Problem 16.5

This problem is kind of unusual in that we are given the block diagram and asked to come up with the transfer function to describe it. All of our examples and homework have gone in the opposite direction. Let's start with the AR plot and see what we can get from that...

We can see that there are two break frequencies in the diagram. They are at 0.1 rad/min and the other one looks like it is at 1 or 2 rad/min. We'll say that it's at 1 and see what else we can figure out. There is another function that starts at about 10 rad/min (It's kind of hard to see, but the slope decreases again after that point.) At very low frequencies, we have a slope of -1. This implies that there is a pure 1/s function in our transfer function (recall how the integral only controller responded at low frequencies...it went to a slope of -1. We know there is some kind of s function in the numerator that starts at $\omega = 0.1$ rad/min. So the time constant for this process is $1/\omega_b = 1/0.1 = 10$. The function that starts at 1 rad/min is in the denominator because the slope is -1 after that point and its time constant is 1. The last part of the transfer function has a time constant of 0.1. At this point we have something that is starting to shape up as:

$$G(s) = \frac{10s+1}{s(s+1)(0.1s+1)}$$

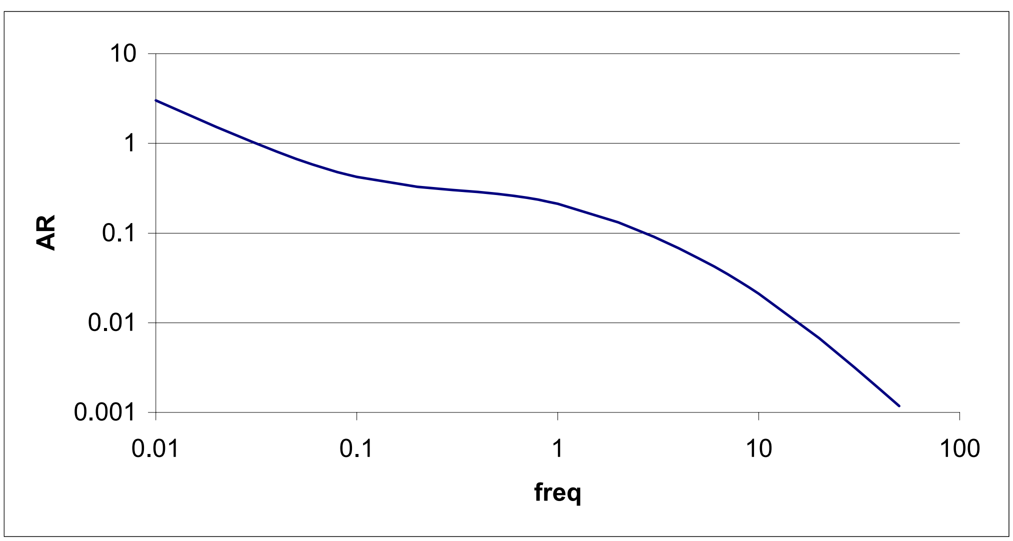
However, there is another piece that we're missing. There is some kind of gain in our process. Notice that our AR is less than one when we hit the first break frequency (0.1) and is about 0.3 at this point. This suggests that we have a constant $K_c = 0.03$. This gives us:

$$G(s) = \frac{0.3(10s+1)}{s(s+1)(0.1s+1)}$$

Let's go through a see what our AR and ϕ functions would be and then see if that gives us good agreement with the plots in the book:

Function	AR equation	ϕ equation	ω_b	AR after break
0.3	0.03	0	---	---
10s+1	sqrt(1+(10ω)^2)	$\tan^{-1}$(10ω)	0.1	+1
1/s	1/ω	-90°	---	-1 everywhere
1/s+1	1/(sqrt(1+ω^2))	$\tan^{-1}$(-ω)	1	-1
1/0.1s+1	1/(sqrt(1+(0.1ω)^2))	$\tan^{-1}$(-0.1ω)	10	-1

Here is what our AR plot looks like:



And here is what our phase angle looks like:

Problem 16.9

First, let's find the open loop function. This is:

$$G_{ol} = G_c G_v G_p G_m$$

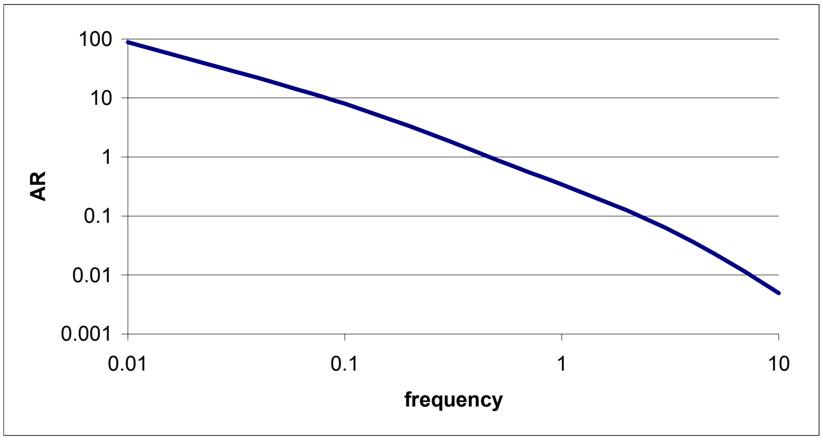
When we plug in the functions for these transfer functions we get:

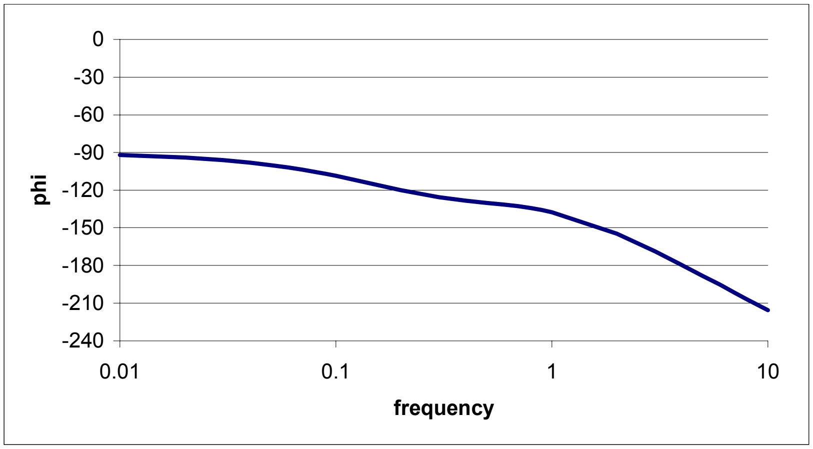
$$G_{ol} = \frac{K_c(2s+1)}{0.1s+1}\frac{2}{0.5s+1}\frac{0.4}{s(5s+1)}(1)$$

We can make a table of the functions that would be needed to find the complete AR and ϕ:

Function	**AR equation**	**ϕ equation**
K_c	K_c	0
2s+1	sqrt(1+4ω^2)	$\tan^{-1}(2\omega)$
1/(0.1s+1)	1/(sqrt(1+0.01ω^2))	$\tan^{-1}(-0.1\omega)$
2	2	0
1/(0.5s+1)	1/(sqrt(1+0.25ω^2))	$\tan^{-1}(-0.5\omega)$
0.4	0.4	0
s	1/ω	-90
1/(5s+1)	1/(sqrt(1+25ω^2))	$\tan^{-1}(-5\omega)$

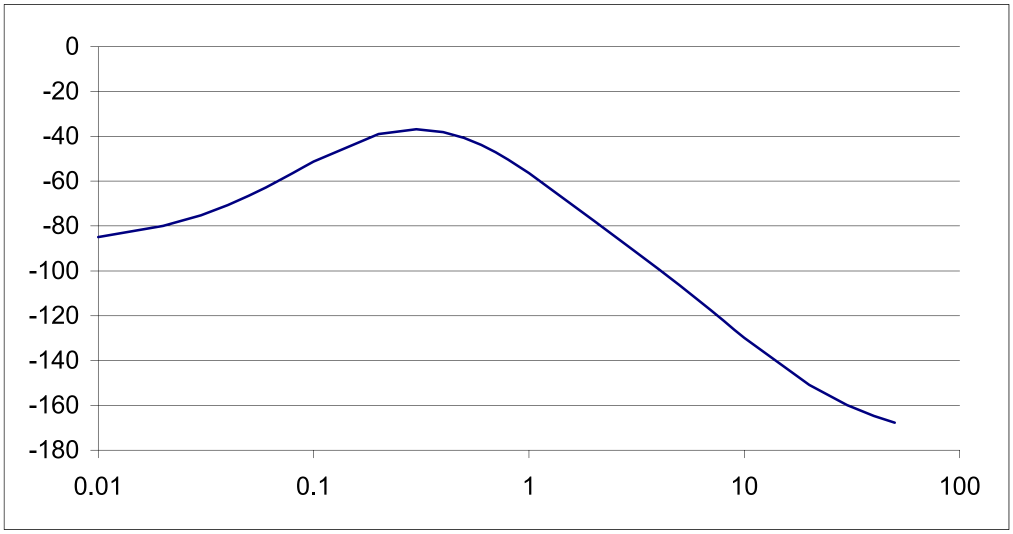
We can plot these functions for the Bode plot to get:





b) We now need to find the value of K_c that would give a phase margin of 30^o using the figure on the previous page. PM = 30 = 180 + ϕ_g gives $\phi_g = -150^o$. Here the critical frequency is 1.72. This leads to an AR at this frequency = 0.1438 To get to the phase margin, $AR_{total} = 1.0$ So, $AR_{ol} * AR_{Kc} = 1$ and $K_c = 1/0.1438 = 6.954$

c) When $K_c = 10$, we can find the new gain margin
Using our functions again with $K_c = 10$, we see that the frequency is equal to 4.053 to get the AR = 0.3584 This leads to GM = 2.79.



We're a little off on this one, but I think that's because we plotted the true numbers instead of quickly sketching out the function.

b) We now choose a value of K_c that gives a gain margin of 1.7. Let's see what that would have to be first. To find GM, we start with the phase angle equal to -180 and find the critical frequency. From the data in the book's plot we see that ω_c is about 50 rad/min, leading to an AR = 0.003. This leads to:

$$1.7 = GM = \frac{1}{AR_{-180^o} K_c}$$

$$K_c = \frac{1}{1.7 * 0.003} = 196.08$$

With this Kc added in, our whole amplitude ratio is going to shift down by a factor of 196.08. To find phase margin we start with AR_{total} = 1.0 and find the frequency where that occurs. This leads to a ϕ_g = -159.85 where AR_{total} = 1.0. So PM = 180 -159.85 = 20.15^o.

Problem 16.12

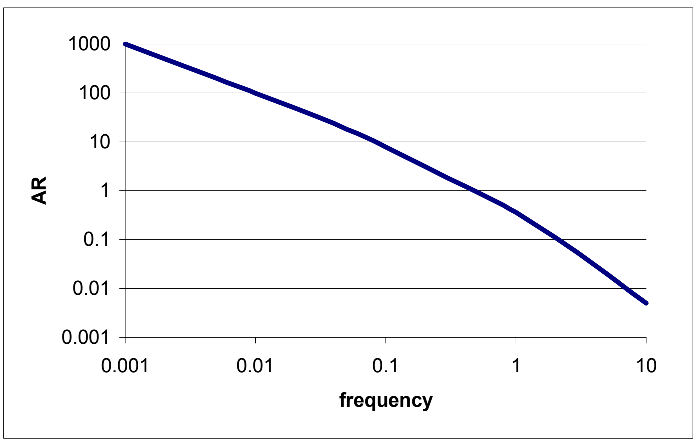
We can write the standard $G_{ol} = G_c G_v G_p G_m$ and get:

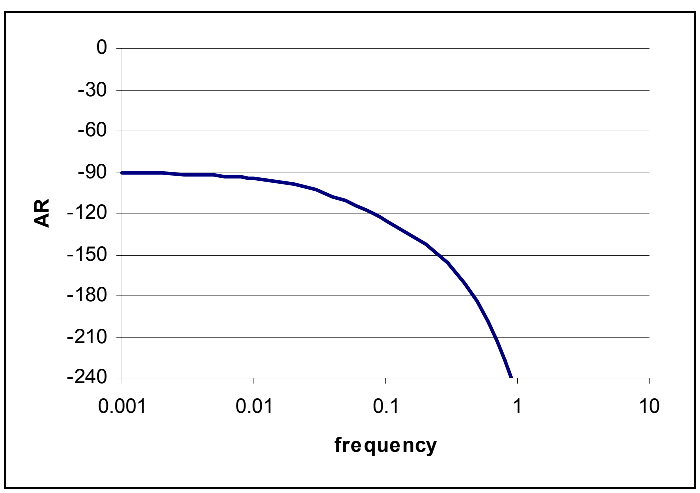
$$G_{ol} = K_c\left(1+\frac{1}{5s}\right)(1)\left(\frac{1}{s+1}\right)\frac{5e^{-2s}}{10s+1}$$
$$= K_c\left(\frac{5s+1}{5s}\right)\left(\frac{1}{s+1}\right)\frac{5e^{-2s}}{10s+1}$$

We can rewrote this slightly and now can make our table of functions:

Function	**AR equation**	**ϕ equation**
5s+1	sqrt(1+25ω^2)	$\tan^{-1}(5\omega)$
1/5s	1/5ω	-90^o
1/(s+1)	1/(sqrt(1+ω^2))	$\tan^{-1}(-\omega)$
5	5	0
e^{-2s}	1	-2ω
1/(10s+1)	1/(sqrt(1+100ω^2))	$\tan^{-1}(-10\omega)$

Now we'll plot the total AR and ϕ:





b) We can look at the Bode stability criteria to find the limits on K_c for stability. At -180°, $\omega_c = 0.47$. Here, the AR is equal to 1.023. This tells us that the K_c we put in must be less than 1/1.023 = 0.9775 for this system to be stable.

c) Now we use $K_c = 0.2$ and see that the frequency when AR =1 is 0.14. At this point, $\phi = -133.48$. PM = 180 - 133.48 = 46.52°.

d) Again, we start with $\phi = 180°$ and find the critical frequency, which is still 0.47. To get a GM = 1.7 we can write:

$$GM = 1.7 = \frac{1}{1.023 * AR_{K_c}}$$

$$K_c = 0.71301$$

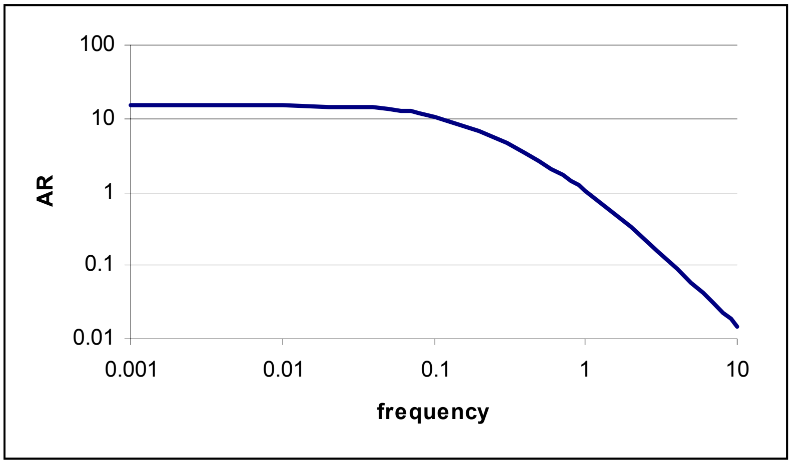
Problem 16.15

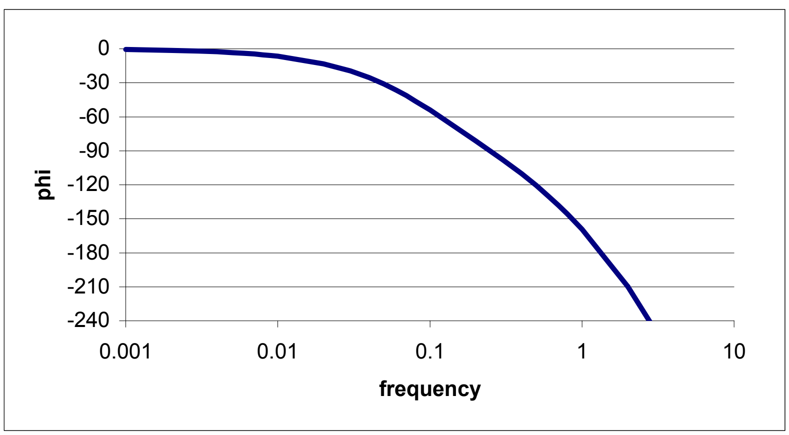
We'll do the same thing we've done on all the other problems where $G_{ol}=G_mG_vG_p$:

$$G_{ol} = e^{-0.5s}\left(\frac{-10}{s+1}\right)\left(\frac{1.5}{10s+1}\right)$$

We'll make our table of functions to get:

Function	AR equation	φ equation
$e^{-0.5s}$	1	-0.5ω
-10	10	0
1.5	1.5	0
1/(s+1)	1/sqrt(1+ω²)	tan⁻¹(-ω)
1/(10s+1)	1/(sqrt(1+100ω²)	tan⁻¹(-10ω)



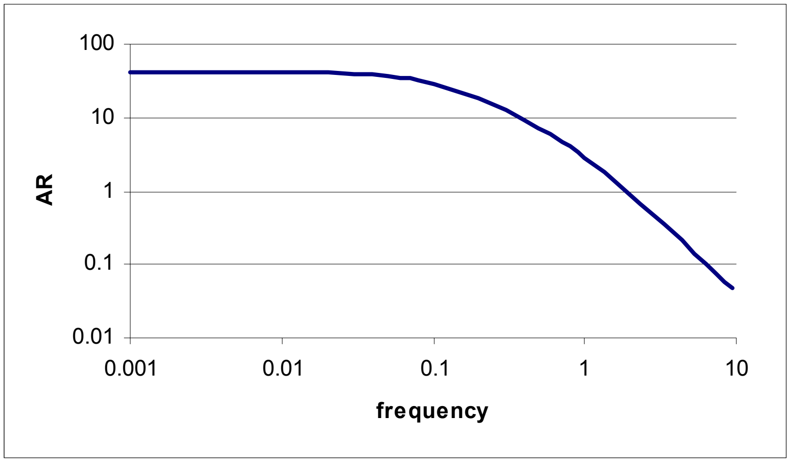


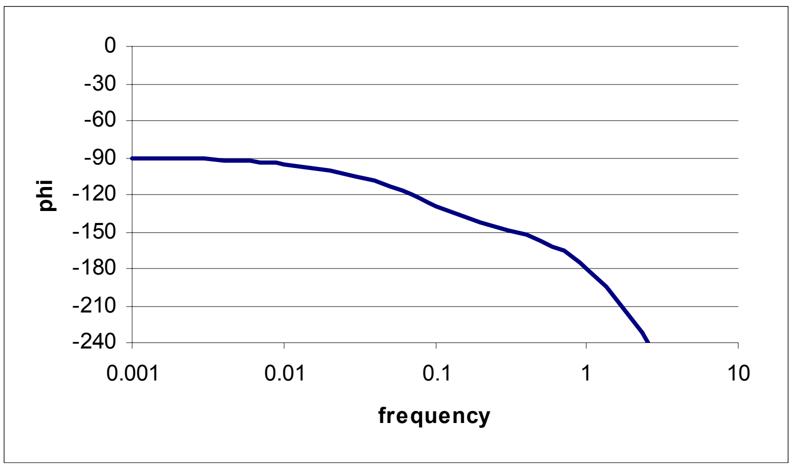
b) The simplest way to design a controller is to use the Ziegler-Nichols criteria from page 298. We need to find the K_u from our plots using the phase angle -180°. The frequency at -180° is 1.36 and this gives an AR of 0.6516. The K_u is 1/AR = 1/0.6516 = 1.5347. The P_u (period) is 2*pi/ω_c = 4.619 sec. We look at page 298 for a PI controller and see that $K_c = 0.45K_u = 0.6906$ and $\tau_I = P_u/1.2 = 3.849$ sec.

Using these settings we'll add a PI controller to our Bode plot. The transfer function is:

$$G_c = 0.6906\left(1+\frac{1}{3.849s}\right)=\frac{2.6581s+1}{3.849s}$$

And the Bode plot contributions are:
AR = sqrt(1+7.066ω^2)/3.849ω and ϕ = $\tan^{-1}$(-2.6581ω) - 90°. We'll add these to our Bode plots to get:



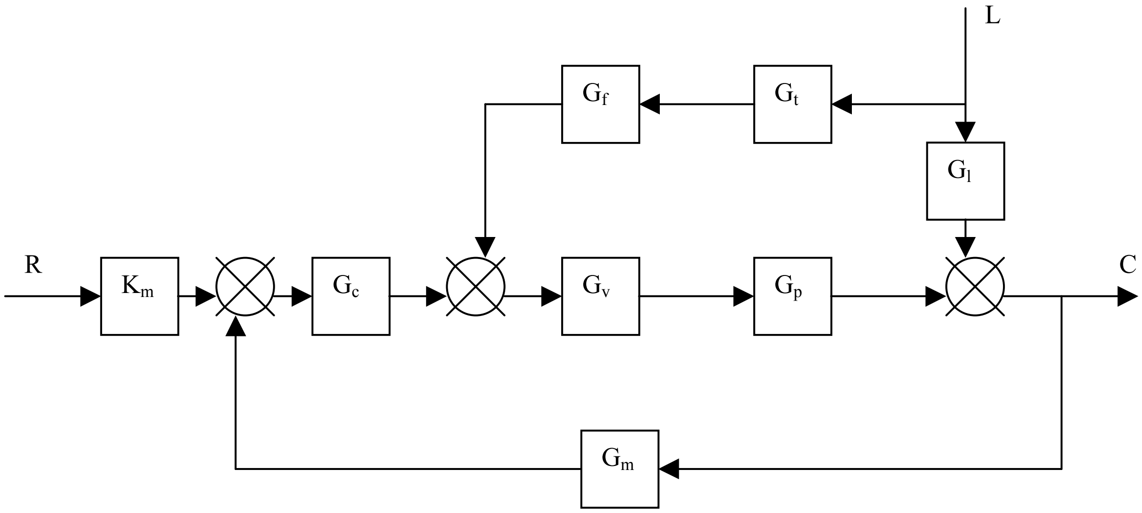


c) We can use these plots to evaluate the stability of the feedback controller now. The critical frequency where ϕ = -180 is 1.015. This frequency gives an AR = 0.458 and a GM = 1/0.458 = 2.183. Using an AR of 1 leads to a ϕ_g = 0.829. The phase angle for this is -165.375 which gives a PM = 14.625.
So, our GM is right about where we want it even though the PM is a bit low. This system will be moderately stable.

d) Finally, we put in a sinusoidal forcing function of 1.5sin0.5t. With this, we can plug in the frequency to our function to see what our AR is at that frequency. It is 2.0478. This leads to an output amplitude of 2.0478*1.5 = 3.0717.

Problem 17.7

b) We have a standard block diagram for a feedforward process that looks like:



In this case, $G_v = G_m = G_t = 1$. We are left with only G_p and G_l (assuming that we have no controller at first).

We know that our C/L can be written as:

$$\frac{C}{L} = \frac{G_t G_f G_v G_p + G_l}{1 + G_c G_v G_p G_m}$$

We want C to be equal to zero for any value of L so we know the numerator must be equal to zero. This gives us:

$$(1)G_f(1)G_p + G_l = 0$$

$$G_f = -\frac{G_l}{G_p} = -\frac{\frac{2}{(s+1)(5s+1)}}{\frac{1}{s+1}}$$

$$G_f = -\frac{2(s+1)}{(s+1)(5s+1)} = -\frac{2}{5s+1}$$

We can rearrange this to solve for G_f:

d) We plug the G_f into our big C/L transfer function and start to add the unit step change in L:

$$\frac{C}{L} = \frac{G_f G_p + G_l}{1 + G_p}$$

$$\frac{C}{L} = \frac{-\frac{2}{5s+1}\left(\frac{1}{s+1}\right) + \frac{2}{(s+1)(5s+1)}}{1 + \frac{1}{s+1}}$$

We see that the top part will cancel out (just like we wanted) so C = 0 for all time.