

**ChEE 201
Fall 2005
University of Arizona
Computer HW 6
Solving Linear Systems of Equations**

This homework is to familiarize students with how to solve a system of linear equations by hand. Students should not use a linear equation solver for this homework, but should instead write out each step so they will be able to follow our discussion of gaussian elimination in lecture since this homework will be referred to. Students should NOT write a VBA program or use Excel for this problem either.

Learning Objectives:

At the end of this homework, students will be able to:

- 1) use the method of combining equations to solve systems of linear equations

Solve the system of equations below

$$\begin{aligned}x_1 + 2x_2 - 3x_3 + 4x_4 &= 4 \\2x_1 - x_2 - 4x_3 + 2x_4 &= 1 \\-4x_1 + 2x_2 + 3x_3 + 2x_4 &= 2 \\-x_1 - 3x_2 + x_3 - 2x_4 &= -2\end{aligned}$$

Students should not switch any of the rows with other rows (which will be allowed when we start performing gaussian elimination). Also, students should not switch columns. As discussed in the handout for this homework, the students should instead work towards transforming their set of equations into the upper triangular form.

For their final answers, students should put a box around their upper triangular form of the system of equations and also list the values for x_1 , x_2 , x_3 , and x_4 .

Solution: We'll multiply the top row by -2 and add it to the second row:

$$\begin{aligned}-2x_1 - 4x_2 + 6x_3 - 8x_4 &= -8 \\2x_1 - x_2 - 4x_3 + 2x_4 &= 1 \\ \hline 0x_1 - 5x_2 + 2x_3 - 6x_4 &= -7\end{aligned}$$

Multiply the top row by four and add to the third row:

$$\begin{aligned}4x_1 + 8x_2 - 12x_3 + 16x_4 &= +16 \\-4x_1 + 2x_2 + 3x_3 + 2x_4 &= 2 \\ \hline 0x_1 + 10x_2 - 9x_3 + 18x_4 &= 18\end{aligned}$$

And simply add the top row with the bottom row to get:

$$\begin{aligned}x_1 + 2x_2 - 3x_3 + 4x_4 &= 4 \\-x_1 - 3x_2 + x_3 - 2x_4 &= -2 \\ \hline 0x_1 - x_2 - 2x_3 + 2x_4 &= 2\end{aligned}$$

Now rewrite our original set of equations to get:

$$\begin{aligned}x_1 + 2x_2 - 3x_3 + 4x_4 &= 4 \\0x_1 - 5x_2 + 2x_3 - 6x_4 &= -7 \\0x_1 + 10x_2 - 9x_3 + 18x_4 &= 18 \\0x_1 - x_2 - 2x_3 + 2x_4 &= 2\end{aligned}$$

Working on x_2 , we'll do two in parallel. On the left, we'll multiply the second equation by two and add it to the third equation. On the right, we'll multiply the second equation by -1/5th and add it to the bottom equation:

$$\begin{array}{r}
 -10x_2 + 4x_3 - 12x_4 = -14 \\
 +10x_2 - 9x_3 + 18x_4 = 18 \\
 \hline
 0x_2 - 5x_3 + 6x_4 = 4
 \end{array}
 \qquad
 \begin{array}{r}
 x_2 - \frac{2}{5}x_3 + \frac{6}{5}x_4 = \frac{7}{5} \\
 -x_2 - 2x_3 + 2x_4 = 2 \\
 \hline
 0x_2 - \frac{12}{5}x_3 + \frac{16}{5}x_4 = \frac{17}{5}
 \end{array}$$

We'll also multiply the right equation by 5 before we plug it back into our system of equations:

$$\begin{array}{r}
 x_1 + 2x_2 - 3x_3 + 4x_4 = 4 \\
 0x_1 - 5x_2 + 2x_3 - 6x_4 = -7 \\
 0x_1 + 0x_2 - 5x_3 + 6x_4 = 4 \\
 0x_1 + 0x_2 - 12x_3 + 16x_4 = 17
 \end{array}$$

For our last one, we'll multiply the third row by $-12/5$ this and add it to the bottom row to get:

$$\begin{array}{r}
 5\frac{12}{5}x_3 - 6\frac{12}{5}x_4 = -4\frac{12}{5} \\
 -12x_3 + 16\frac{5}{5}x_4 = \frac{5}{5}17 \\
 \hline
 0x_3 + \frac{8}{5}x_4 = \frac{37}{5}
 \end{array}$$

And we multiply the bottom one by 5 and plug that into our system of equations to get the upper triangular form, which should be work 10 points. Students may have multiplied some of the rows by factors or divided by them and still get full credit:

$$\begin{array}{r}
 x_1 + 2x_2 - 3x_3 + 4x_4 = 4 \\
 0x_1 - 5x_2 + 2x_3 - 6x_4 = -7 \\
 0x_1 + 0x_2 - 5x_3 + 6x_4 = 4 \\
 0x_1 + 0x_2 + 0x_3 + 8x_4 = 37
 \end{array}$$

Now, we zipper up the equations from the bottom to the top to get:

$$x_4 = \frac{37}{8} = 4.625$$

Then,

$$\begin{array}{r}
 x_1 + 2x_2 - 3x_3 = -14.5 \\
 0x_1 - 5x_2 + 2x_3 = 20.75 \\
 0x_1 + 0x_2 - 5x_3 = -23.75
 \end{array}$$

So,

$$x_3 = \frac{23.75}{5} = 4.75$$

And,

$$\begin{array}{r}
 x_1 + 2x_2 = -0.25 \\
 0x_1 - 5x_2 = 11.25
 \end{array}$$

Leading to:

$$x_2 = \frac{11.25}{-5} = -2.25$$

And, finally,

$$x_1 = 4.25$$

Our final answers, then, are: $x_1 = 4.25$, $x_2 = -2.25$, $x_3 = 4.75$, and $x_4 = 4.625$. This is worth 10 points. We plug these into our original equations and see that these are indeed the correct answers.