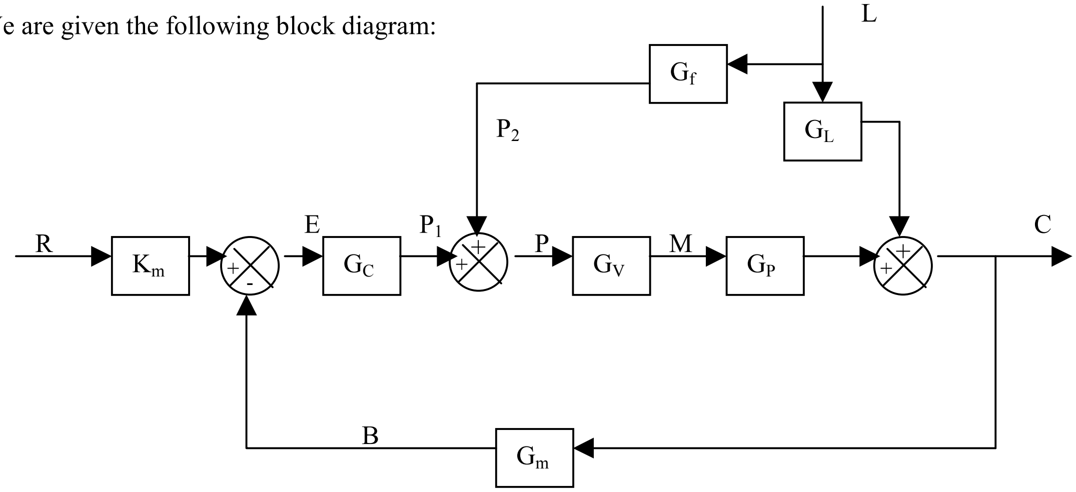
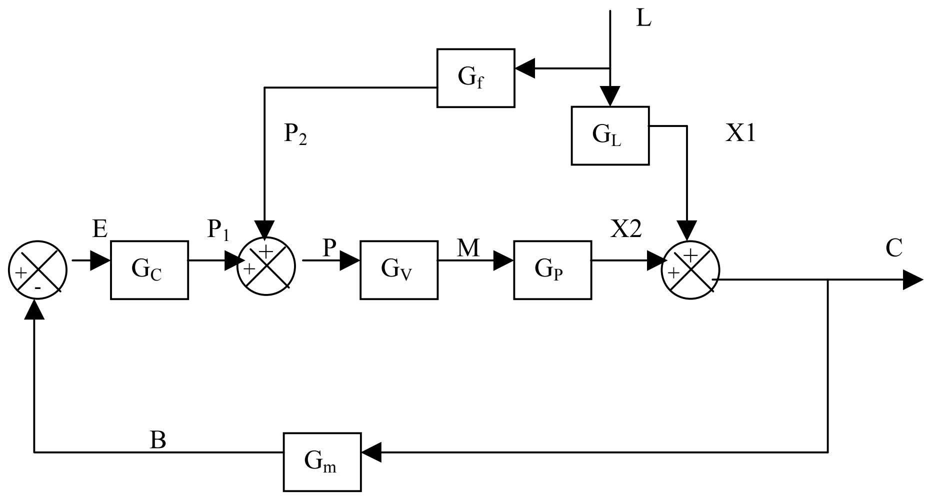
Problem 10.5

We are given the following block diagram:



This looks a little complex so we might not be able to use our short-cut methods of solving this problem. Let's add a few extra variables and go from there (here, we've let R = 0 because we are doing the regulator problem):



We start at C:

$$C = X1 + X2$$
$$X1 = LG_L$$
$$X2 = MG_p$$

But $M = PG_v$ and,

$$P = P_1 + P_2$$

So,

$$C = LG_L + (P_1 + P_2)G_v G_p$$

But,

$$P_2 = LG_f$$
$$P_1 = EG_c$$

So,

$$C = LG_L + (EG_c + LG_f)G_v G_p$$

And, E = R - B = 0 - B = -B. Finally, B = CG_m. Plug this all in to get:

$$C = LG_L + \left(-CG_mG_c + LG_f\right)G_vG_p$$

Now we need to separate C and L from each other:

$$C = LG_L + -CG_mG_cG_vG_p + LG_fG_vG_p$$

$$C + CG_mG_cG_vG_p = LG_L + LG_fG_vG_p$$

$$C\left(1 + G_mG_cG_vG_p\right) = L\left(G_L + G_fG_vG_p\right)$$

So, C/L is:

$$\frac{C}{L} = \frac{G_L + G_fG_vG_p}{1 + G_mG_cG_vG_p}$$

Well, what do you know? This does end up the same as if we had used the short-cut method, but had used both forward paths to reach the output point...

b) Now we want to find out what G_f would have to be to make C = 0 regardless of what L is. Let's keep L as an arbitrary variable for now... We know we want C = 0, so let's plug that in and start to rearrange:

$$0 = \frac{\left(G_L + G_fG_vG_p\right)L}{1 + G_mG_cG_vG_p}$$

$$0 = \left(G_L + G_fG_vG_p\right)L$$

$$0 = G_L + G_fG_vG_p$$

$$G_f = -\frac{G_L}{G_vG_p}$$

That wasn't that bad to do. And, we now see why we talked about needing to be able to model the process correctly in order to use feedforward control. Without knowing the process dynamics, we won't know G_L or G_p so we wouldn't be able to have G_f.

Problem 10.8

We need to introduce a few extra variables in order to make solving this problem easier. We will call X_1 the stream after G_p, X_2 the stream after G_p(tilde), and B the signal from the right comparator heading back into the feedback loop. We will solve for the general case first and then simplify for the servo and regulator parts.

$$C = MG_p + LG_L$$
$$M = EG_c$$
$$E = R - B$$
$$B = C - M\tilde{G}_p$$

So,

$$E = R - \left(C - M\tilde{G}_p\right)$$
$$C = \left(R - \left(C - M\tilde{G}_p\right)\right)G_c G_p + LG_L$$
$$C = RG_c G_p - CG_c G_p + MG_c G_p \tilde{G}_p + LG_L$$

Recall that $C = MG_p + LG_L$ so,

$$M = \frac{C - LG_L}{G_p}$$

Combine everything to get:

$$C = RG_c G_p - CG_c G_p + \left(\frac{C - LG_L}{G_p}\right)G_c G_p \tilde{G}_p + LG_L$$
$$C = RG_c G_p - CG_c G_p + CG_c \tilde{G}_p - LG_L G_c \tilde{G}_p + LG_L$$

Now group like terms:

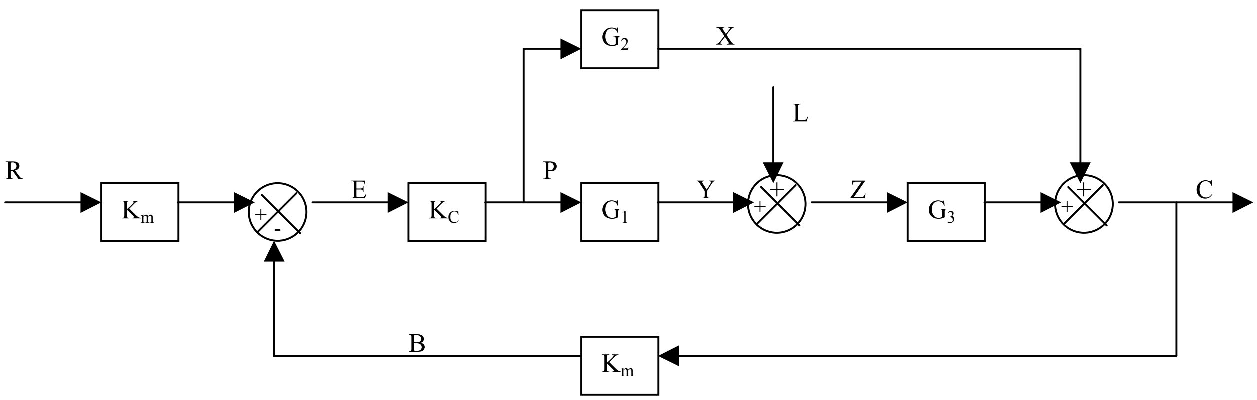
$$C\left(1 + G_c G_p - G_c \tilde{G}_p\right) = RG_c G_p + L\left(G_L - G_L G_c \tilde{G}_p\right)$$

So, we can finally write:

$$\frac{C}{R} = \frac{G_c G_p}{1 + G_c G_p - G_c \tilde{G}_p}$$
$$\frac{C}{L} = \frac{G_L - G_L G_c \tilde{G}_p}{1 + G_c G_p - G_c \tilde{G}_p}$$

Problem 10.12

We are given the following block diagram:



a) we must find C(s)/L(s) for this part. Let's start at C and write down a few equations to get us started:

$$C = X + ZG_3$$
$$Z = L + Y$$
$$Y = PG_1$$
$$X = PG_2$$

Now let's combine all this to get something simpler before we move on:

$$C = PG_2 + (L + PG_1)G_3$$

Let's now write more equations to eliminate P:

$$P = EK_c$$
$$E = RK_m - B$$
$$B = CK_m$$

But recall that we are not interested in R (changes in the set point) right now because we are solving the regulator problem. Let R = 0 and combine these terms:

$$C = -CK_mK_cG_2 + (L + -CK_mK_cG_1)G_3$$

Now rearrange to group the C and L terms on different sides of the equals sign:

$$C + CK_mK_cG_2 + CK_mK_cG_1G_3 = LG_3]$$
$$C(1 + K_mK_cG_2 + K_mK_cG_1G_3) = LG_3$$
$$\frac{C}{L} = \frac{G_3}{1 + K_mK_cG_2 + K_mK_cG_1G_3}$$

b) now that we have a transfer function that describes this complex block diagram, we'll plug in the following functions:

$$G_1(s) = \frac{0.3}{s+1} \qquad G_3(s) = \frac{0.9e^{-s}}{12s+1}$$
$$G_2(s) = \frac{1}{10s+1} \qquad K_m = 0.6$$

Let's plug these functions into our transfer function:

$$\frac{C}{L}=\frac{\left(\dfrac{0.9e^{-s}}{12s+1}\right)}{1+0.6K_c\left(\dfrac{1}{10s+1}\right)+K_c0.6\left(\dfrac{0.3}{s+1}\right)\left(\dfrac{0.9e^{-s}}{12s+1}\right)}$$

Now we'll try to simplify by multiplying by (10s + 1) in the top and bottom, and by (s + 1) and (12s + 1) as well:

$$\frac{C}{L}=\frac{0.9(s+1)(10s+1)e^{-s}}{(10s+1)(s+1)(12s+1)+0.6K_c(s+1)(12s+1)+K_c0.162(10s+1)e^{-s}}$$

We are putting in a load of -2, so L(s) = -2/s. Plugging this in gives:

$$C=\frac{-1.8(s+1)(10s+1)e^{-s}}{s\left((10s+1)(s+1)(12s+1)+0.6K_c(s+1)(12s+1)+K_c0.162(10s+1)e^{-s}\right)}$$

Hmmm....We were asked to find the value of K_c that would give an offset of 0.4 after this step change. One way would be to now expand this out and put it into the time domain before evaluating that function as time goes to infinity. This would also involve using the Pade approximation to find this solution. That gives us at least fourth order in the denominator, which I don't feel like dealing with. Isn't there a shorter way?

Of course! Let's use the final value theorem to find what C goes to as time goes to infinity:

$$\lim_{s\to 0}sC(s)=\frac{-1.8((0)+1)(10(0)+1)e^{-(0)}}{\left((10(0)+1)((0)+1)(12(0)+1)+0.6K_c((0)+1)(12(0)+1)+K_c0.162(10(0)+1)e^{-(0)}\right)}$$

$$=\frac{-1.8(1)(1)(1)}{((1)(1)(1)+0.6K_c(1)(1)+K_c0.162(1)(1))}=\frac{-1.8}{1+0.6K_c+0.162K_c}$$

$$=\frac{-1.8}{1+0.762K_c}$$

Now we have to find the offset using it's definition. offset = desired value - final steady state value. Our problem is that we need to know what the desired value is. Do we want our output (the variable we are trying to control) to change when we have a load come in or not? Let's think of an analogy: Do we want the temperature in our living room to drop just because it's cold outside? No! So, we want C = 0. This gives us:
We really didn't have to make any assumptions to solve this problem.

$$offset=0.4=0-new\ s.s.\ value$$

$$-0.4=\frac{-1.8}{1+0.762K_c}$$

$$0.4(1+0.762K_c)=1.8$$

$$0.4+0.3048K_c=1.8$$

$$K_c=4.593$$