Problem 14.1

Fine the response of the first order system:

$$G(s) = \frac{2}{0.2s+1}$$

to a sinusoidal input x(t) = sin 2t and plot it.
We'll start by finding X(s):

$$X(s) = \frac{2}{s^2+4}$$

Now find Y(s) = G(s)X(s):

$$Y(s) = \left(\frac{2}{0.2s+1}\right)\frac{2}{s^2+4}$$

Now do our partial fraction expansion:

$$Y(s) = \left(\frac{2}{0.2s+1}\right)\frac{2}{s^2+4} = \frac{A}{0.2s+1} + \frac{Bs+C}{s^2+4}$$

Do Heaviside expansion to find A:

$$Y(s) = \frac{4}{(-5)^2+4} = A = \frac{4}{29}$$

We can't do Heaviside for B and C so let's choose a value for s and then solve for one of them. Let s = 0

$$\left(\frac{2}{0.2(0)+1}\right)\frac{2}{0^2+4} = \frac{\frac{4}{29}}{0.2(0)+1} + \frac{B(0)+C}{0^2+4}$$

$$\frac{4}{4} = 1 = \frac{4}{29} + \frac{C}{4}$$

$$C = \frac{100}{29}$$

Now we'll let s = 5 and then find B (I chose 5 to keep my fractions tidy:

$$\left(\frac{2}{0.2(5)+1}\right)\frac{2}{5^2+4} = \frac{\frac{4}{29}}{0.2(5)+1} + \frac{B(5)+\frac{100}{29}}{5^2+4}$$

$$\frac{2}{29} = \frac{2}{29} + \frac{B(5)+\frac{100}{29}}{29}$$

$$0 = B(5) + \frac{100}{29}$$

$$B = \frac{-20}{29}$$

We can plug everything in to get:

$$Y(s) = \frac{\frac{4}{29}}{0.2s+1} + \frac{-20}{29}\frac{s}{s^2+4} + \frac{50}{29}\frac{2}{s^2+4}$$

Remember the tricks we used to do this before?

Problem 14.3

We vary the input into our system as P' = 0.5 sin 0.2t. Let's take this into the Laplace domain:

$$P'(s) = \frac{(0.5)0.2}{s^2 + 0.04}$$

We're going to multiply by the first two transfer functions to find the function to describe T'(s):

$$T'(s) = \left(\frac{0.1}{s^2 + 0.04}\right)\left(\frac{10}{s+1}\right)\left(\frac{1}{5s+1}\right)$$

$$T'(s) = \frac{0.2}{(s^2 + 0.04)(s+1)(s+0.2)}$$

Do partial fraction expansion to get:

$$T'(s) = \frac{0.2}{(s^2 + 0.04)(s+1)(s+0.2)} = \frac{A}{s+1} + \frac{B}{s+0.2} + \frac{Cs+D}{s^2 + 0.04}$$

Now use Heaviside to get A and B:

$$\frac{0.2}{(-1^2 + 0.04)(-1+0.2)} = A = -0.240$$

$$\frac{0.2}{(-0.2^2 + 0.04)(-0.2+1)} = B = 3.125$$

Now we'll plug these two values in and let s = 0 to find D:

$$\frac{0.2}{(0^2 + 0.04)(0+1)(0+0.2)} = \frac{-0.24}{0+1} + \frac{3.125}{0+0.2} + \frac{C(0)+D}{0^2 + 0.04}$$

$$\frac{0.2}{(0.04)(1)(0.2)} = \frac{-0.24}{1} + \frac{3.125}{0.2} + \frac{D}{0.04}$$

$$9.615 = \frac{D}{0.04}$$

$$D = 0.3846$$

Now we'll just let s = 1 to find C:

$$\frac{0.2}{(1^2 + 0.04)(1+1)(1+0.2)} = \frac{-0.24}{1+1} + \frac{3.125}{1+0.2} + \frac{C(1)+0.3846}{1^2 + 0.04}$$

$$\frac{0.2}{(1.04)(2)(1.2)} = \frac{-0.24}{2} + \frac{3.125}{1.2} + \frac{C+0.3846}{1.04}$$

$$-2.404 = \frac{C+0.3846}{1.04}$$

$$C = -2.8848$$

So,

$$T'(s) = \frac{-0.24}{s+1} + \frac{3.125}{s+0.2} + \frac{-2.8848s + 0.3846}{s^2 + 0.04}$$

We need to rewrite this a little bit before we invert back into the time domain:

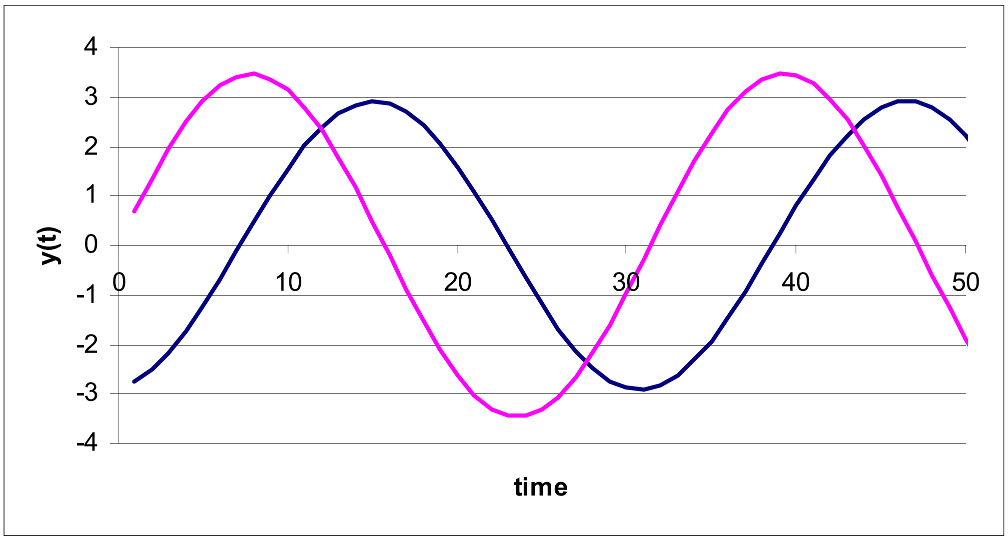
$$T'(s) = \frac{-0.24}{s+1} + \frac{3.125}{s+0.2} + \frac{-2.8848s}{s^2+0.04} + \frac{0.3846}{s^2+0.04}$$

Invert to get:

At long times, both exponential terms will go to zero and we'll just be left with the sin and cos functions. These

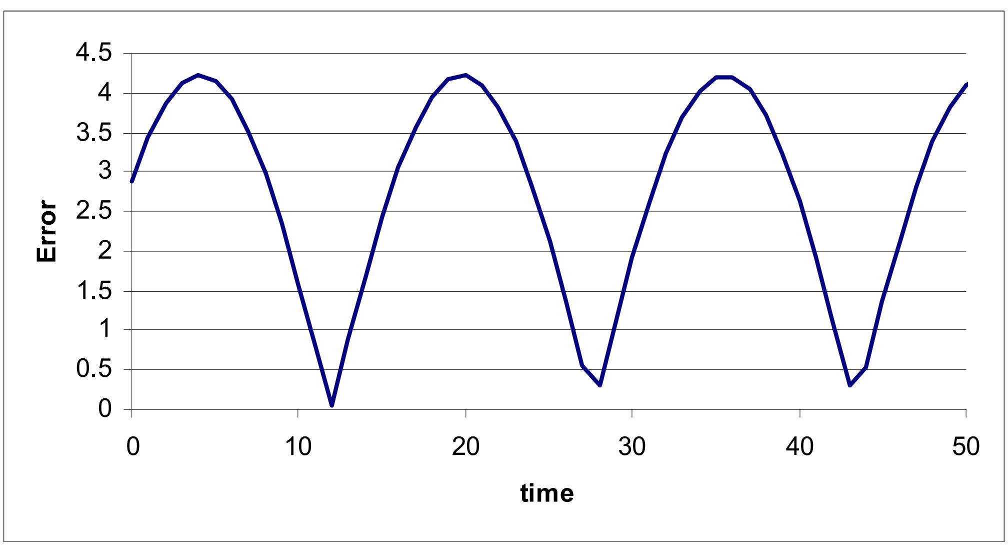
$$T'(t) = -0.24e^{-t} + 3.125e^{-0.2t} - 2.8848\cos(0.2t) + 0.3846\sin(0.2t)$$

remaining terms can be compared to the function given in the problem statement. Let's plot both of these and see what we get:



We see that the T (black curve with large amplitude) and T_m (red curve with smaller amplitude) curves differ quite a bit in phase and size. This would lead to an error. Let's find the absolute difference between these two curves and plot that:

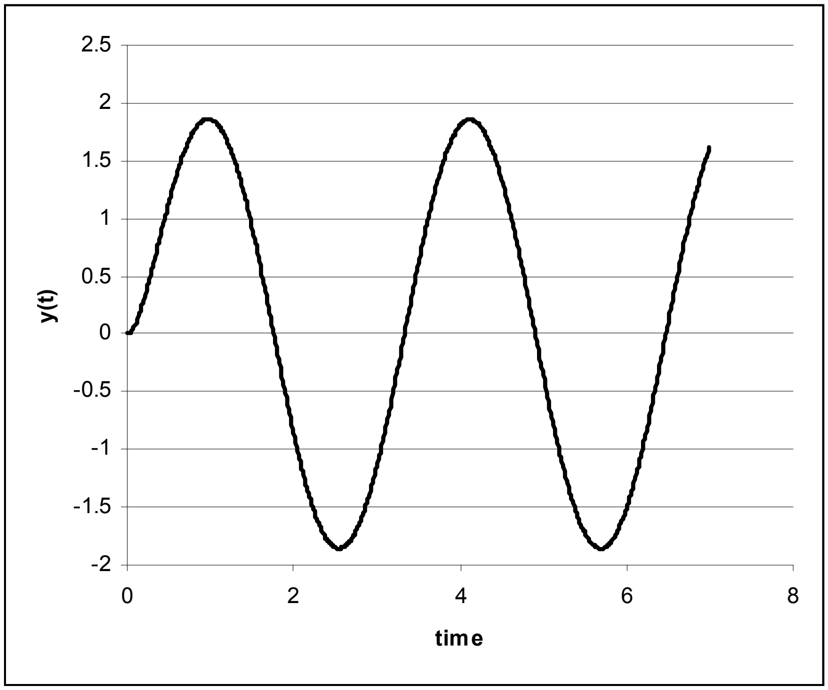
Now we see that there is a maximum error of about 12.4 between the two curves at long times.



Now we invert into the time domain to get:

$$y(t) = \frac{20}{29}e^{-5t} + \frac{50}{29}\sin 2t - \frac{20}{29}\cos 2t$$

We can make a plot of this versus t to get:



From the chart, we see that putting a sin function in with an amplitude of 1 gives an amplitude of 1.85 out. We can use the trig identities from page 316 to change our function to:

$$y(long\, t) = \frac{2}{\sqrt{1+(\omega\tau)^2}}\sin(2t+\phi)$$

$$where\ \phi = \tan^{-1}(-\omega\tau)$$

We have ω = 2 and τ = 0.2 so we get:

$$y(long\, t) = \frac{2}{\sqrt{1+(0.4)^2}}\sin(2t+\phi)$$

$$where\ \phi = \tan^{-1}(-0.4) = -21^{o}$$

Now that we've found this, let's go on to the shortcut method of finding amplitude ratio. We'll start with G(s) and plug in s = jω:

$$G(j\omega) = \frac{2}{(1+0.2j\omega)}$$

Now rationalize the complex number:

$$G(j\omega) = \frac{2}{(1+0.2j\omega)}\left(\frac{1-0.2j\omega}{1-0.2j\omega}\right) = \frac{2-0.4\omega j}{1+0.04\omega^2}$$

In our problem, w = 0.2 so we get:

$$G(j\omega) = \frac{2-0.4(2)j}{1+0.04(4)^2} = \frac{2}{1.16} - \frac{0.8}{1.16}j$$

We can now find the amplitude ratio:

$$AR = \sqrt{R^2 + I^2} = \sqrt{\left(\frac{2}{1.16}\right)^2 + \left(\frac{0.8}{1.16}\right)^2} = 1.857$$

And finally:

$$\phi = \tan^{-1}\left(-\frac{0.8}{2}\right) = -21^o$$

We see that we got the same answer as we did with the long drawn out method.

Problem 14.4

a) We start with the transfer function for an integrating function and plug in s = jω:

$$G(s) = \frac{1}{s} = \frac{1}{j\omega}$$

Multiply by the complex conjugate to get:

$$G(j\omega) = \frac{1}{j\omega}\left(\frac{-j\omega}{-j\omega}\right) = \frac{-j\omega}{\omega^2} = \frac{-j}{\omega}$$

Now find AR and f:

$$AR = \sqrt{R^2 + I^2} = \sqrt{\left(\frac{-1}{\omega}\right)^2} = \frac{1}{\omega}$$

$$\phi = \tan^{-1}\left(\frac{I}{R}\right) = \tan^{-1}\left(\frac{-1}{0\omega}\right) = \tan^{-1}(-\infty) = -90^o$$

b) Now start with the transfer function for a purely derivative portion. G(s) = s

$$G(j\omega) = j\omega$$

$$AR = \sqrt{\omega^2} = \omega$$

$$\phi = \tan^{-1}\left(\frac{\omega}{0}\right) = \tan^{-1}(\infty) = 90^o$$

Problem 14.8

Starting with the following transfer function, plot AR and ϕ:

$$G(s) = \frac{5(s+1)e^{-2s}}{(3s+1)(2s+1)}$$

We'll break this up into 5 separate functions and plot each of them. Then we'll plot the composite picture. Here are our functions and their amplitude ratios and phase angles:

$$G_a(s) = 5$$
$$AR = 5$$
$$\phi = \tan^{-1}(0) = 0^o C$$

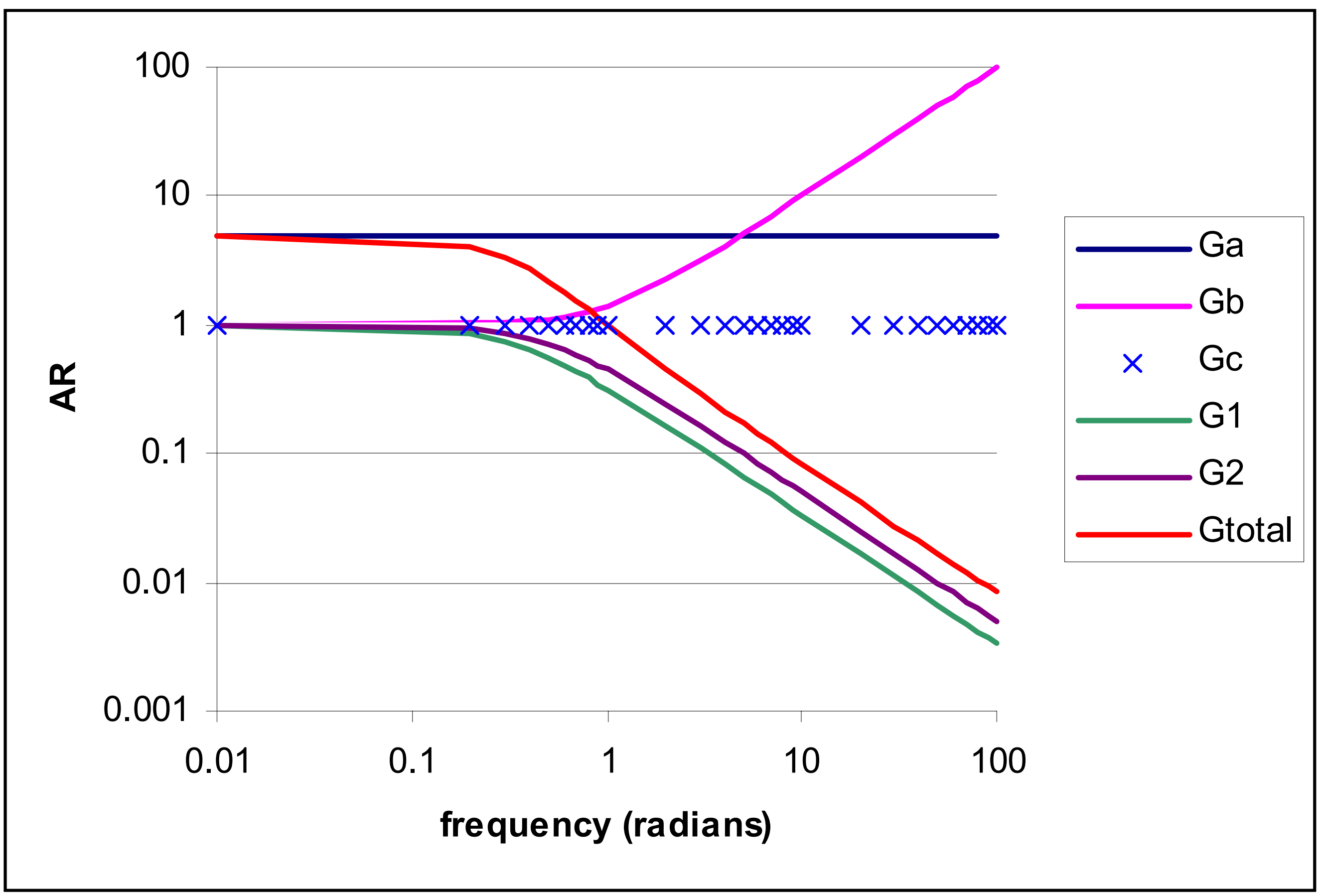
$$G_b(s) = s+1$$
$$AR = \sqrt{1+\omega^2}$$
$$\phi = \tan^{-1}(\omega)$$

$$G_c(s) = e^{-2s}$$
$$AR = 1$$
$$\phi = -2\omega$$

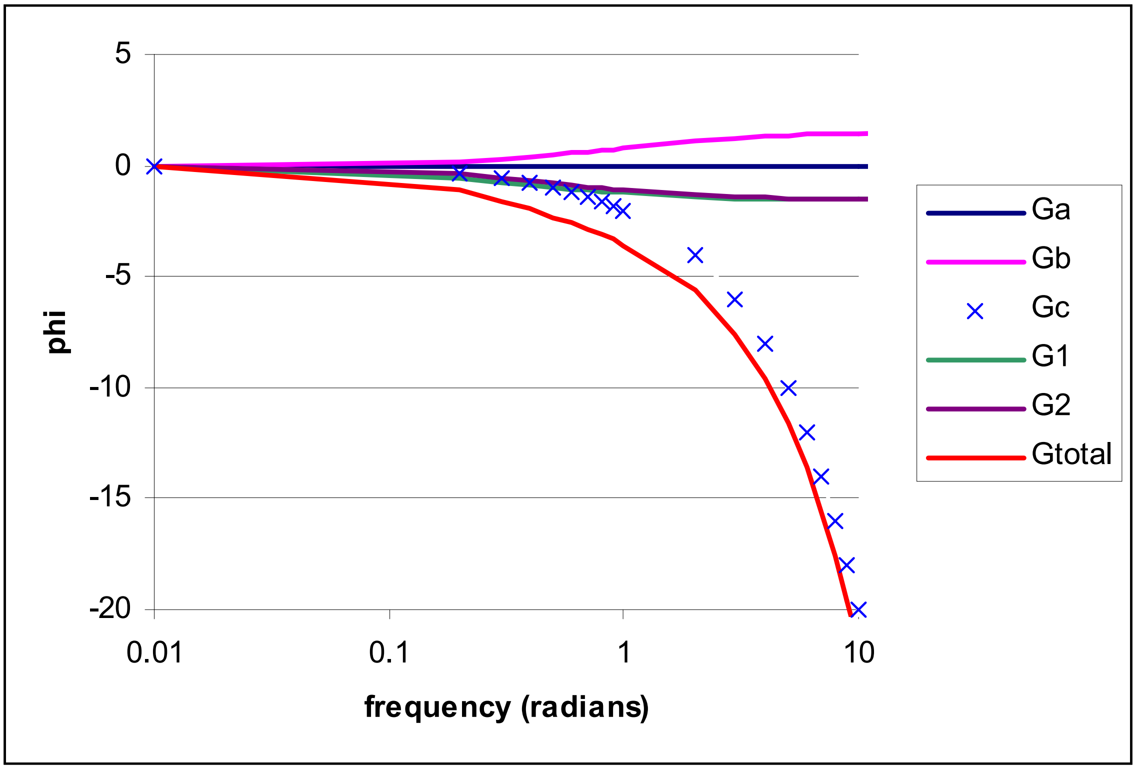
$$G_1(s) = \frac{1}{3s+1}$$
$$AR = \frac{1}{\sqrt{1+9\omega^2}}$$
$$\phi = -\tan^{-1}(3\omega)$$

$$G_1(s) = \frac{1}{2s+1}$$
$$AR = \frac{1}{\sqrt{1+4\omega^2}}$$
$$\phi = -\tan^{-1}(2\omega)$$

Let's plot each one of these now:



And we can also plot the phase angle for all of them as well:



Done!