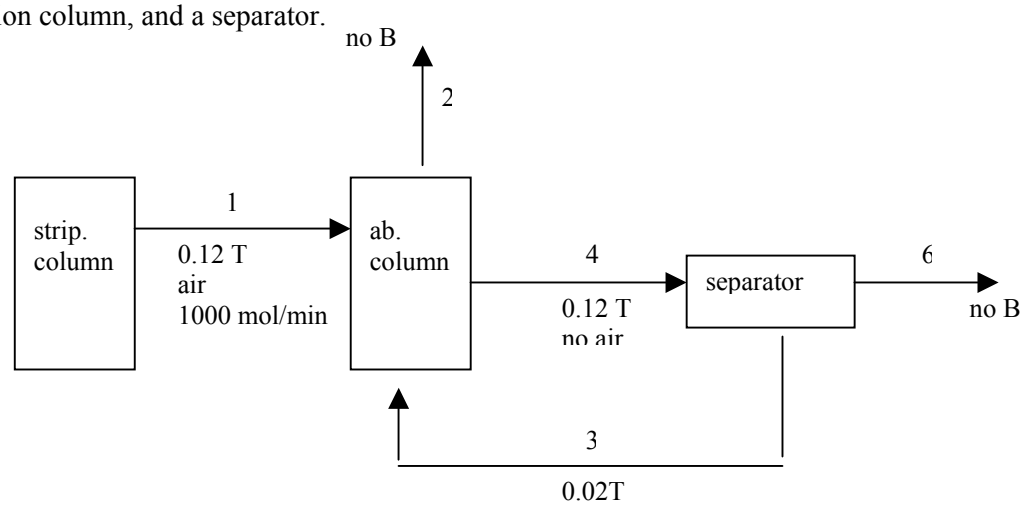
Another more complicated example: Environmental Applicatoins

You are an engineer that is working with and environmental consulting firm to design a process for removing TCE (trichloroethylene) from an air stream. The TCE entered the air stream during an air stripping process where it was removed from water on the south side of Tucson. In the process you have designed to remediate the air stream, 1000 mol/min of air that has 12 mol% TCE in it enters an absorption column where it is contacted with a benzene stream containing 2 percent TCE. The air stream leaving the absorption column has only 5% of the TCE that entered the column while the benzene has 12 mol %, and none of the air. The contaminated benzene stream enters a novel separator where all of the benzene is recycled back to the column, while the bulk of the TCE is sent off site. How much TCE is removed per day?

What might you do with the removed TCE?

The first thing we did in beginning the problem is to draw a sketch of the process, label it, and then add additional information to it. We have a stripping column sending something into our process, an absorption column, and a separator.



With the figure now done, we can translate some of the sentences into math. We are told that 5% of the TCE that enters the absorption column leaves it in stream 2. In other words:

$$F_{2,T} = 0.05\left(F_{1,T} + F_{3,T}\right)$$

Now let's write the mass balances for our process:
absorption column: $F_1 + F_3 = F_2 + F_4$
separator: $F_4 = F_3 + F_5$
overall: $F_1 = F_2 + F_6$

Now let's create our table to keep track of information in:

Stream	total moles	air moles	B moles	TCE moles
1	100	880	0	120
2		880	0	
3		0		
4		0		
6		0	0	

We've also filled in the things we can easily get to above, including the zeroes.

Now things get interesting because we need to write some equations to start solving. And in this case, we can't just write one equation with one unknown.

Let's rewrite our given equation about the 5% thing from above:

$$F_{2,T} = 0.05\left(F_{1,T} + F_{3,T}\right)$$

$$F_{2,T} = 0.05\left(120 + 0.02F_3\right)$$

Let's next do a T balance around the separator to get:

$$0.12F_4 = 0.02F_3 + F_6$$

And a B balance around the separator gives:

$$0.88F_4 = 0.98F_3$$

While an overal T balance gives:

$$120 = F_{2,T} + F_6$$

We now have $F_{2,T}$, F_3, F_4, and F_6 as our four unknowns with four equations. We should be able to solve this using Gauss elimination.

Our four equations are:

$$F_{2,T} - 0.02F_3 = 6$$

$$0.02F_3 - 0.12F_4 + F_6 = 0$$

$$0.98F_3 - 0.88F_4 = 0$$

$$F_{2,T} + F_6 = 120$$

Let's expand this out into the full matrix representation like in the previous example we did. Here, we'll have the variables listed as $F_{2,T}$, F_3, F_4, and then F_6, followed by the right hand side:

$$\left[A\right] = \left[\begin{array}{cccc|c} 1 & -0.02 & 0 & 0 & 6 \\ 0 & 0.02 & -0.12 & 1 & 0 \\ 0 & 0.98 & -0.88 & 0 & 0 \\ 1 & 0 & 0 & 1 & 120 \end{array}\right]$$

Remember, our goal is to get to an "upper right triangle" so we can quickly solve for all of the variables. (Keep in mind that the upper right triangle is just a formality. What we really want is to reduce our matric down to having one row with only one variable in it, another row with no more than two variables, another row with no more than three…and so on until we have a row that could possibly have all of the variables in it.) Let's multiply the top row by -1 and add it to the fourth row to get:

$$\left[A\right] = \left[\begin{array}{cccc|c} 1 & -0.02 & 0 & 0 & 6 \\ 0 & 0.02 & -0.12 & 1 & 0 \\ 0 & 0.98 & -0.88 & 0 & 0 \\ 0 & 0.02 & 0 & 1 & 114 \end{array}\right]$$

Now, we'll multiply the second row by -1 and add to the fourth row to get:

$$[A] = \left[\begin{array}{cccc|c} 1 & -0.02 & 0 & 0 & 6 \\ 0 & 0.02 & -0.12 & 1 & 0 \\ 0 & 0.98 & -0.88 & 0 & 0 \\ 0 & 0 & 0.12 & 0 & 114 \end{array}\right]$$

Although this is not in upper right triangle form, I can now easily solve for all of the variables at this point.

The bottom row gives us that $0.12F_4 = 114$ so $F_4 = 950$ mol

And, $0.98F_3 - 0.88F_4 = 0$. So we can get $F_3 = 0.88*950/0.98 = 853.06$ mol

Now, $0.02F_3 - 0.12F_4 + F_6 = 0$. So we get $0.02(853.06) - 0.12(950) + F_6 = 0$
F6 = 96.94 mol

Finally, $F_{2,T} - 0.02F_3 = 6$. So, $F_{2,T} = 0.02(853.06) + 6 = 23.0612$