the spherical wave across an (x,y) plane located normal to the z axis at coordinate \( z = 0 \).
   (b) Using a paraxial approximation, express the phase distribution of the parabolic wavefront that approximates this spherical wavefront.
   (c) Find an exact expression for the phase by which the spherical wavefront lags or leads the phase of the parabolic wavefront.

4. Assuming unit-amplitude normally incident plane-wave illumination:
   (a) Find the intensity distribution of the Fraunhofer diffraction pattern of the double slit aperture shown below.
   (b) Sketch normalized cross sections of this pattern that appear along the x and y axes in the observation plane, assuming \( X/\lambda z = 10/m \), \( Y/\lambda z = 1/m \), and \( D/\lambda z = 1.5/m \), z being the observation distance and \( \lambda \) the wavelength.
5. Find an expression for the intensity distribution of the Fraunhofer diffraction pattern of the aperture shown below. Assume unit-amplitude, normally incident plane wave illumination. The aperture is square and has a square central obscuration.

![Diagram of an aperture with a central obscuration](image)

6. Two discrete spectral lines of a source are said to be “just resolved” by a diffraction grating if the peak of the qth order diffraction component due to source wavelength $\lambda_1$ falls exactly on the first zero of the qth order diffraction component due to source wavelength $\lambda_2$. The resolving power of the grating is defined as the ratio of the mean wavelength $\lambda$ to the minimum resolvable wavelength difference $\Delta\lambda$. Show that the resolving power of a sinusoidal phase grating is $\lambda/\Delta\lambda = 2qw_0 = qM$, where $q$ is the diffraction order, $w_0$ is the grating spatial frequency, $2w$ is the width of the square grating, and $M$ is the number of spatial periods of the grating contained in the aperture. What phenomenon limits the use of arbitrarily high diffraction orders?

7. The amplitude transmittance function of a thin square wave absorption grating is shown below. Find the following properties of this grating:
   (a) The fraction of incident light that is absorbed by the grating.
   (b) The fraction of incident light that is transmitted by the grating.
   (c) The fraction of light that is transmitted into a single first order.

![Diagram of a square wave absorption grating](image)