Chemical Engineering 201
Fall 2002
Gaussian Elimination versus Substitution

Let's briefly review how to solve a system of equations using the following three equations with three unknowns:

$$F_1 + F_2 + F_3 = 6$$
$$F_1 + 2F_2 + F_3 = 8$$
$$2F_1 + F_2 + 3F_3 = 13$$

We are all familiar with solving one equation in terms of the other and plugging that into the other equations to reduce the unknowns. We'll solve the top equation for F_1 and plug into the other two equations to get:

$$F_1 = 6 - F_2 - F_3$$
$$6 - F_2 - F_3 + 2F_2 + F_3 = 8$$
$$2(6 - F_2 - F_3) + F_2 + 3F_3 = 13$$

Now, we'll simplify the bottom two equations to get:

$$F_2 = 2$$
$$-F_2 + F_3 = 1$$

So we get $F_2 = 2$ by substitution while F_3 then equals 3 and $F_1 = 1$. Done!

Another way to solve this problem is by Gauss elimination so let's solve it that way now just to compare. The book explained how to apply Gauss elimination to problems so this will just be a quick review. First, lets transform the original 3 equations into matrix form:

$$[A] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 1 & 2 & 1 & 8 \\ 2 & 1 & 3 & 13 \end{array}\right]$$

We'll do this one bit by bit and leave you some blanks to fill in so that you can follow along. First we'll multiply the top equation by -1 and add it to the second equation. We'll rewrite this now so you can see how this leads to the first column second row becoming a zero:

$$\begin{array}{rcccc} 2nd\ equation: & 1 & 2 & 1 & 8 \\ -1st\ equation: & -1 & -1 & -1 & -6 \\ \hline result\ to\ replace\ 2nd\ equation: & 0 & 1 & 0 & 2 \end{array}$$

Now rewrite the matrix with this new second row replacing the original row:

$$[A] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 0 & 2 \\ 2 & 1 & 3 & 13 \end{array}\right]$$

Now you do the next one by turning the first column, third row, into a zero. Multiply the top row by:_______. Then ________________ it to the bottom row. Writing it so we can see just these steps will give us:

$$\begin{array}{rcccc} 3rd\ equation: & 2 & 1 & 3 & 13 \\ -1st\ equation: & & & & \\ \hline result\ to\ replace\ 3rd\ equation: & & & & \end{array}$$

Now we can rewrite the matrix to replace the third row with this row to get:

$$[A] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 0 & 2 \\ 0 & & & \end{array}\right]$$

Now we need to get rid of the number in the bottom middle row so we will have an upper right triangle and can solve the equations from the bottom up like a zipper. So we want to multiply the ______ equation by __________ and ______________ it to the ____________ equation. This gives:

$$\begin{array}{c} \_\_\_\_\_\_\_\_\_\_\_\_ \; equation: \\ \underline{\_\_\_\_\_\_\_\_\_\_\_\_ \; equation: \qquad\qquad\qquad\qquad} \\ result\ to\ replace\ \_\_\_\_\_\_\_\_\_\_ \; equation: \end{array}$$

Then we can rewrite the matrix as:

$$[A] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 0 & 2 \\ 0 & & & \end{array}\right]$$

Let's turn this back into equation form so you can see what we've ended up with:

$$\begin{array}{rrrcl} 1F_1 + & 1F_2 + & 1F_3 & = & 6 \\ & 1F_2 & & = & 2 \\ & & F_3 & = & 3 \end{array}$$

We can easily solve this system of equations to get that $F_1 = 1$, $F_2 = 2$, and $F_3 = 3$.

General rules for matrices:

You can always switch two rows around to make the math simpler on yourself.

You can always multiply or divide a row by a constant to make the math simpler as well.

You **cannot** switch two columns.

The number of columns (with the b column added) must be only one number higher than the number of rows.

Your goal is to get to a point where you have an upper right triangle form for your matrix.