Problem 11.1

A PI controller is to be used in a temperature control loop. For nominal conditions, it has been determined that the closed-loop system is stable when $\tau_I = 10$ min and $-10 < K_c < 0$. Would you expect these stability limits to change for any of the following instrumentation changes? Justify your answers using qualitative arguments.

a) The span on the temperature transmitter is reduced from 40 ^oC to 20 ^oC.

If we go back to chapter 9, we see that the span really affects the gain of the transmitter. The span is the change in input that leads to a change in output. We aren't changing the output, but we are decreasing the input by half. This will lead to a doubling of the transmitter gain. If this happens, we would expect the range of acceptable K_c values to decrease to keep the process in control.

b) The zero on the temperature transmitter is increased from 110 to 130 ^oC.

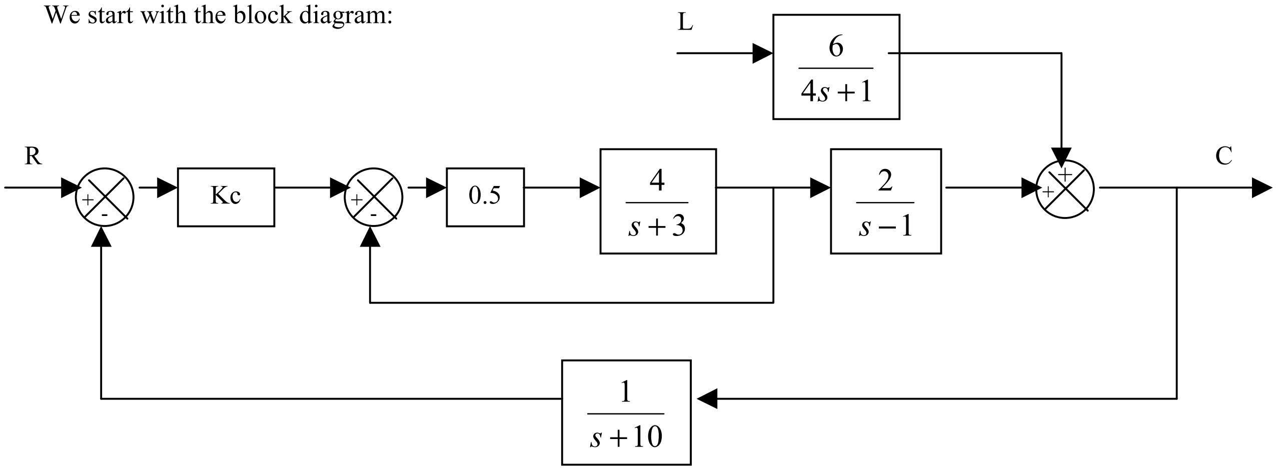
Changing the zero of the transmitter does not change its gain. Therefore, we would not expect the overall stability limits to be affected by this change.

c) The control valve "trim" is changed from linear to equal percentage.

We don't have enough information from our book to explain how this change will change the stability.

Problem 11.2

We start with the block diagram:



Now we will replace the inner loop with an appropriate simplification:

$$\frac{output}{input} = \frac{\Pi_{forward}}{1+\Pi_{loop}} = \frac{0.5\left(\frac{4}{s+3}\right)}{1+0.5\left(\frac{4}{s+3}\right)} = \frac{\frac{2}{s+3}}{1+\frac{2}{s+3}} = \frac{2}{s+3+2} = \frac{2}{s+5}$$

Now we need to find the overall characteristic equation for the entire block diagram, which is equal to 1 + G_{ol}:

$$1+G_{ol} = 1+K_c\left(\frac{2}{s+5}\right)\left(\frac{2}{s-1}\right)\left(\frac{1}{s+10}\right)$$

To determine the stability limits, we have to find the roots of this equation (i.e., when 1 + G_{ol} = 0):

$$1+K_c\left(\frac{2}{s+5}\right)\left(\frac{2}{s-1}\right)\left(\frac{1}{s+10}\right) = 0$$

$$(s+5)(s-1)(s+10)+4K_c = 0$$

$$s^3+14s^2+35s+(4K_c-50) = 0$$

All the powers of s must be positive for the controller to be stable. This gives us:

$$4K_c - 50 > 0$$

$$4K_c > 50$$

$$K_c > \frac{50}{4} = 12.5$$

This gives us one limit of stability. Now we'll apply the Routh stability criteria to see if there are any other limits on K_c to keep the system stable:

1	1	35
2	14	$4K_c - 50$
3	$\frac{-4K_c+540}{14}$	0
4	$4K_c - 50$	

We only got one more criteria from all of the left-hand column's terms needing to be equal to zero. We can now write:

$$\frac{-4K_c + 540}{14} > 0$$
$$-4K_c + 540 > 0$$
$$540 > 4K_c$$
$$135 > K_c$$

The total limits on our stability are $12.5 < K_c < 135$.

Problem 11.4

We can use the figure from page 243 to find the block diagram for this process. It will have the standard controller, transmitter, final control element, and process transfer function blocks. I won't bother sketching it here since we've already done 4 of those already in chapter 10.

We are given that the area is 3 ft^2
$q_{3,ss}$ = 10 gal/min
K_v = -1.3 gal/min/mA
K_m = 4 mA/ft.
τ_v = 10 sec.

We need to find out the values of K_c and τ_I that result in a stable system.

I already see that we are going to have a units problem so let's convert all times to minutes and all volumes to feet so we'll have min, ft, and mA as our units. (Note that we aren't going to have to adjust $q_{3,ss}$ because it isn't going to actually enter into our problem solution).
τ_v = 0.167 min.
K_v = -0.173 ft^3/min mA

We know we are going to need our G_p for our block diagram. Looking at Fig 10.22 and back to page 242 we week that they give us G_p = -1/As

Now let's start:

$$1+G_{ol}=1+G_pG_vG_mG_c=0$$

$$G_m=K_m$$

$$G_c=K_c\left(1+\frac{1}{\tau_I s}\right)$$

$$G_v=\frac{K_v}{\tau_t s+a}$$

Plug everything in to get:

$$1+G_{ol}=1+\frac{-1}{As}\left(\frac{K_v}{\tau_t s+a}\right)(K_m)K_c\left(1+\frac{1}{\tau_I s}\right)4=0$$

$$1-\frac{1}{3s}\left(\frac{-0.173}{0.167s+1}\right)K_c\left(\frac{\tau_I s+1}{\tau_I s}\right)4=0$$

$$1+\frac{1}{3s}\left(\frac{0.173}{0.167s+1}\right)K_c\left(\frac{\tau_I s+1}{\tau_I s}\right)4=0$$

$$3s(0.167s+1)(\tau_I s)+0.692K_c\tau_I s+0.692K_c=0$$

$$0.501\tau_I s^3+3\tau_I s^2+0.0.692K_c\tau_I s+0.0.692K_c=0$$

All of the coefficients must be zero. This tells us that both τ_I and K_c must be greater than zero to make the first, second, and last terms greater than zero.

Now let's apply the Routh stability test to see if there are any other hidden conditions we don't already know about:

1	$0.501\tau_I$	$0.692K_c\tau_I$	0
2	$3\tau_I$	$0.692K_c$	0
3	$\dfrac{2.076K_c\tau_I^{\ 2} - 0.3467K_c\tau_I}{3}$	0	
4	$0.692K_c$	0	

So, we only got one new criteria from the left column. Let's look at it now:

$$\frac{2.076K_c\tau_I^{\ 2} - 0.3467K_c\tau_I}{3} > 0$$

$$0.692K_c\tau_I - 0.1156K_c > 0$$

$$0.692\tau_I - 0.1156 > 0$$

$$0.692\tau_I > 0.1156$$

$$\tau_I > 0.167$$

So, the only requirements that we find for this process are that $K_c > 0$ and $\tau_I > 0.167$ for this to be stable.