ChEE 413
Spring 2005
Homework Handout 3

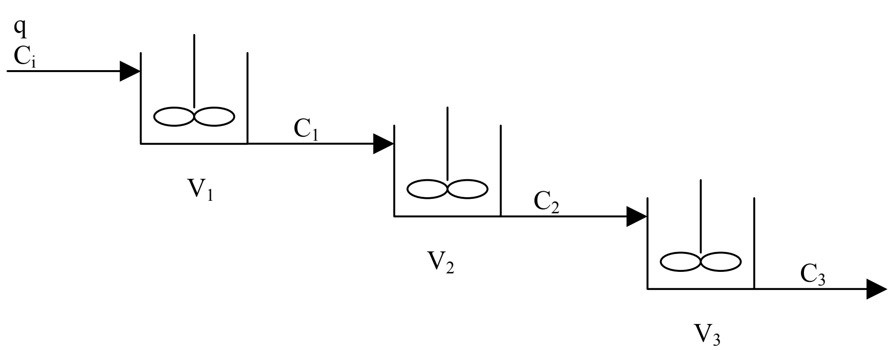
1) Problem 3.14 from Edition One:
Find the solution to the following differential equation:

$$\ddot{x} + \dot{x} + x = \sin t$$

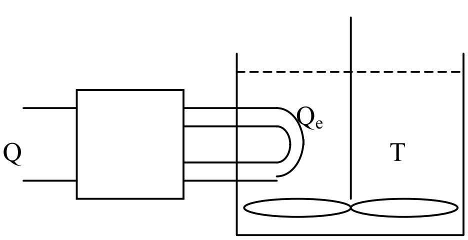
wherex(0) = 0 and dx/dt at x(0) = 0

Use the final value theorem to determine x(t) as t goes to infinity from X(s). What can you say about this result.
Note: dots denote differentiation with respect to time.

2) Problem 3.16 from Edition One:
Three stirred tanks in series are used in a reactor train (see drawing). The flow rate into the system of some inert species is maintained constant while tracer tests are conducted. Assuming that mixing in each tank is perfect and volumes are constant:
a) derive model expressions for the conentration of tracer leaving each tank. C_i is the concentration of tracer entering the first tank.
b) If C_i has been constant and equal to zero for a long time and an operator suddently injects a large amount of tracer material in the inlet to tank 1, what will be the form of $C_3(t)$ (i.e., what kind of time functions will be involved) if
i) $V_1 = V_2 = V_3$
or, ii) V_1, V_2, and V_3 are all not equal to each other
c) If the amount of tracer injected is unknown, is it possible to back-calculate the amount from experimental data? If so, how?



3) Problem 4.2 from Edition One:
A perfectly stirred, constant mass liquid system is heated by an electrical immersion heater element as shown in the drawing (just like an aquarium). If the thermal capacitance of the element is not small or the effective heat transfer coefficient to the liquid is not large, the dynamics of the heater must be considered in developing a model of the system.



a) write an unsteady-state model for this system using more that one differential equation. You may assume that:
   i) The only input is the electrical heating rate of the element Q, which initially is zero
   ii) The rate of energy transfer to the liquid Q_e is given by the usual transport expression.
   iii) Losses to ambient are zero.
b) Find transfer fucntions relating Q_e to Q and T to Q. Put the relations in standard form.

c) Check the units on all gains and time constants in the transfer functions.

Variables you should use:
Q = Electrical heating rate, kW
T_e = temperature of the heater, ^oF
T = temperature of the liquid, ^oF
m_e = mass of the heater, lb
m = mass of the liquid, lb
Ce = specific heat of the heater, Btu/lb ^oF
C = specific heat of the liquid, Btu/lb ^oF
P = conversion factor, kW to Btu/min
h_e = heat transfer coefficient for the heater, Btu/ ^oF min ft^2
A = heat transfer area of the heater, ft^2
Q_e = heat transfer rate, Btu/min

4) Problem 4.4 from Edition One:
A process follows the differential equation model:

$$\frac{d^3y}{dt^3} + 5\frac{d^2y}{dt^2} + 8\frac{dy}{dt} + 4y = 2\frac{dx}{dt} + 3x$$

Find the transfer function relating Y(s) to X(s). Both y and x are deviation variables with zero initial values.