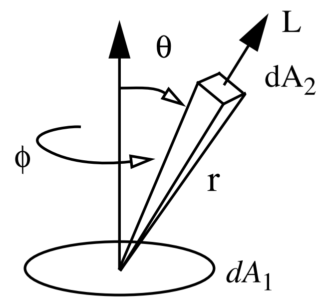
RADIOMETRY OF LAMBERTIAN SOURCES

Relate M_λ and L_λ

- **Valuable exercise in spherical coordinate integration**



- **Area element of sphere, radius r**

$$dA_2 = r d\phi r \sin\theta d\theta$$
$$= r^2 \sin\theta d\theta d\phi$$

- **Start with radiance to get flux within area dA_2**

DR. ROBERT A. SCHOWENGERDT schowengerdt@ece.arizona.edu 520 621-2706 (voice), 520 621-8076 (fax)

$$L(\theta, \phi) = \frac{d^2\Phi}{dA_1 \cos\theta d\Omega}$$

$$d^2\Phi = L(\theta, \phi)\cos\theta dA_1 d\Omega$$

$$= L(\theta, \phi)\cos\theta dA_1 \frac{dA_2}{r^2}$$

$$= L(\theta, \phi)\cos\theta dA_1 \sin\theta d\theta d\phi$$

- **Total flux in hemisphere above source**

$$\Phi = dA_1 \int_0^{2\pi} d\phi \int_0^{\pi/2} L(\theta, \phi)\cos\theta \sin\theta d\theta$$

Now, in general $L = L(\theta,\phi)$, but for a Lambertian source

DR. ROBERT A. SCHOWENGERDT schowengerdt@ece.arizona.edu 520 621-2706 (voice), 520 621-8076 (fax)

L = constant

- **Therefore**

$$\Phi = dA_1 2\pi L \frac{(\sin\theta)^2}{2}\bigg|_0^{\pi/2}$$

$$= \pi L dA_1$$

Radiant exitance definition

$$M = \frac{\Phi}{dA_1}$$

$$= \pi L$$

DR. ROBERT A. SCHOWENGERDT schowengerdt@ece.arizona.edu 520 621-2706 (voice), 520 621-8076 (fax)

Include spectral variation

$$M_\lambda = \pi L_\lambda$$

$$\text{or} \quad L_\lambda = M_\lambda / \pi$$

- Can think of π as having units of *sr*, which cancels the sr^{-1} units of *L*

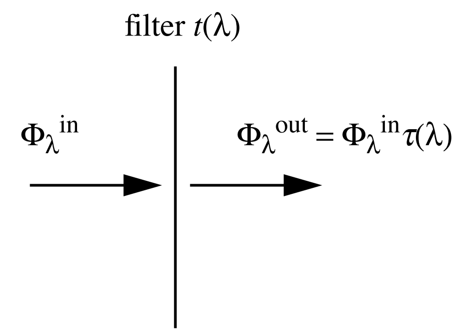
- Note: $M \neq 2\pi L$, as one might guess since hemisphere is 2π *sr*

  Why?

DR. ROBERT A. SCHOWENGERDT schowengerdt@ece.arizona.edu 520 621-2706 (voice), 520 621-8076 (fax)

TRANSMITTANCE

Defined as ratio of transmitted output flux to input flux

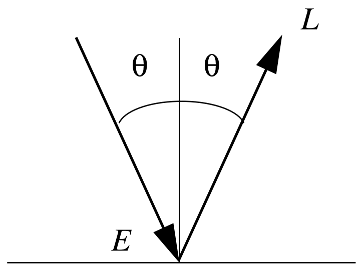


$$t(\lambda) = \Phi_\lambda^{out} / \Phi_\lambda^{in}$$

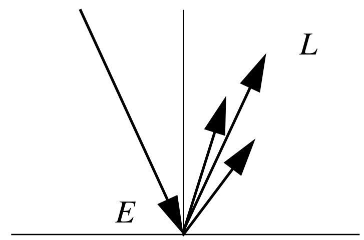
DR. ROBERT A. SCHOWENGERDT schowengerdt@ece.arizona.edu 520 621-2706 (voice), 520 621-8076 (fax)

REFLECTANCE

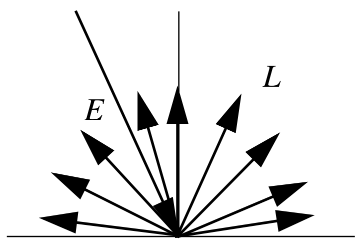
Three types:



specular (mirror)



directional



diffuse (Lambertian)

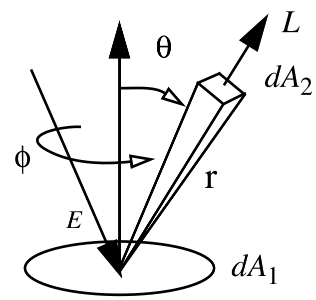
Most natural surfaces are approximately Lambertian

DR. ROBERT A. SCHOWENGERDT schowengerdt@ece.arizona.edu 520 621-2706 (voice), 520 621-8076 (fax)

for θ < 40°, snow and sand for θ < 60°

- At larger θ, natural surfaces tend to become directional

Reflectance of Lambertian surface



- Reflectance defined similarly to transmittance

$$\rho(\lambda) = \Phi_\lambda^{out} / \Phi_\lambda^{in}$$

- From earlier derivation

DR. ROBERT A. SCHOWENGERDT schowengerdt@ece.arizona.edu 520 621-2706 (voice), 520 621-8076 (fax)

$$\Phi_\lambda^{out} = \pi L_\lambda dA_1$$

and

$$\Phi_\lambda^{in} = E_\lambda dA_1$$

Therefore,

$$\rho(\lambda) = \pi L_\lambda / E_\lambda, \quad 0 \le \rho(\lambda) \le 1$$

Given reflectance of surface and incident irrradiance, can get radiance

$$L_\lambda = \rho(\lambda) E_\lambda / \pi$$

DR. ROBERT A. SCHOWENGERDT schowengerdt@ece.arizona.edu 520 621-2706 (voice), 520 621-8076 (fax)

BAND-AVERAGED IRRADIANCE

Cascade spectral quantitites from source-to-surface-to-surface . . .

$$E_\lambda = (\text{geometric factors}) \cdot L_\lambda \cdot t_1(\lambda) \cdot t_2(\lambda) \ldots \rho_1(\lambda) \ldots$$

- **until arriving at a spectrally-integrating element, i.e. a detector,**

$$E_{total} = \int E_\lambda \cdot S(\lambda) d\lambda$$

where $S(\lambda)$ is the detector's spectral sensitivity

Example of a specific detector $S(\lambda)$ is human vision system sensitivity $V(\lambda)$

E_{total} is the effective irradiance, because it is what is

DR. ROBERT A. SCHOWENGERDT schowengerdt@ece.arizona.edu 520 621-2706 (voice), 520 621-8076 (fax)

detected and measured

Band-averaged spectral irradiance

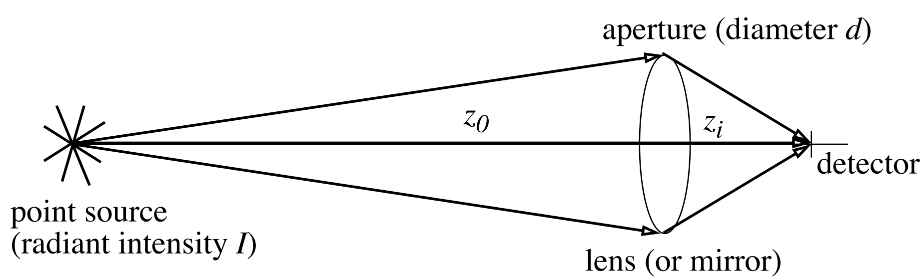
$$E_b = \frac{E_{total}}{\int_0^{\infty} S(\lambda)d\lambda}$$

DR. ROBERT A. SCHOWENGERDT schowengerdt@ece.arizona.edu 520 621-2706 (voice), 520 621-8076 (fax)

RADIOMETRY OF OPTICAL SYSTEMS

telescope

- collects light from point source
- "light bucket"



- assume:
  - source and detector on optical axis
  - object-to-sensor distance much greater than aperture

DR. ROBERT A. SCHOWENGERDT schowengerdt@ece.arizona.edu 520 621-2706 (voice), 520 621-8076 (fax)

diameter $z_0 \gg d$

- no transmission losses

- irradiance at lens aperture $E = I/z_0^2$ (inverse square law)

- flux collected by aperture

$$\Phi_c = E \cdot A_{aperture}$$
$$= E \cdot \frac{\pi d^2}{4}$$
$$= \frac{I}{z_0^2} \cdot \frac{\pi d^2}{4}$$

- flux at detector (no losses) $\Phi_i = \Phi_c$
  - proportional to radiant intensity of source
  - proportional to square of aperture diameter

DR. ROBERT A. SCHOWENGERDT schowengerdt@ece.arizona.edu 520 621-2706 (voice), 520 621-8076 (fax)

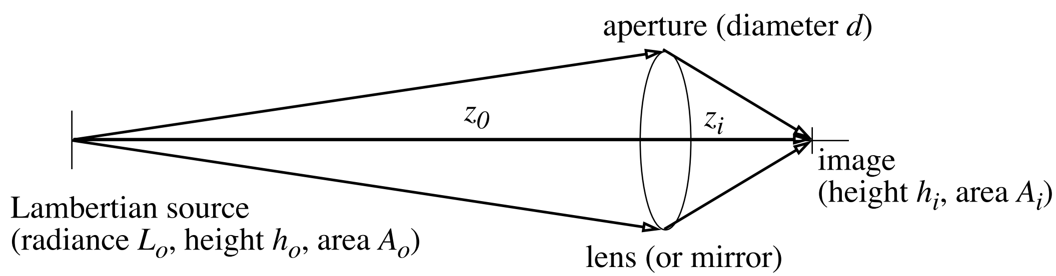
- *inversely proportional to square of object-to-sensor distance*

What is the only way to increase the amount of light collected by a telescope?

Camera

- *assume:*
  - *Lambertian source and detector plane normal to optical axis*
  - *magnification* $m = h_i / h_o = z_i / z_o$

DR. ROBERT A. SCHOWENGERDT schowengerdt@ece.arizona.edu 520 621-2706 (voice), 520 621-8076 (fax)



- ***flux collected by aperture***

$$\Phi_c = L_o A_o \Omega$$

$$= L_o A_o \frac{A}{z_o^2}$$

$$= \frac{L_o A_o}{z_o^2} \cdot \frac{\pi d^2}{4}$$

  - ***NOTE: similarity to "light bucket" equation***

- ***flux at detector (no losses)*** $\Phi_i = \Phi_c$

- ***irradiance at detector***

$$E_i = \Phi_c / A_i$$

$$= \frac{\Phi_c}{m^2 A_o}$$

$$= L_o \frac{\pi d^2}{m^2 4 z_o^2}$$

- ***by definition of magnification (see Geometrical Optics)***

$$E_i = L_o \frac{\pi d^2}{4 z_i^2}$$

- ***define effective f-number N of sensor,*** $N = z_i / d$
  - ***then,*** $E_i = L_o \frac{\pi}{4N^2}$ ***Camera Equation***

- ***NOTE:***
  - ***proportional to radiance of source***

DR. ROBERT A. SCHOWENGERDT schowengerdt@ece.arizona.edu 520 621-2706 (voice), 520 621-8076 (fax)

- *inversely proportional to square of sensor f-number*
- *does* not *depend on* z_o, *the source-to-sensor distance*

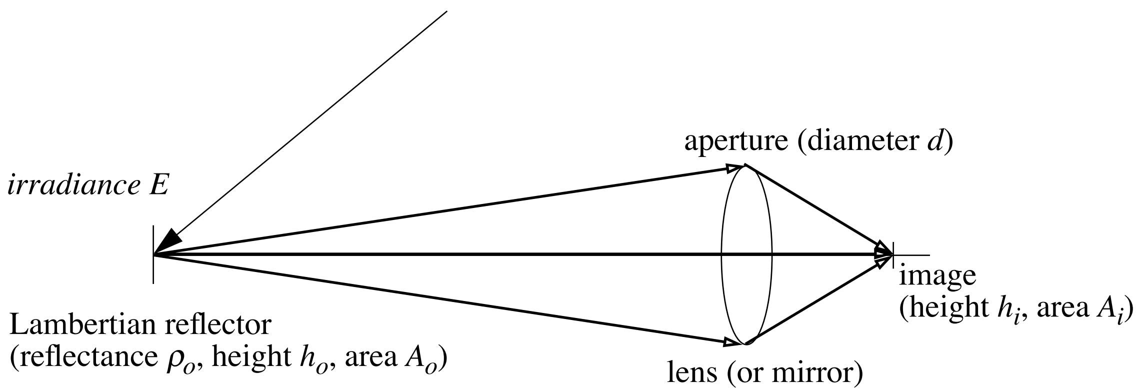
- *f-number N typically preset on camera to 1.4, 2, 2.8, 4, 5.6, 8, 11, 16, 22*

What is rationale for the above preset values of N?

DR. ROBERT A. SCHOWENGERDT schowengerdt@ece.arizona.edu 520 621-2706 (voice), 520 621-8076 (fax)

Camera imaging reflecting object

- **Irradiance on object *E***
- **Reflectance of object ρ_o**

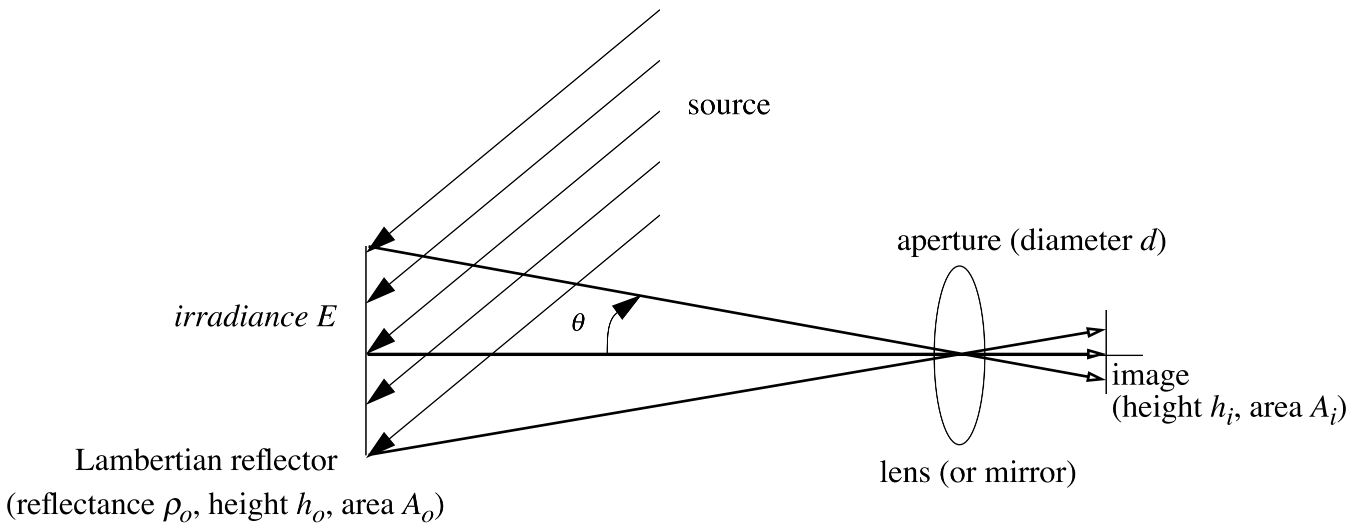


- **From earlier derivation:**

$$E_i = \frac{E_o \rho}{4N^2}$$

DR. ROBERT A. SCHOWENGERDT schowengerdt@ece.arizona.edu 520 621-2706 (voice), 520 621-8076 (fax)

Camera imaging extended reflecting object



- **Cos$^4\theta$ Law applies:**

$$E_i(\theta) = \frac{E_o \rho}{4N^2}(\cos\theta)^4$$

With object irradiance and reflectance fixed, what is the only way to increase the

DR. ROBERT A. SCHOWENGERDT schowengerdt@ece.arizona.edu 520 621-2706 (voice), 520 621-8076 (fax)

amount of light collected by a camera?

DR. ROBERT A. SCHOWENGERDT schowengerdt@ece.arizona.edu 520 621-2706 (voice), 520 621-8076 (fax)