CheE 413
Spring 2000

Exam #1: February 22, 2000

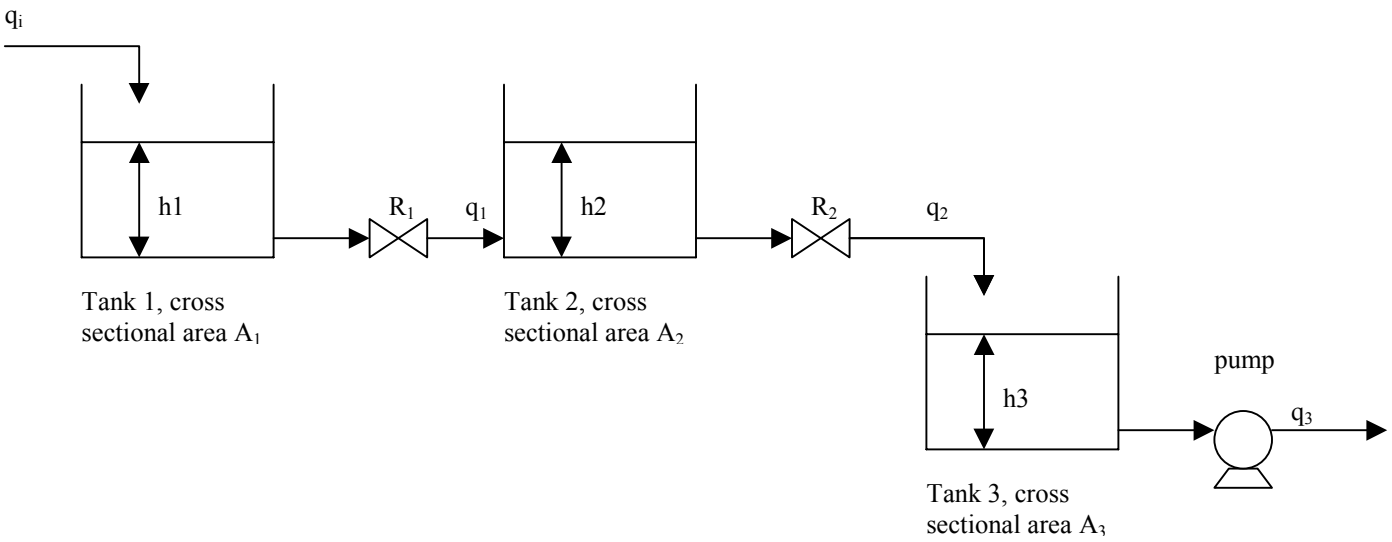
1. Consider a system with the following transfer function:

$$\frac{Y(s)}{X(s)} = \frac{30}{s^2 + 8s + 7}$$

a) (5 pts) Is the system underdamped, overdamped, or critically damped?

b) (20 pts) If $x(t) = e^{-6t}$, find an expression for y(t). Show all work.

2) Consider the 3-tank system shown below:



(20 pts) Find the transfer function H_3/Q_i. R_1 and R_2 are linear resistances to flow.

3. (20 pts). The Calrod heating element transfers heat largely by a radiation mechanism. If the rate of electrical energy input to the heater is Q and the rot temperature and ambient temperatures are T and T_a, respectively, then an appropriate unsteady-state model for the system is:

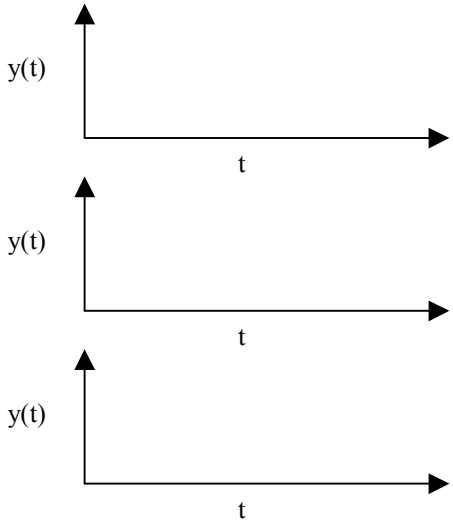
$$mC_p \frac{dT}{dt} = Q - k(T^4 - T_a^4)$$

Find the transfer functions relating T' to Q' and T' to T_a'. (Be sure they are both in standard form.)

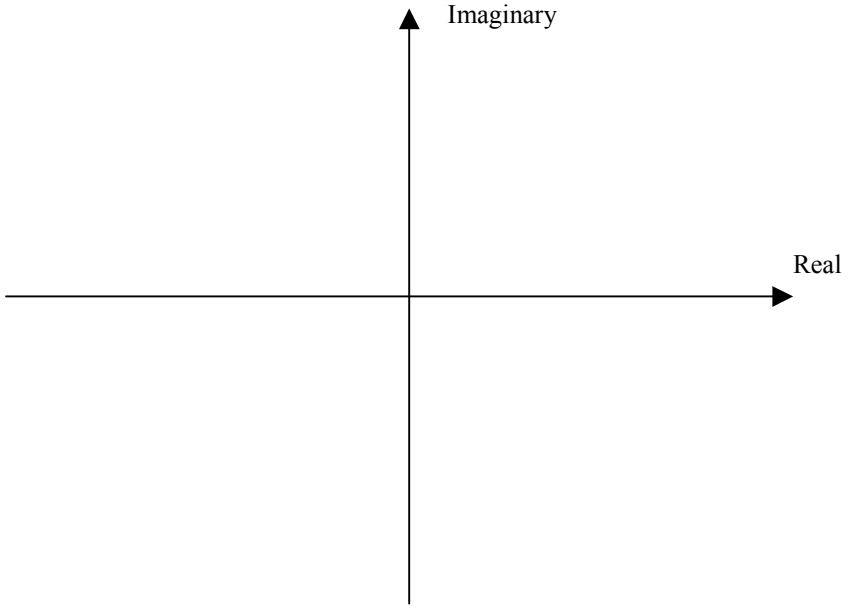
4. Consider the three transfer functions shown below:

A. $\frac{Y(s)}{X(s)} = \frac{2}{2s+1}$ B. $\frac{Y(s)}{X(s)} = \frac{2}{(0.5s+1)(0.25s+1)}$ C. $\frac{Y(s)}{X(s)} = \frac{2}{s^2+3s+1}$

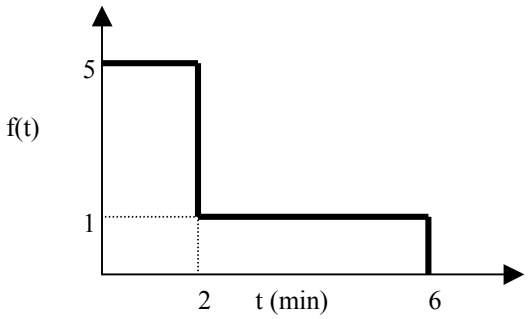
a) (9 pts) Three representative shapes for unit step response curves are shown below (i.e., y(t) for x(t) = a step function). Next to each curve, write the letter of the transfer function that would exhibit a unit step response of the general shape illustrated in the figure.



b) (12 pts). In the complex plane shown below, indicate the locations of the roots of the characteristic equations for the transfer functions shown above. That is, write A, B, and C in the appropriate locations. Clearly label the coordinates of the points.



5. Consider f(t) shown in the figure below:



a) (6 pts) Write a mathematical expression for f(t) in terms of a linear combination of step changes.

b) (8 pts) Write a mathematical expression for the Laplace transform of f(t), which is F(s).