Problem 9.1

For all of these problems, we will use the equation of a line: y = mx + b to find how the transmitters work.

a)

$$m = \frac{15-3}{100-0} = 0.03\frac{psig\ \min}{gal}$$
$$3\ psig = 0(0.03) + b$$
$$b = 3\ psig$$
$$y = 0.03\frac{psig\ \min}{gal}q + 3\,psig$$

b)

$$m = \frac{20-4}{30-10} = 0.8\frac{mA}{in\ Hg}$$
$$4\,in\ Hg = 0.8(10) + b$$
$$b = -4in\ Hg$$
$$y = 0.8\frac{mA}{in\ Hg}P - 4in\ Hg$$

c)

$$m = \frac{5-1}{20-0.5} = 0.2051\frac{VDC}{m}$$
$$1VDC = 0.2501(0.5) + b$$
$$b = 0.8975\,VDC$$
$$y = 0.2051\frac{VDC}{m}h + 0.8975\,VDC$$

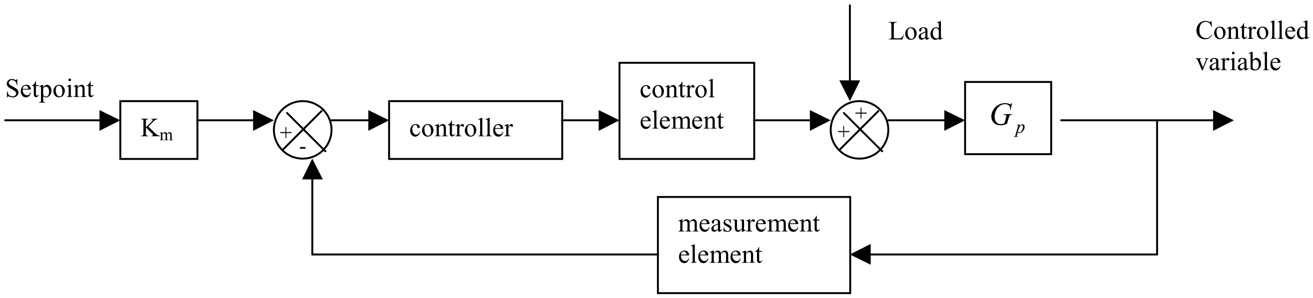
d) The gain of each transmitter can be found as:

K_1 = 0.03 psig/min/gal
K_2 = 0.8 mA/in Hg
K_3 = 0.2051 VDC/m

These are all constant

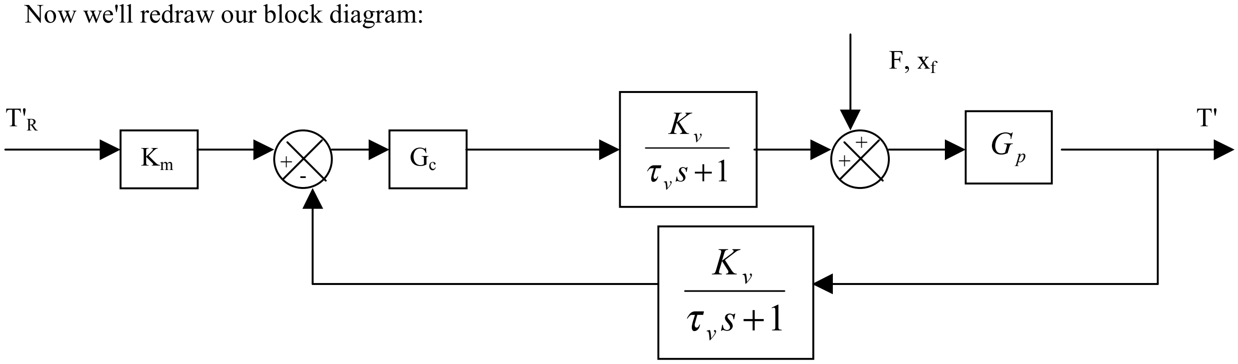
Problem 10.1

We were given a diagram flowsheet for a distillation column in this problem and asked to draw a block diagram. We will start with the normal elements that we must have for a block diagram: a setpoint, a controller, a control element, a measurement element, a load, and a process transfer function. This gives:



We are controlling the temperature of a plate near the top of the column so we have the controlled variable as T' in deviation variable form. We adjust the reflux flow rate R, which is probably done with a pneumatic valve (which is normally first order - Chapter 9). So, our control element transfer function is a standard first order one. Next, we are told that the feed rate, F, and the composition, x_f, are both load variables so we'll have two different loads coming into our process. Since everything is pneumatic, all signals in the loop would have pressure units.

Now we'll redraw our block diagram:

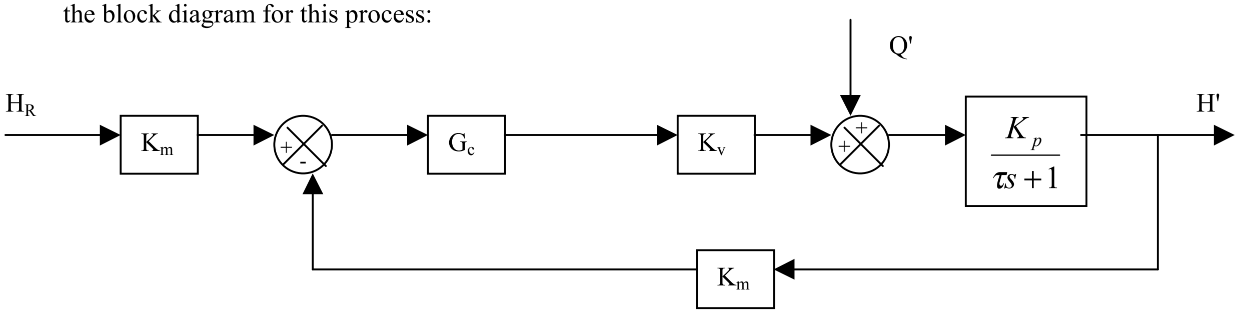


I've been a little bit lazy here in not adding the load transfer functions explicitly, but we don't know what those are anyway. We also do not know what the process dynamics are so we can't replace G_p. The controller was also not described so we don't know if it's proportional only, PI, or PID.

Problem 10.2

We are considering Figure 10.15 in this problem with the following given information:
A = 3 ft^2, R = 1.0 min/ft^2, K_v = 0.2 ft^2/min psi, K_m = 1.7 psi/ft, K_c = 4, and τ_I = 3 min. Suppose that the system is initially at the nominal steady state with a liquid level of 2 ft. If we change the set point from 2 to 3 ft, how long will it take the system to reach 2.5 ft and 3 ft?

Let's start with a redrawing of the block diagram for our process. If we read the chapter, we see that Figure 10.16 is the block diagram for this process:

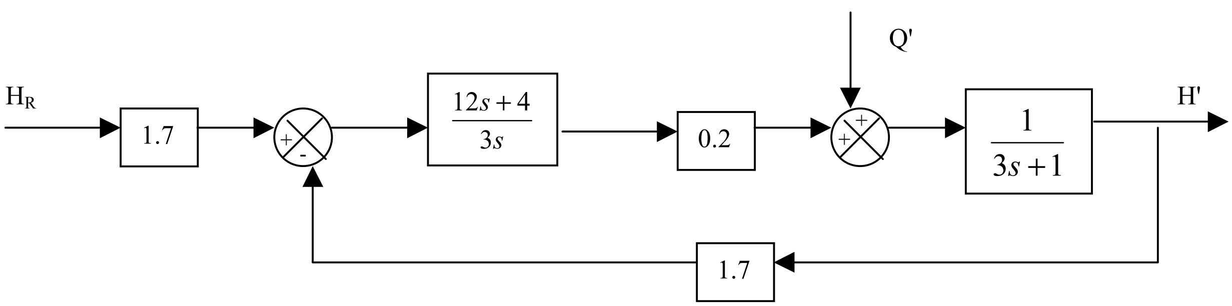


A PI controller has the following transfer function:

$$G_c = K_c\left(1+\frac{1}{\tau_I s}\right) = \frac{\tau_I s K_c + K_c}{\tau_I s} = \frac{12s+4}{3s}$$

We can also use the information from page 236 to show that K_p = R = 1 and τ = RA = 3. You may think we're not being careful of units, but we are...All units are in psi, ft, and min. We'd have to be more careful if we also had some seconds, meters, etc. floating around in the problem statement.

If we add in the transfer functions that were given in the problem statement, we get:



We are trying to find out what happens to H when we change H_R, so we should find H/H_R:

$$\frac{H'}{H'_R} = \frac{\Pi_f}{1+\Pi_{OL}} = \frac{1.7\left(\frac{12s+4}{3s}\right)(0.2)\left(\frac{1}{3s+1}\right)}{1+1.7\left(\frac{12s+4}{3s}\right)(0.2)\left(\frac{1}{3s+1}\right)}$$

We can simplify this somewhat in a few steps:

$$\frac{H'}{H'_R} = \frac{0.34(12s+4)}{3s(3s+1)+0.34(12s+4)}$$

$$= \frac{0.34(4)(3s+1)}{3s(3s+1)+0.34(4)(3s+1)} = \frac{1.36}{3s+1.36}$$

Now that we have found the overall transfer function for this PI controlled process, we can find $H_R(s)$ and plug that in to get:
$h'_R = 3 - 2 = 1$. So, $H_R(s) = 1/s$ and,

$$H' = \frac{1.36}{s(3s + 1.36)}$$

We do partial fraction expansions as:

$$H' = \frac{0.453}{s(s + 0.453)} = \frac{A}{s} + \frac{B}{s + 0.453}$$

$$\frac{0.453}{(0 + 0.453)} = A = 1$$

$$\frac{0.453}{-0.453} = B = -1$$

$$H' == \frac{1}{s} + \frac{-1}{s + 0.453}$$

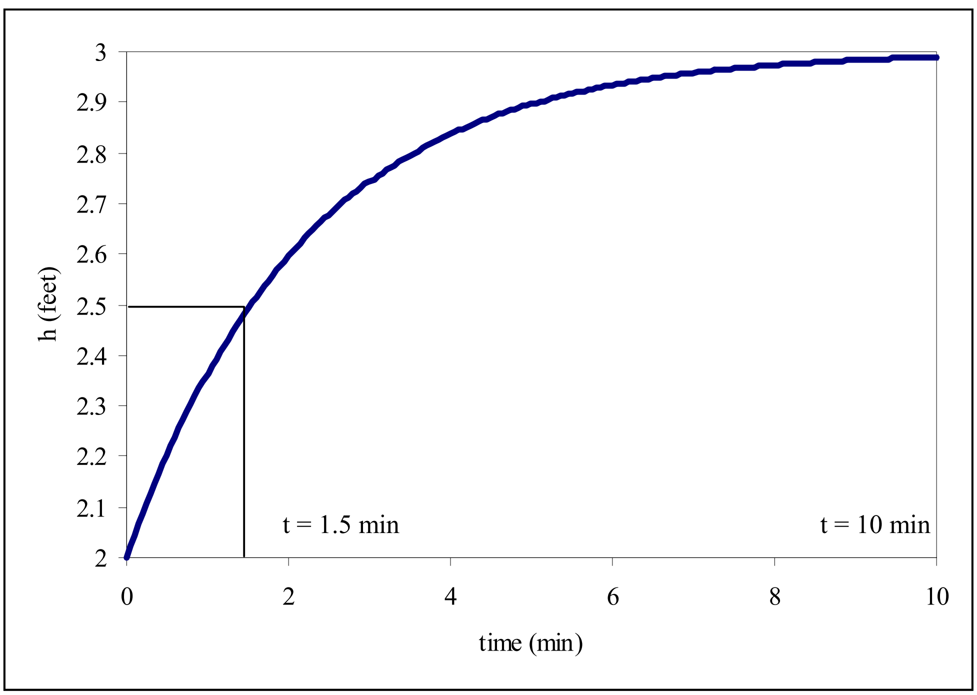
Invert this into the time domain to get:

$$h'(t) = 1 - 1e^{-0.453t}$$

If we assume we were at steady state initially, $h_{ss} = 2$ and h(t) becomes:

$$h(t) = 3 - 1e^{-0.453t}$$

We can plot this function to see when h = 2.5 and 3.0.



Problem 10.3

We'll use Figure 10.7 to solve this problem. Given:
the temperature transmitter has a span of 50 ^oF and a zero of 55 ^oF.
Tss = 80 ^oF
Ti,ss = 65 ^oF.
The controller has a gain of 5
The valve and current to pressure transducer have gains of $K_v = 1.2$
$K_{IP} = 0.75$
The time constant for the tank is 5 min
We change the set-point from 80 to 85 ^oF and the tank temperature eventually reaches a new value of 84.14 ^oF.

a) what is the offset?
Offset is defined as the desired output value - the actual output value.
Here, offset = 85 - 84.14 = 0.86 ^oF.

b) What is the process gain, K_2, using Figure 10.7?

To figure this out, we'll start with the block diagram in the figure and find how T' changes for a change in T'_R.

$$\frac{T'(s)}{T'_R(s)} = \frac{K_m K_c\left(1+\frac{1}{\tau_I s}\right)(K_{IP})\left(\frac{K_v}{\tau_v s+1}\right)\left(\frac{K_2}{\tau s+1}\right)}{1+K_c\left(1+\frac{1}{\tau_I s}\right)(K_{IP})\left(\frac{K_v}{\tau_v s+1}\right)\left(\frac{K_2}{\tau s+1}\right)\left(\frac{K_m}{\tau_m s+1}\right)}$$

We need to make a few assumptions first. We weren't given any information about the time constants for the valve or for the measurement device so we can assume that their dynamics are negligible (i.e, their time constants are zero). This gives:

$$\frac{T'(s)}{T'_R(s)} = \frac{K_m K_c K_v\left(1+\frac{1}{\tau_I s}\right)(K_{IP})\left(\frac{K_2}{\tau s+1}\right)}{1+K_c K_v K_m\left(1+\frac{1}{\tau_I s}\right)(K_{IP})\left(\frac{K_2}{\tau s+1}\right)}$$

We also have a proportional only controller. Now plug everything in and start to rearrange and simplify:

$$\frac{T'(s)}{T'_R(s)} = \frac{5(1.2)(1.2)(0.75)K_2}{5s+1+5(1.2)K_m K_2(0.75)}$$
$$= \frac{5.4K_2}{5s+1+4.5K_m K_2}$$

We have to go back to chapter 9 to see how to find K_m. There, we saw that most measuring elements have a range of output values between 4 and 20 milliamps. The zero of the measuring element is 55 ^oF and it has a span of 50 ^oF. So we can find the gain as:

$$K_m = \frac{output}{input} = \frac{20-4}{(50+55)-50} = 0.32\frac{mA}{^oF}$$

We plug this in and get:

$$\frac{T'(s)}{T'_R(s)} = \frac{5.4K_2}{5s+1+1.44K_2}$$

Now we rearrange this into standard form by dividing the top and the bottom by $1 + 1.44K_2$:

$$\frac{T'(s)}{T'_R(s)} = \frac{\dfrac{K_2}{1.44K_2 + 1}}{\dfrac{5}{1.44K_2 + 1}s + 1}$$

We are changing our set-point from 80 to 85 here so we have $T'_R = 85 - 80 = 5$.

From one our earlier chapters we saw that offset was equal to the change in setpoint*(1 - $K_{overall}$). We'll use this to find K_2:

$$0.86 = 5(1 - K_3)$$

$$K_3 = 0.828 = \frac{K_2}{1.44K_2 + 1}$$

$$K_2 = 3.343$$

c) Now we need to find the pressure signal p_t at the new steady state:

$T'(s) = 84.14 - 80 = 4.14\ ^oF$
and $T'_R(s) = 85 - 80 = 5\ ^oF$

So, $T'_m = K_m T'(s) = 0.32 * 4.14 = 1.3248$ mA.
$T'_R = 5 * 0.32 = 1.6$ mA.
$E = T'_R - T'_m = 1.6 - 1.3248 = 0.2752$ mA.
$P' = 5 * 0.2752 = 1.376$
$P't = 1.376 * 0.75 = 1.032$ psia.