1. The actual load is the parallel combination of Z_L and the impedance of the shunt.

$$Z_{shunt} = Z_0 \frac{Z_{L,shunt} + j Z_0 \tan kl}{Z_0 + j Z_{L,shunt} \tan kl}$$

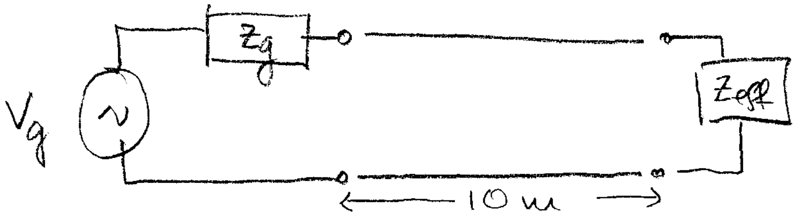
since shunt is a shorted line, $Z_{L,shunt} = 0$
$\Rightarrow$

$$Z_{shunt} = j Z_0 \tan kl$$

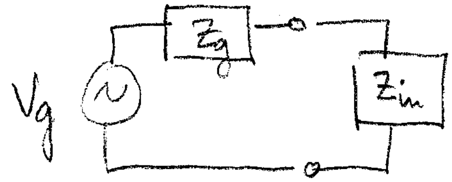
The effective load is thus

$$\frac{1}{Z_{eff}} = \frac{1}{Z_L} + \frac{1}{Z_{shunt}}$$

The First Equivalent circuit is thus



Second Equivalent circuit is thus



$l_2 = 10\,m$

where

$$Z_{in} = Z_0 \frac{Z_{eff} + j Z_0 \tan k l_2}{Z_0 + j Z_{eff} \tan k l_2}$$

Know $Z_0 = 100\,\Omega$ $\quad Z_g = 100\,\Omega$

$Z_L = 0.5 - 0.5j$ $\quad V_g = 100$ $\quad f = 3 \times 10^7$

$C_l = \frac{1}{3} \times 10^{-10}$

$k = \omega\sqrt{L_\ell C_\ell}$ $\qquad Z_0 = \sqrt{\frac{L_\ell}{C_\ell}}$ $\qquad L_\ell = Z_0^2 C_\ell = \frac{1}{3}\times 10^{-6}$

$$k = 2\pi\, 3\times10^{7}\sqrt{\frac{1}{3}\times10^{-10}\times\frac{1}{3}\times10^{-6}} = 2\pi\, 3\times10^{7}\,\frac{1}{3}\times10^{-8}$$

$$= 0.1 \times 2\pi = .628$$

Thus

$$k\ell_z = .1\times 2\pi \times 10 = 2\pi$$

Therefore

$$Z_{in} = Z_{eff}$$

To maximize the power transfer to the load, we know that

$$\mathrm{Im}(Z_{eff}) = 0$$

$$\frac{1}{Z_{eff}} = \frac{1}{0.5-0.5j} + \frac{1}{j100\tan k\ell}$$

$$= \frac{0.5+0.5j}{0.5} - j\frac{1}{100\tan k\ell}$$

$$= 1 + j\left[1 - \frac{1}{100\tan k\ell}\right]$$

$\Rightarrow$ need

$$\tan k\ell = 1/100 = 0.01$$

or

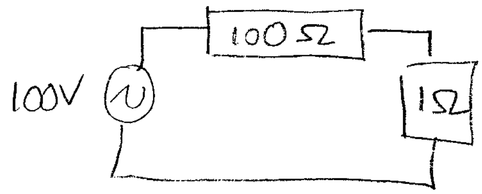
$$\ell = \frac{1}{k}\tan^{-1}(0.01) = \frac{.01}{.1\times 2\pi}$$

$$\ell = 1.592\times10^{-2}\ m$$

$$= \underline{\underline{1.592\ cm}}$$

Thus $Z_{eff} = 1.0\,\Omega$ if $l = 1.592\,cm$.

$\Rightarrow$



is the equivalent circuit $\Leftrightarrow$ simple voltage divider

Thus

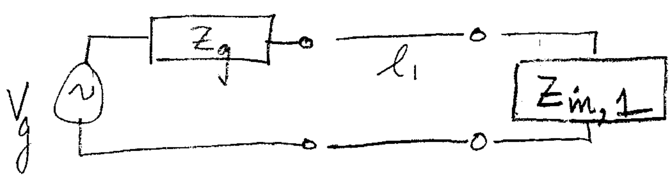
$$P_{load,\,max} = V_{max}^{Load}\, I_{max}^{load}$$

IN phase since impedances are real

$$= \left(I_{max}^{load}\right)^2 Z_{load}$$

$$= \left(\frac{100}{100+1}\right)^2 1 = \underline{0.98 \text{ Watts}}$$

2. Cascaded transmission line :



$$Z_{in,1} = Z_2 \frac{Z_L + j Z_2 \tan k_2 l_2}{Z_2 + j Z_L \tan k_2 l_2}$$

we know

$$k_2 l_2 = 2\pi \frac{1}{8} = \frac{\pi}{4}$$

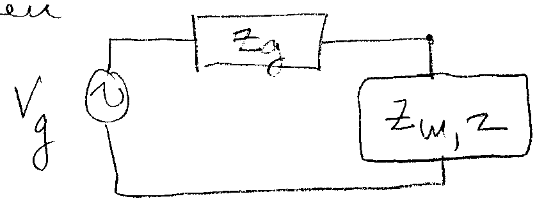
$$\tan k_2 l_2 = \underline{1}$$

$\Rightarrow$

$$Z_{in,1} = 50 \frac{100 + j50 + j50}{50 + j(100 + j50)}$$

$$= 50 \frac{100(1+j)}{j100} = 50(1-j)$$

Then



$$Z_{in,2} = Z_1 \frac{Z_{in,1} + j Z_1 \tan k_1 l_1}{Z_1 + j Z_{in,1} \tan k_1 l_1}$$

$$k_1 l_1 = \frac{2\pi}{\lambda} \frac{\lambda}{4} = \frac{\pi}{2} \qquad \tan k_1 l_1 = \infty$$

$$Z_{in,2} = \frac{Z_1^2}{Z_{in,1}} = \frac{(100)^2}{50(1-j)}$$

a. $$Z_{in,2} = 100(1+j)$$

b. Voltage at the input of the transmission line

$$V(-l_1 - l_2) = Z_{in,1} \; I(-l_1 - l_2)$$

$$= \frac{Z_{in,1} \; V_g}{Z_{in,1} + Z_g}$$

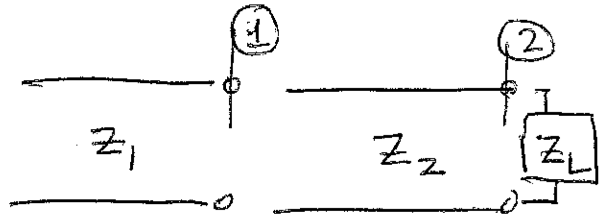
$$= \frac{100(1+j) \quad 200}{100(1-j) + 100(1+j)}$$

$$= 100(1+j)$$

C. $I(-l_1-l_2) = \dfrac{V_g}{Z_{in,1}+Z_g} = \dfrac{200}{100(1-j)+100(1+j)}$

$= \dfrac{200}{100+100} = \underline{\underline{1.0 \text{ amp}}}$

D.



Because lines are lossless, power along each line is constant

Power at point ②:

$P_{Load} = (1-|\Gamma_L|^2) P_{\text{at point} ①}$

$P_{\text{at point} ①} = \left(1-|\Gamma_{\text{at point } 1}|^2\right) P_{Ave, input}$

$\Gamma_{\text{at point } 1} = \dfrac{Z_{in,1}-Z_1}{Z_{in,1}+Z_1} \qquad \Gamma_L = \dfrac{Z_L - Z_2}{Z_L + Z_2}$

$\Gamma_L = \dfrac{100+j50-50}{100+j50+50} = \dfrac{50+j50}{150+j50} = \dfrac{1+j}{3+j} = \dfrac{(1+j)(3-j)}{10}$

$= \dfrac{3+3j-j+1}{10} = \dfrac{4+2j}{10} = .4+.2j \Rightarrow |\Gamma_L|^2 = 0.2$

$\Gamma_{\text{at point } 1} = \dfrac{50-50j-100}{50-50j+100} = \dfrac{-50-50j}{150-50j} = \dfrac{-(1+j)}{3-j} = .2-.4j$

$\Rightarrow |\Gamma_{\text{at point}_1}|^2 = 0.2$

$\underline{P_{Load}} = (0.8)(0.8)\,50W = \underline{\underline{32W}}$

Problem 2.27 At an operating frequency of 300 MHz, a lossless 50-Ω air-spaced transmission line 2.5 m in length is terminated with an impedance $Z_L = (40 + j20)$ Ω. Find the input impedance.

Solution: Given a lossless transmission line, $Z_0 = 50$ Ω, $f = 300$ MHz, $l = 2.5$ m, and $Z_L = (40 + j20)$ Ω. Since the line is air filled, $u_p = c$ and therefore, from Eq. (2.48),

$$\beta = \frac{\omega}{u_p} = \frac{2\pi \times 300 \times 10^6}{3 \times 10^8} = 2\pi \text{ rad/m}.$$

Since the line is lossless, Eq. (2.79) is valid:

$$\begin{aligned} Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) &= 50 \left[\frac{(40 + j20) + j50 \tan(2\pi \text{ rad/m} \times 2.5 \text{ m})}{50 + j(40 + j20) \tan(2\pi \text{ rad/m} \times 2.5 \text{ m})} \right] \\ &= 50 \, [(40 + j20) + j50 \times 0] \, 50 + j(40 + j20) \times 0 \\ &= (40 + j20) \ \Omega. \end{aligned}$$

Problem 2.32 A 6-m section of 150-Ω lossless line is driven by a source with

$$v_g(t) = 5\cos(8\pi \times 10^7 t - 30°) \quad \text{(V)}$$

and $Z_g = 150$ Ω. If the line, which has a relative permittivity $\varepsilon_r = 2.25$, is terminated in a load $Z_L = (150 - j50)$ Ω, determine:

(a) λ on the line.

(b) The reflection coefficient at the load.

(c) The input impedance.

(d) The input voltage $\widetilde{V}_i$.

(e) The time-domain input voltage $v_i(t)$.

(f) Quantities in (a) to (d) using CD Modules 2.4 or 2.5.

Solution:

$$v_g(t) = 5\cos(8\pi \times 10^7 t - 30°) \text{ V},$$
$$\widetilde{V}_g = 5e^{-j30°} \text{ V}.$$

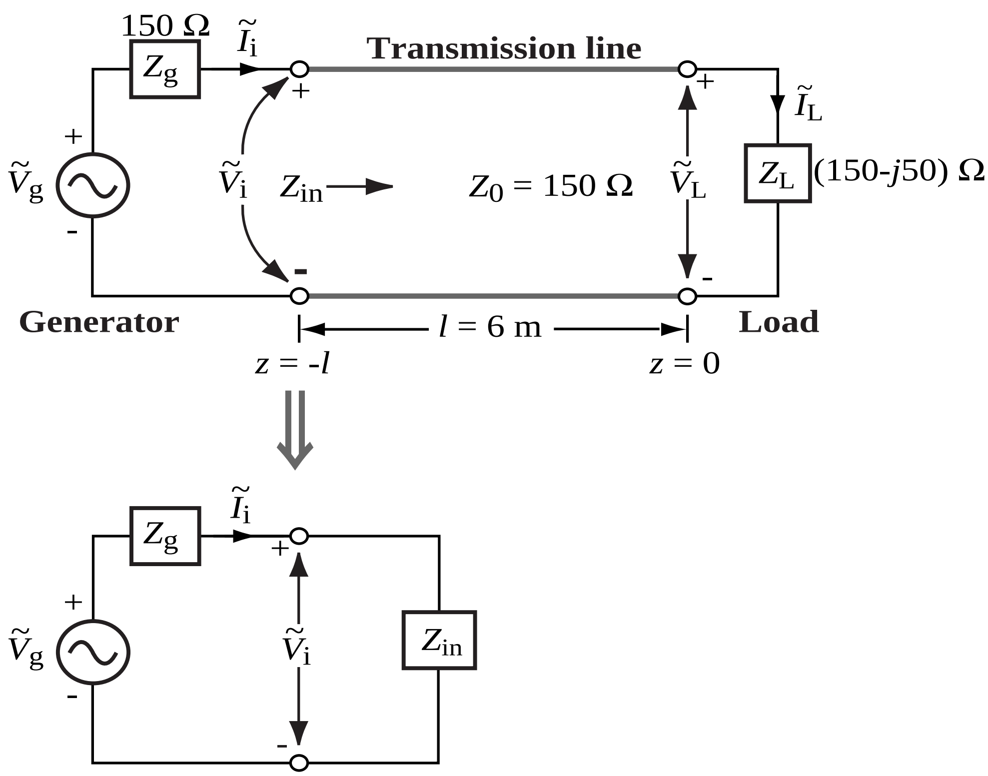


Figure P2.32: Circuit for Problem 2.32.

(a)

$$\begin{aligned}
u_{\mathrm{p}} &= \frac{c}{\sqrt{\varepsilon_{\mathrm{r}}}} = \frac{3\times10^8}{\sqrt{2.25}} = 2\times10^8 \quad \text{(m/s)},\\
\lambda &= \frac{u_{\mathrm{p}}}{f} = \frac{2\pi u_{\mathrm{p}}}{\omega} = \frac{2\pi\times2\times10^8}{8\pi\times10^7} = 5 \ \ \text{m},\\
\beta &= \frac{\omega}{u_{\mathrm{p}}} = \frac{8\pi\times10^7}{2\times10^8} = 0.4\pi \quad \text{(rad/m)},\\
\beta l &= 0.4\pi\times6 = 2.4\pi \quad \text{(rad)}.
\end{aligned}$$

Since this exceeds 2π (rad), we can subtract 2π, which leaves a remainder $\beta l = 0.4\pi$ (rad).

(b) $\Gamma = \dfrac{Z_{\mathrm{L}}-Z_0}{Z_{\mathrm{L}}+Z_0} = \dfrac{150-j50-150}{150-j50+150} = \dfrac{-j50}{300-j50} = 0.16\,e^{-j80.54^\circ}$.

(c)

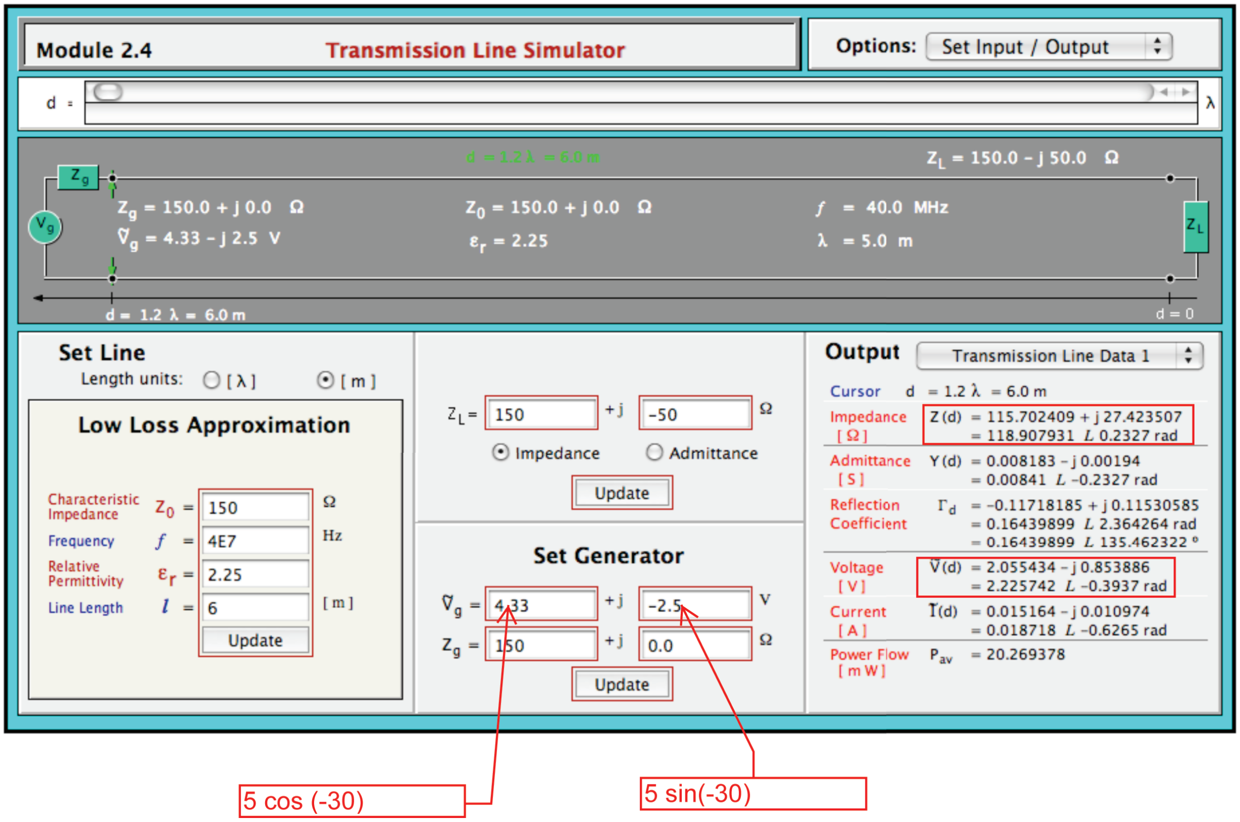
$$\begin{aligned}
Z_{\mathrm{in}} &= Z_0\left[\frac{Z_{\mathrm{L}}+jZ_0\tan\beta l}{Z_0+jZ_L\tan\beta l}\right]\\
&= 150\left[\frac{(150-j50)+j150\tan(0.4\pi)}{150+j(150-j50)\tan(0.4\pi)}\right] = (115.70+j27.42)\ \Omega.
\end{aligned}$$

(d)

$$\begin{aligned}
\widetilde{V}_{\mathrm{i}} = \frac{\widetilde{V}_{\mathrm{g}}Z_{\mathrm{in}}}{Z_{\mathrm{g}}+Z_{\mathrm{in}}} &= \frac{5e^{-j30^\circ}(115.7+j27.42)}{150+115.7+j27.42}\\
&= 5e^{-j30^\circ}\left(\frac{115.7+j27.42}{265.7+j27.42}\right)\\
&= 5e^{-j30^\circ}\times0.44\,e^{j7.44^\circ} = 2.2\,e^{-j22.56^\circ} \quad \text{(V)}.
\end{aligned}$$

(e)

$$v_{\mathrm{i}}(t) = \mathfrak{Re}[\widetilde{V}_{\mathrm{i}}e^{j\omega t}] = \mathfrak{Re}[2.2\,e^{-j22.56^\circ}e^{j\omega t}] = 2.2\cos(8\pi\times10^7 t-22.56^\circ)\ \ \text{V}.$$



Module 2.4
Transmission Line Simulator
Options: Set Input / Output
Z_L = 150.0 – j 50.0 Ω
Z_g = 150.0 + j 0.0 Ω
V_g = 4.33 – j 2.5 V
Z_0 = 150.0 + j 0.0 Ω
ε_r = 2.25
f = 40.0 MHz
λ = 5.0 m
d = 1.2 λ = 6.0 m
d = 0
Set Line
Length units: [λ] [m]
Low Loss Approximation
Characteristic Impedance Z_0 = 150 Ω
Frequency f = 4E7 Hz
Relative Permittivity ε_r = 2.25
Line Length l = 6 [m]
Update
Z_L = 150 + j -50 Ω
Impedance Admittance
Update
Set Generator
V_g = 4.33 + j -2.5 V
Z_g = 150 + j 0.0 Ω
Update
Output Transmission Line Data 1
Cursor d = 1.2 λ = 6.0 m
Impedance [Ω] Z (d) = 115.702409 + j 27.423507 = 118.907931 ∠ 0.2327 rad
Admittance [S] Y (d) = 0.008183 – j 0.00194 = 0.00841 ∠ -0.2327 rad
Reflection Coefficient Γ_d = -0.11718185 + j 0.11530585 = 0.16439899 ∠ 2.364264 rad = 0.16439899 ∠ 135.462322 °
Voltage [V] V (d) = 2.055434 – j 0.853886 = 2.225742 ∠ -0.3937 rad
Current [A] I (d) = 0.015164 – j 0.010974 = 0.018718 ∠ -0.6265 rad
Power Flow [m W] P_av = 20.269378
5 cos (-30)
5 sin(-30)

Problem 2.42 A generator with $\widetilde{V}_g = 300$ V and $Z_g = 50\ \Omega$ is connected to a load $Z_L = 75\ \Omega$ through a 50-Ω lossless line of length $l = 0.15\lambda$.

(a) Compute Z_{in}, the input impedance of the line at the generator end.

(b) Compute $\widetilde{I}_i$ and $\widetilde{V}_i$.

(c) Compute the time-average power delivered to the line, $P_{in} = \frac{1}{2}\mathfrak{Re}[\widetilde{V}_i\widetilde{I}_i^*]$.

(d) Compute $\widetilde{V}_L$, $\widetilde{I}_L$, and the time-average power delivered to the load, $P_L = \frac{1}{2}\mathfrak{Re}[\widetilde{V}_L\widetilde{I}_L^*]$. How does P_{in} compare to P_L? Explain.

(e) Compute the time-average power delivered by the generator, P_g, and the time-average power dissipated in Z_g. Is conservation of power satisfied?

Solution:

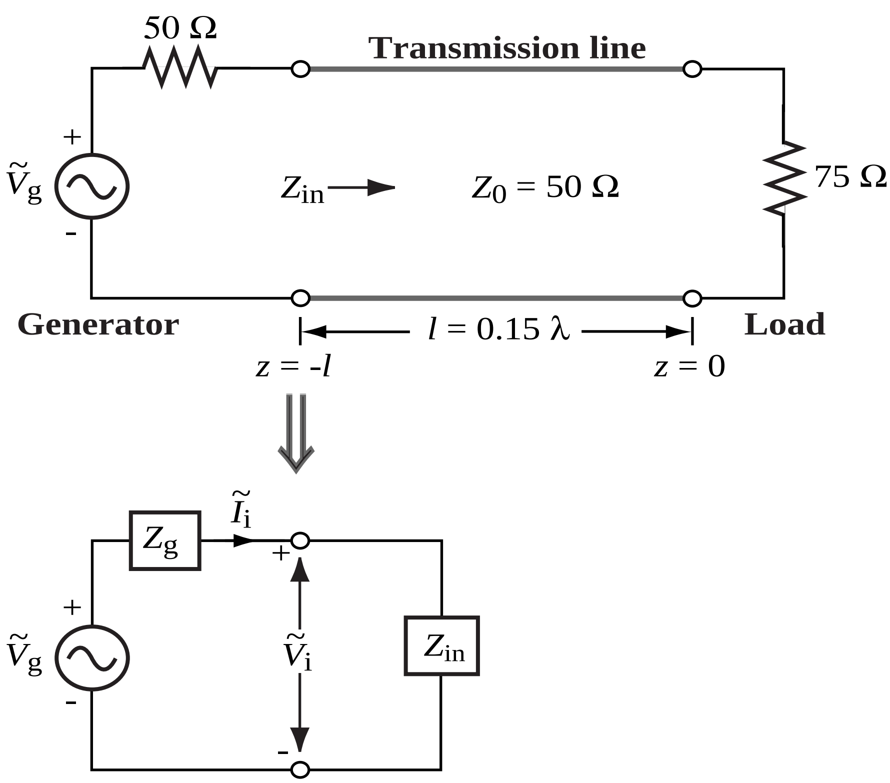


Figure P2.42: Circuit for Problem 2.42.

(a)

$$\beta l = \frac{2\pi}{\lambda} \times 0.15\lambda = 54^\circ,$$

$$Z_{\text{in}} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right] = 50 \left[\frac{75 + j50 \tan 54^\circ}{50 + j75 \tan 54^\circ}\right] = (41.25 - j16.35)\ \Omega.$$

(b)

$$\widetilde{I}_i = \frac{\widetilde{V}_g}{Z_g + Z_{\text{in}}} = \frac{300}{50 + (41.25 - j16.35)} = 3.24 e^{j10.16^\circ} \quad \text{(A)},$$

$$\widetilde{V}_i = \widetilde{I}_i Z_{\text{in}} = 3.24 e^{j10.16^\circ}(41.25 - j16.35) = 143.6 e^{-j11.46^\circ} \quad \text{(V)}.$$

(c)

$$P_{\text{in}} = \frac{1}{2}\mathfrak{Re}[\widetilde{V}_i \widetilde{I}_i^*] = \frac{1}{2}\mathfrak{Re}[143.6 e^{-j11.46^\circ} \times 3.24 e^{-j10.16^\circ}]$$

$$= \frac{143.6 \times 3.24}{2} \cos(21.62^\circ) = 216 \quad \text{(W)}.$$

(d)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{75 - 50}{75 + 50} = 0.2,$$

$$V_0^+ = \widetilde{V}_i \left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right) = \frac{143.6 e^{-j11.46^\circ}}{e^{j54^\circ} + 0.2 e^{-j54^\circ}} = 150 e^{-j54^\circ} \quad \text{(V)},$$

$$\widetilde{V}_L = V_0^+ (1 + \Gamma) = 150 e^{-j54^\circ}(1 + 0.2) = 180 e^{-j54^\circ} \quad \text{(V)},$$

$$\widetilde{I}_L = \frac{V_0^+}{Z_0}(1 - \Gamma) = \frac{150 e^{-j54^\circ}}{50}(1 - 0.2) = 2.4 e^{-j54^\circ} \quad \text{(A)},$$

$$P_L = \frac{1}{2}\mathfrak{Re}[\widetilde{V}_L \widetilde{I}_L^*] = \frac{1}{2}\mathfrak{Re}[180 e^{-j54^\circ} \times 2.4 e^{j54^\circ}] = 216 \quad \text{(W)}.$$

$P_L = P_{\text{in}}$, which is as expected because the line is lossless; power input to the line ends up in the load.

(e)

Power delivered by generator:

$$P_g = \frac{1}{2}\mathfrak{Re}[\widetilde{V}_g \widetilde{I}_i] = \frac{1}{2}\mathfrak{Re}[300 \times 3.24 e^{j10.16^\circ}] = 486 \cos(10.16^\circ) = 478.4 \quad \text{(W)}.$$

Power dissipated in Z_g:

$$P_{Z_g} = \frac{1}{2}\mathfrak{Re}[\widetilde{I}_i \widetilde{V}_{Z_g}] = \frac{1}{2}\mathfrak{Re}[\widetilde{I}_i \widetilde{I}_i^* Z_g] = \frac{1}{2}|\widetilde{I}_i|^2 Z_g = \frac{1}{2}(3.24)^2 \times 50 = 262.4 \quad \text{(W)}.$$

Note 1: $P_g = P_{Z_g} + P_{\text{in}} = 478.4$ W.

Problem 2.45 The circuit shown in Fig. P2.45 consists of a 100-Ω lossless transmission line terminated in a load with $Z_L = (50 + j100)$ Ω. If the peak value of the load voltage was measured to be $|\widetilde{V}_L| = 12$ V, determine:

(a) the time-average power dissipated in the load,

(b) the time-average power incident on the line,

(c) the time-average power reflected by the load.

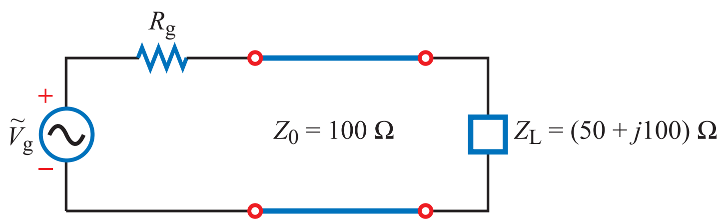


Figure P2.45: Circuit for Problem 2.45.

Solution:

(a)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 + j100 - 100}{50 + j100 + 100} = \frac{-50 + j100}{150 + j100} = 0.62e^{j82.9^\circ}.$$

The time average power dissipated in the load is:

$$\begin{aligned} P_{av} &= \frac{1}{2}|\widetilde{I}_L|^2 R_L \\ &= \frac{1}{2}\left|\frac{\widetilde{V}_L}{Z_L}\right|^2 R_L \\ &= \frac{1}{2}\frac{|\widetilde{V}_L|^2}{|Z_L|^2} R_L = \frac{1}{2} \times 12^2 \times \frac{50}{50^2 + 100^2} = 0.29 \text{ W}. \end{aligned}$$

(b)

$$P_{av} = P^i_{av}(1 - |\Gamma|^2)$$

Hence,

$$P^i_{av} = \frac{P_{av}}{1 - |\Gamma|^2} = \frac{0.29}{1 - 0.62^2} = 0.47 \text{ W}.$$

(c)

$$P^r_{av} = -|\Gamma|^2 P^i_{av} = -(0.62)^2 \times 0.47 = -0.18 \text{ W}.$$

Problem 2.66 A 200-Ω transmission line is to be matched to a computer terminal with $Z_L = (50 - j25)$ Ω by inserting an appropriate reactance in parallel with the line. If $f = 800$ MHz and $\varepsilon_r = 4$, determine the location nearest to the load at which inserting:

(a) A capacitor can achieve the required matching, and the value of the capacitor.

(b) An inductor can achieve the required matching, and the value of the inductor.

Solution:

(a) After entering the specified values for Z_L and Z_0 into Module 2.6, we have z_L represented by the red dot in Fig. P2.66(a), and y_L represented by the blue dot. By moving the cursor a distance $d = 0.093\lambda$, the blue dot arrives at the intersection point between the SWR circle and the $S = 1$ circle. At that point

$$y(d) = 1.026126 - j1.5402026.$$

To cancel the imaginary part, we need to add a reactive element whose admittance is positive, such as a capacitor. That is:

$$\begin{aligned} \omega C &= (1.54206) \times Y_0 \\ &= \frac{1.54206}{Z_0} = \frac{1.54206}{200} = 7.71 \times 10^{-3}, \end{aligned}$$

which leads to

$$C = \frac{7.71 \times 10^{-3}}{2\pi \times 8 \times 10^8} = 1.53 \times 10^{-12} \text{ F}.$$

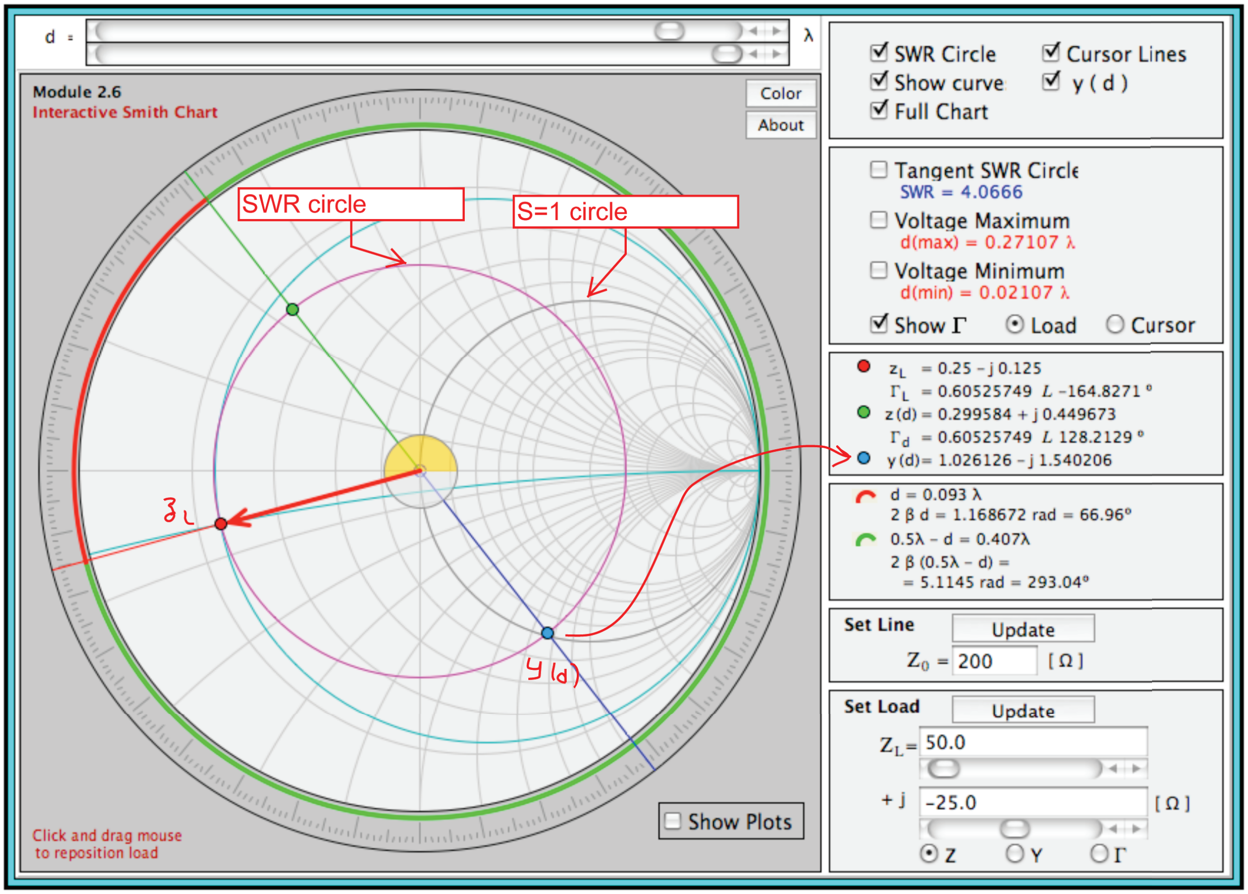


Figure P2.66(a)

(b) Repeating the procedure for the second intersection point [Fig. P2.66(b)] leads to

$$y(d) = 1.000001 + j1.520691,$$

at $d_2 = 0.447806\lambda$.

To cancel the imaginary part, we add an inductor in parallel such that

$$\frac{1}{\omega L} = \frac{1.520691}{200},$$

from which we obtain

$$L = \frac{200}{1.52 \times 2\pi \times 8 \times 10^8} = 2.618 \times 10^{-8} \text{ H}.$$

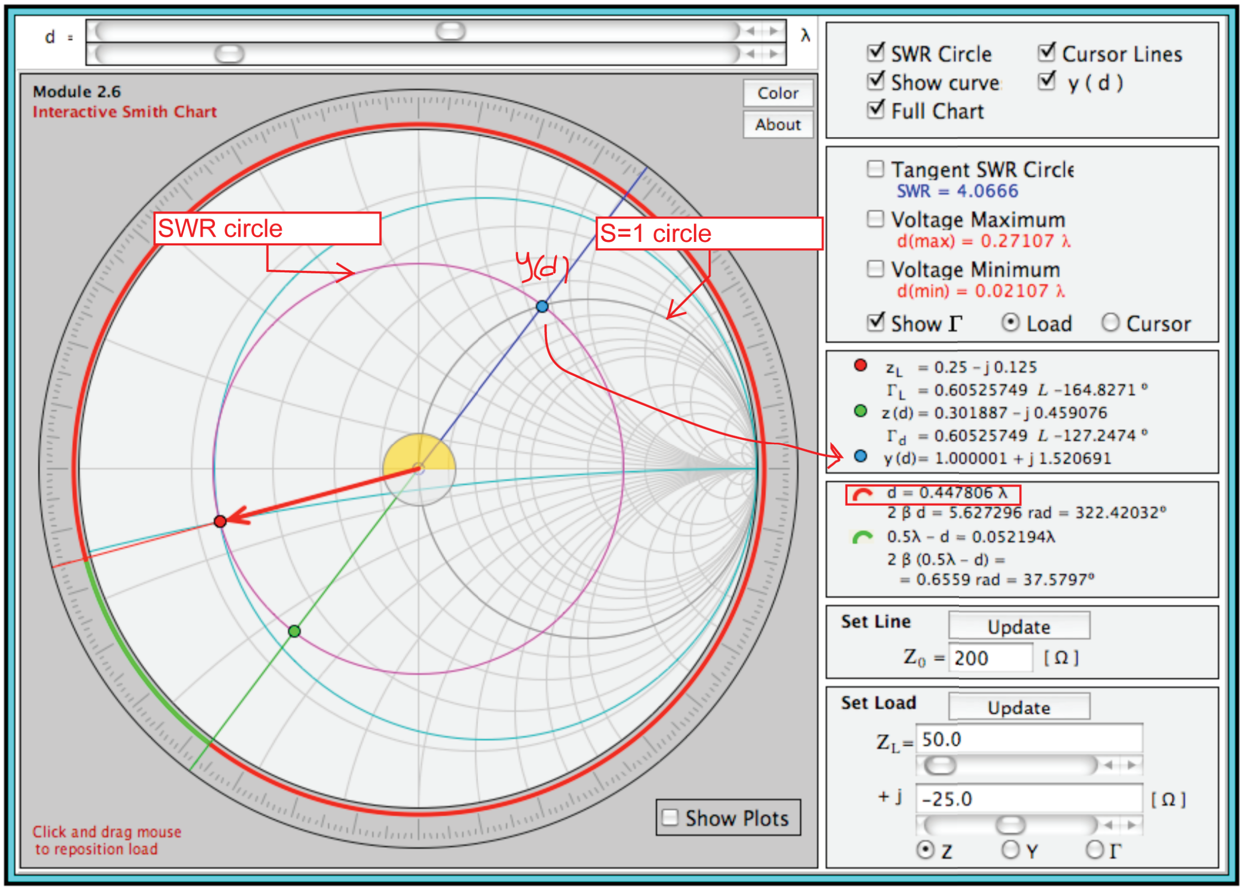


Figure P2.66(b)

Problem 2.31 A voltage generator with

$$v_g(t) = 5\cos(2\pi \times 10^9 t) \text{ V}$$

and internal impedance $Z_g = 50\ \Omega$ is connected to a 50-Ω lossless air-spaced transmission line. The line length is 5 cm and the line is terminated in a load with impedance $Z_L = (100 - j100)\ \Omega$. Determine:

(a) Γ at the load.

(b) Z_{in} at the input to the transmission line.

(c) The input voltage $\widetilde{V}_i$ and input current $\widetilde{I}_i$.

(d) The quantities in (a)–(c) using CD Modules 2.4 or 2.5.

Solution:

(a) From Eq. (2.59),

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(100 - j100) - 50}{(100 - j100) + 50} = 0.62e^{-j29.7^\circ}.$$

(b) All formulae for Z_{in} require knowledge of $\beta = \omega/u_p$. Since the line is an air line, $u_p = c$, and from the expression for $v_g(t)$ we conclude $\omega = 2\pi \times 10^9$ rad/s. Therefore

$$\beta = \frac{2\pi \times 10^9 \text{ rad/s}}{3 \times 10^8 \text{ m/s}} = \frac{20\pi}{3} \text{ rad/m.}$$

Then, using Eq. (2.79),

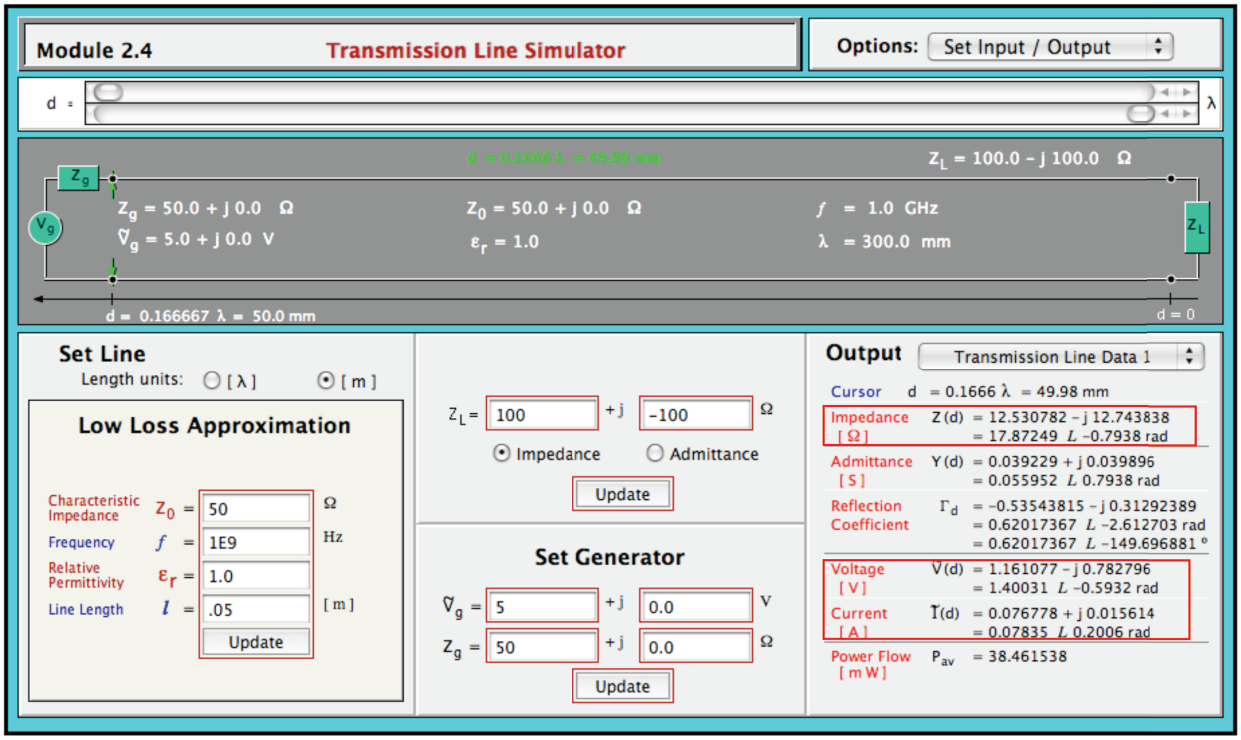
$$\begin{aligned}
Z_{in} &= Z_0 \left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) \\
&= 50 \left[\frac{(100 - j100) + j50 \tan\left(\frac{20\pi}{3} \text{ rad/m} \times 5 \text{ cm}\right)}{50 + j(100 - j100) \tan\left(\frac{20\pi}{3} \text{ rad/m} \times 5 \text{ cm}\right)} \right] \\
&= 50 \left[\frac{(100 - j100) + j50 \tan\left(\frac{\pi}{3} \text{ rad}\right)}{50 + j(100 - j100) \tan\left(\frac{\pi}{3} \text{ rad}\right)} \right] = (12.5 - j12.7)\ \Omega.
\end{aligned}$$

(c) In phasor domain, $\widetilde{V}_g = 5 \text{ V}e^{j0^\circ}$. From Eq. (2.80),

$$\widetilde{V}_i = \frac{\widetilde{V}_g Z_{in}}{Z_g + Z_{in}} = \frac{5 \times (12.5 - j12.7)}{50 + (12.5 - j12.7)} = 1.40e^{-j34.0^\circ} \quad \text{(V)},$$

and also from Eq. (2.80),

$$\widetilde{I}_i = \frac{\widetilde{V}_i}{Z_{in}} = \frac{1.4e^{-j34.0^\circ}}{(12.5 - j12.7)} = 78.4e^{j11.5^\circ} \quad \text{(mA)}.$$


Module 2.4
Transmission Line Simulator
Options: Set Input / Output
d =
λ
d = 0.1666 λ = 49.98 mm
Z_L = 100.0 – j 100.0 Ω
Z_g
V_g
Z_g = 50.0 + j 0.0 Ω
$\tilde{V}_g$ = 5.0 + j 0.0 V
Z_0 = 50.0 + j 0.0 Ω
ε_r = 1.0
f = 1.0 GHz
λ = 300.0 mm
Z_L
d = 0.166667 λ = 50.0 mm
d = 0
Set Line
Length units: [λ] [m]
Low Loss Approximation
Characteristic Impedance Z_0 = 50 Ω
Frequency f = 1E9 Hz
Relative Permittivity ε_r = 1.0
Line Length l = .05 [m]
Update
Z_L = 100 + j -100 Ω
Impedance
Admittance
Update
Set Generator
$\tilde{V}_g$ = 5 + j 0.0 V
Z_g = 50 + j 0.0 Ω
Update
Output
Transmission Line Data 1
Cursor d = 0.1666 λ = 49.98 mm
Impedance [Ω] Z (d) = 12.530782 – j 12.743838 = 17.87249 ∠ –0.7938 rad
Admittance [S] Y (d) = 0.039229 + j 0.039896 = 0.055952 ∠ 0.7938 rad
Reflection Coefficient Γ_d = –0.53543815 – j 0.31292389 = 0.62017367 ∠ –2.612703 rad = 0.62017367 ∠ –149.696881 °
Voltage [V] $\tilde{V}$(d) = 1.161077 – j 0.782796 = 1.40031 ∠ –0.5932 rad
Current [A] $\tilde{I}$(d) = 0.076778 + j 0.015614 = 0.07835 ∠ 0.2006 rad
Power Flow [m W] P_{av} = 38.461538

Problem 2.33 Two half-wave dipole antennas, each with an impedance of 75 Ω, are connected in parallel through a pair of transmission lines, and the combination is connected to a feed transmission line, as shown in Fig. P2.33.

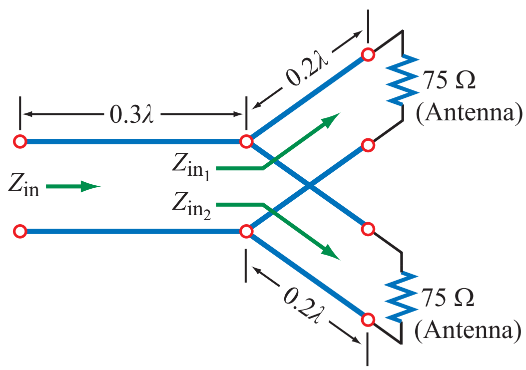


Figure P2.33: Circuit for Problem 2.33.

All lines are 50 Ω and lossless.

(a) Calculate Z_{in_1}, the input impedance of the antenna-terminated line, at the parallel juncture.

(b) Combine Z_{in_1} and Z_{in_2} in parallel to obtain Z'_L, the effective load impedance of the feedline.

(c) Calculate Z_{in} of the feedline.

Solution:

(a)

$$\begin{aligned} Z_{in_1} &= Z_0 \left[\frac{Z_{L_1} + jZ_0 \tan \beta l_1}{Z_0 + jZ_{L_1} \tan \beta l_1} \right] \\ &= 50 \left\{ \frac{75 + j50 \tan[(2\pi/\lambda)(0.2\lambda)]}{50 + j75 \tan[(2\pi/\lambda)(0.2\lambda)]} \right\} = (35.20 - j8.62)\ \Omega. \end{aligned}$$

(b)

$$Z'_L = \frac{Z_{in_1} Z_{in_2}}{Z_{in_1} + Z_{in_2}} = \frac{(35.20 - j8.62)^2}{2(35.20 - j8.62)} = (17.60 - j4.31)\ \Omega.$$

(c)

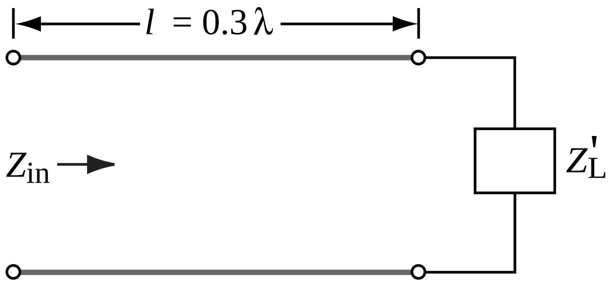


Figure P2.33: (b) Equivalent circuit.

$$Z_{\text{in}} = 50\left\{\frac{(17.60 - j4.31) + j50\tan[(2\pi/\lambda)(0.3\lambda)]}{50 + j(17.60 - j4.31)\tan[(2\pi/\lambda)(0.3\lambda)]}\right\} = (107.57 - j56.7)\ \Omega.$$

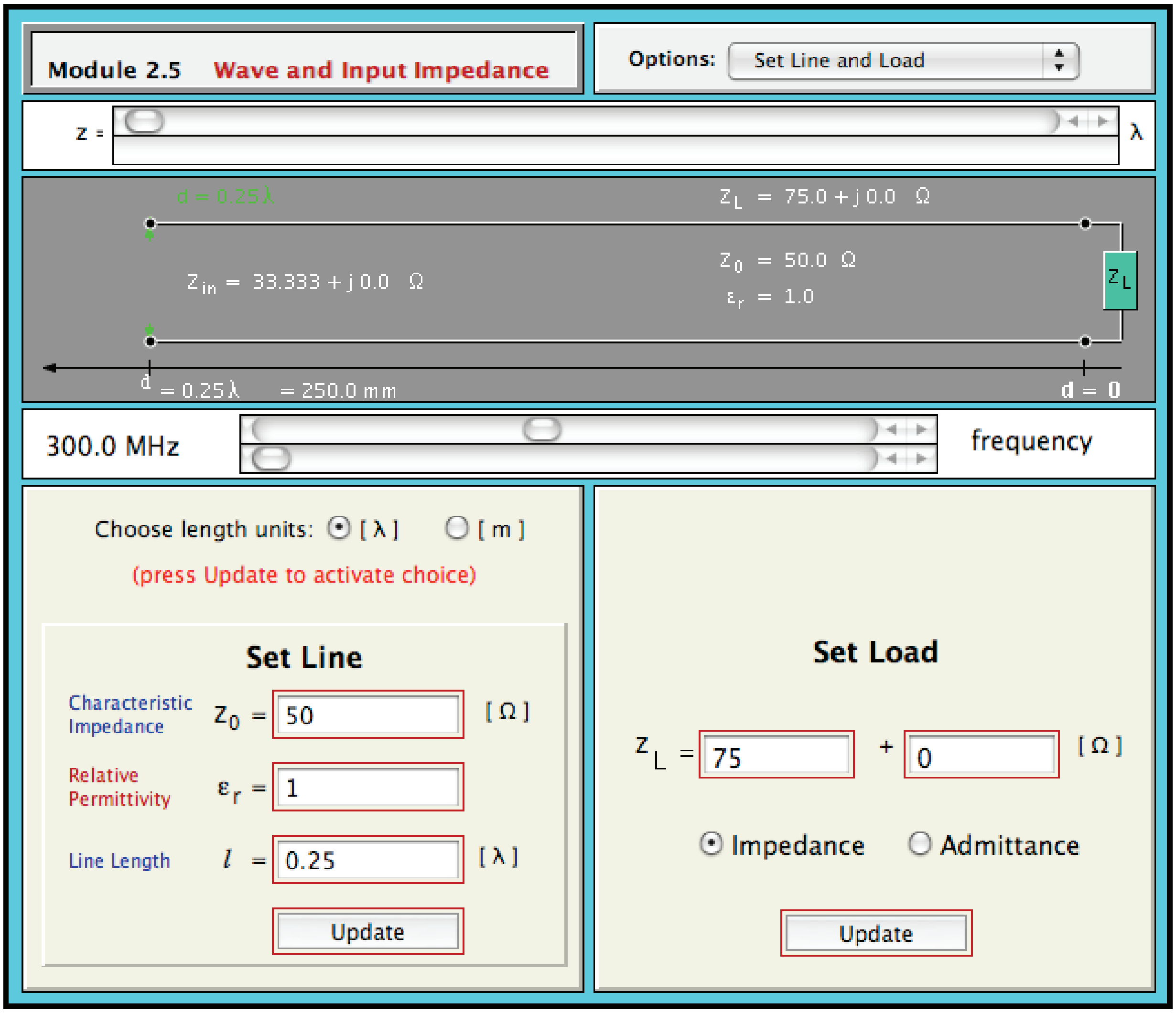
Problem 2.39 A 75-Ω resistive load is preceded by a $\lambda/4$ section of a 50-Ω lossless line, which itself is preceded by another $\lambda/4$ section of a 100-Ω line. What is the input impedance? Compare your result with that obtained through two successive applications of CD Module 2.5.

Solution: The input impedance of the $\lambda/4$ section of line closest to the load is found from Eq. (2.97):

$$Z_{\text{in}} = \frac{Z_0^2}{Z_{\text{L}}} = \frac{50^2}{75} = 33.33\ \Omega.$$

The input impedance of the line section closest to the load can be considered as the load impedance of the next section of the line. By reapplying Eq. (2.97), the next section of $\lambda/4$ line is taken into account:

$$Z_{\text{in}} = \frac{Z_0^2}{Z_{\text{L}}} = \frac{100^2}{33.33} = 300\ \Omega.$$


Module 2.5 Wave and Input Impedance
Options: Set Line and Load
z =
λ
d = 0.25 λ
Z_L = 75.0 + j 0.0 Ω
Z_0 = 50.0 Ω
Z_in = 33.333 + j 0.0 Ω
ε_r = 1.0
Z_L
d = 0.25 λ = 250.0 mm
d = 0
300.0 MHz
frequency
Choose length units: [λ] [m]
(press Update to activate choice)
Set Line
Characteristic Impedance Z_0 = 50 [Ω]
Relative Permittivity ε_r = 1
Line Length l = 0.25 [λ]
Update
Set Load
Z_L = 75 + 0 [Ω]
Impedance Admittance
Update

Problem 2.43 If the two-antenna configuration shown in Fig. P2.43 is connected to a generator with $\widetilde{V}_g = 250$ V and $Z_g = 50\ \Omega$, how much average power is delivered to each antenna?

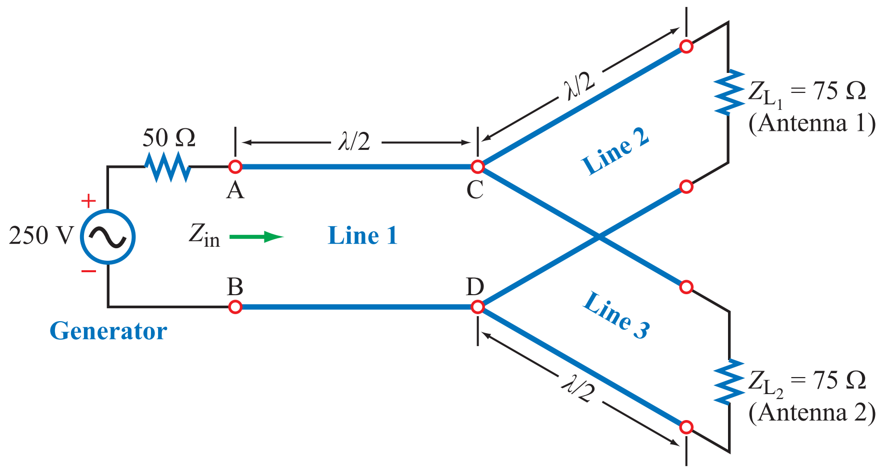


Figure P2.43: Antenna configuration for Problem 2.43.

Solution: Since line 2 is $\lambda/2$ in length, the input impedance is the same as $Z_{L_1} = 75\ \Omega$. The same is true for line 3. At junction C–D, we now have two 75-Ω impedances in parallel, whose combination is $75/2 = 37.5\ \Omega$. Line 1 is $\lambda/2$ long. Hence at A–C, input impedance of line 1 is 37.5 Ω, and

$$\widetilde{I}_i = \frac{\widetilde{V}_g}{Z_g + Z_{in}} = \frac{250}{50 + 37.5} = 2.86 \quad \text{(A)},$$

$$P_{in} = \frac{1}{2}\mathfrak{Re}[\widetilde{I}_i \widetilde{V}_i^*] = \frac{1}{2}\mathfrak{Re}[\widetilde{I}_i \widetilde{I}_i^* \widetilde{Z}_{in}^*] = \frac{(2.86)^2 \times 37.5}{2} = 153.37 \quad \text{(W)}.$$

This is divided equally between the two antennas. Hence, each antenna receives $\frac{153.37}{2} = 76.68$ (W).

Problem 2.74 A 25-Ω antenna is connected to a 75-Ω lossless transmission line. Reflections back toward the generator can be eliminated by placing a shunt impedance Z at a distance l from the load (Fig. P2.74). Determine the values of Z and l.

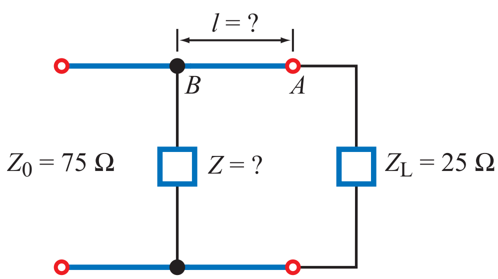
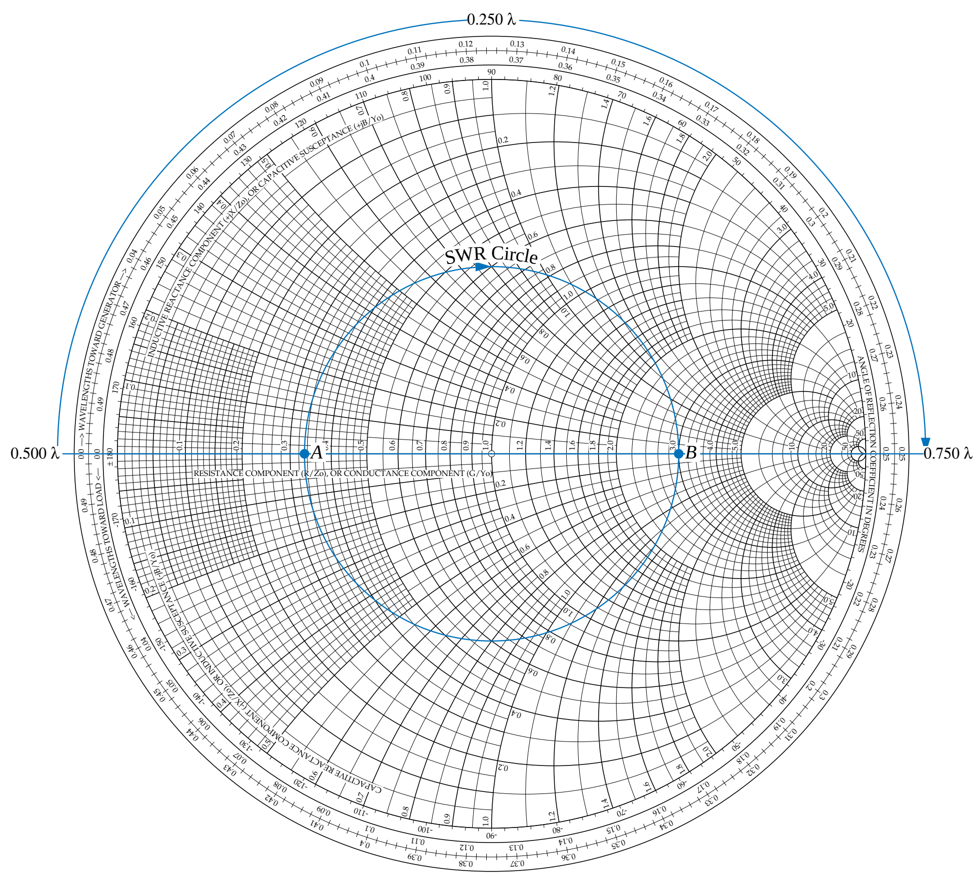


Figure P2.74: Circuit for Problem 2.74.

Solution:



The normalized load impedance is:

$$z_L = \frac{25}{75} = 0.33 \qquad \text{(point } A \text{ on Smith chart)}$$

The Smith chart shows A and the SWR circle. The goal is to have an equivalent impedance of 75 Ω to the left of B. That equivalent impedance is the parallel combination of Z_{in} at B (to the right of the shunt impedance Z) and the shunt element Z. Since we need for this to be purely real, it's best to choose l such that Z_{in} is purely real, thereby choosing Z to be simply a resistor. Adding two resistors in parallel generates a sum smaller in magnitude than either one of them. So we need for Z_{in} to be larger than Z_0, not smaller. On the Smith chart, that point is B, at a distance $l = \lambda/4$ from the load. At that point:

$$z_{in} = 3,$$

which corresponds to

$$y_{in} = 0.33.$$

Hence, we need y, the normalized admittance corresponding to the shunt impedance Z, to have a value that satisfies:

$$\begin{aligned} y_{in} + y &= 1 \\ y &= 1 - y_{in} = 1 - 0.33 = 0.66 \\ z &= \frac{1}{y} = \frac{1}{0.66} = 1.5 \\ Z &= 75 \times 1.5 = 112.5\ \Omega. \end{aligned}$$

In summary,

$$\begin{aligned} l &= \frac{\lambda}{4}, \\ Z &= 112.5\ \Omega. \end{aligned}$$