Gain and Phase Margin Problem

Problem
Select the frequency $f_1$ in the gain expression of EQ. 1 below to obtain a two-pole Butterworth step response for a voltage feedback amplifier with $\beta_{FB} = 10 \text{ mV/V}$.

EQ. 1

$$A_v(f) = \frac{A_v0}{\left(1 + j\frac{f}{f_1}\right)\left(1 + j\frac{f}{f_2}\right)\left(1 + j\frac{f}{f_3}\right)}.$$ 

The frequencies $f_2 = 10^6 \text{Hz}$ and $f_3 = 10^7 \text{Hz}$, and the low-frequency gain is $A_{v0} = 10^5 \text{ V/V}$. Also, determine the gain and phase margins of this amplifier.

Schematic
The PSPICE circuit for this amplifier is shown in Figure 1.

**FIGURE 1**

PSPICE representation of three-pole amplifier

Using the circuit of Figure 1 we can find the gain and phase plots, as shown in Figure 2 and Figure 3 below.
FIGURE 3
PSpice phase plots for open loop and closed loop amplifier; this plot determines the frequency $f_{180}$ where the phase is $-180^\circ$; phase margin ($63^\circ$) is labeled with double arrow

Transient response

FIGURE 4
Step response of closed loop amplifier; time to first maximum is 0.98 $\mu$s and overshoot is 5.7%

The step response in Figure 4 can be compared with the two-pole estimates for a Butterworth design of $t_{\text{MAX}} = 1/(f_1 + f_2) = 1 \mu s$ and overshoot of 4.3%.

Design procedure
For a Butterworth response we require a time constant separation factor of $2\beta \text{FB}A_\nu_0$, so

\[ \frac{t_1}{t_2} = \frac{f_2}{f_1} = 2\beta_{\text{FB}}A_\nu_0. \]

That is, $f_1 = f_2/(2 \times 10^{-2} \times 10^5) = 500$ Hz

We then draw the Bode plots for EQ. 1 with the poles $f_1, f_2, f_3$ that approximate Figure 2 and Figure 3, and determine the frequencies $f_{180_{\text{FB}}}$ and $f_{180}$. The gain and phase margins are then

\[ \text{phase margin} = \arg\left(A_\nu(f_1/f_{\text{FB}})\right) - (-180^\circ) = 63^\circ \]

\[ \text{EQ. 4} \]

\[ \text{gain margin} = 20\log_{10}\left(\frac{1}{\beta_{\text{FB}}} \left| A_\nu(f_{180})\right|\right) = 26.9 \text{ dB} \]

The numerical work can be done very conveniently using a spreadsheet, as shown in Figure 5 below.
Open-Loop Amplifier

<table>
<thead>
<tr>
<th>Input</th>
<th>pi</th>
<th>3.1415926</th>
</tr>
</thead>
</table>
| A_v0  | 1.00E+05 | f_1/Bfb  | 4.55E+05  
| f_1   | 500 | f_180    | 3.16E+06  
| f_2   | 1.00E+06 | f_3dB    | 5.00E+02  
| f_3   | 1.00E+07 |          |          
| C_1   | 1.00E-09 |          |          
| B_FB  | 1.00E-02 |          |          

Feedback Amplifier

<table>
<thead>
<tr>
<th>Calculated</th>
<th>R_1</th>
<th>318309.89</th>
<th>Phase margin</th>
<th>63.00875</th>
</tr>
</thead>
<tbody>
<tr>
<td>R_2</td>
<td>159.15495</td>
<td>Gain Margin (dB)</td>
<td>26.85318</td>
<td></td>
</tr>
<tr>
<td>R_3</td>
<td>15.915495</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A_v0 (dB) | 100
A_v (f_3dB) | 96.9897
A_vFB (f_3dB) | 36.9897

Phase=(ATAN2(1,Frequency/f_1)+ATAN2(1,Frequency/f_2)+ATAN2(1,Frequency/f_3))180/pi
Gain=A_v0/(SQRT(1+(Frequency/f_1)^2)*SQRT(1+(Frequency/f_2)^2)*SQRT(1+(Frequency/f_3)^2))

**FIGURE 5**
Spreadsheet for gain and phase margin calculations

With the spreadsheet of Figure 5 the frequencies $f_{1/\beta_{FB}}$ and $f_{180}$ are readily found using GOAL SEEK to set the magnitude to $1/\beta_{FB}$ and the phase to $-180^\circ$ by varying the frequency.

**Comment on the two-pole approximation**

Figure 4 shows that the two-pole approximation to design for a Butterworth amplifier provides a good approximation for setting the lowest pole at $f_1$ provided the higher poles are not too close to $f_2$. In this example the overshoot in step response (Figure 4) isconstantly larger than Butterworth because the third pole makes the gain margin a little lower than for the two-pole system comprised of only of poles at $f_1$ and $f_2$. The time to maximum overshoot is very low nearly as expected.

If we move $f_3$ to very high frequency, the modified system approaches a two-pole system with phase margin $65.6^\circ$, a bit larger than our original system with $63^\circ$ margin. So a two-pole estimate of phase margin is not a bad approximation, and accounts for the success of the two-pole Butterworth design.

However, the two-pole estimate of gain margin is terrible, as explained next. You may recall that a two-pole system is always stable, with $f_{180} = \infty$. Of course, no real amplifier has $f_{180} = \infty$, so this two-pole estimate of $f_{180}$ is hopelessly inaccurate. Because the gain of any amplifier tends to zero at very high frequencies, the gain $\to 0$ as $f_{180} \to \infty$. That is, the poor estimate of $f_{180}$ using a two-pole system makes the two-pole estimate of gain margin hopelessly inaccurate for any real amplifier ($\log_{10}(0) = -\infty$), even if the two-pole system approximates the gain curve quite well over a range of frequencies from low values to somewhere above the second pole.

For these reasons, it is more useful to focus on phase margin as a stability estimate when using a two-pole approximation, not gain margin.