ChEE 201
Fall 2007
Computer Homework 5
Taylor Series Approximations

Students need to have read Computer Reading 4 and 5 along with having completed Computer HW 4 before attempting this homework.

At the end of this section, students will be able to:
1) use a computer program with a Taylor series approximation to estimate a function at a point
2) apply the approximate relative error can be used to decide when they have included enough terms in their infinite series to have come close enough to a converged answer to stop
3) understand how double and single precision can affect answers
4) compute a Taylor series expansion and approximate a function that has more than one variable

1) In the previous homework, you found an infinite series representation for ln(x) where you expanded about the point $x_i = 1$ because you know that ln(1) is zero. You then found how many terms it would take to find ln(2) using an approximate relative error of 0.01. Now, use your infinite sum equation to find how many terms you would need to have in your series to find the ln of 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, and 2.0 with an approximate relative error of 0.01. To do this, write a short VBA program that takes in x_{i+1} as input and reports the number of terms that you need to reach convergence. Make a table that has x_{i+1} and the number of terms it takes to get convergence for each value of x_{i+1}. For full credit, submit a hard copy of your program and your table of the number of steps it takes for each value.

2) If you tried to use your program to find the ln(2.9), though, it can't with this convergence criteria. Try it with 10000 steps just to prove to yourself that this doesn't converge. The problem is that the power on the $(x_{i+1} - x_i)$ term starts to dominate over the other terms until your function explodes; you're taking 1.01^n and this grows too quickly to be reigned in by dividing by n. Your approximation will diverge for any value of x_{i+1} that is greater than 1.

Let's imagine you wanted to approximate ln(2.9). How could you do this and have your program still converge? For this problem, if x_{i+1} is greater than 2 but less than three, have the computer first approximate the ln(2) with the program you have above and then use that approximation in your new approximation to get to 2.9, rewriting your derivation for the second infinite series. In essense, you'll be doing a Taylor series approximation to get from 1 to 2 and then doing another one from 2 to 3. Use a convergence criteria of 0.01 on the absolute relative error for both approximations. Submit a hardcopy of your program and your approximate answer to ln(3). Email a copy of your Excel file saved as First Name Last Name 5.2.xls to blowers@engr.arizona.edu before 10 am on the day the program is due.

3) This problem is going to be very, very short, but will address objective three above.
The infinite series:

$$f(N) = \sum_{n=1}^{N} \frac{1}{n^2}$$

converges on a value of $f(N)=\pi^2/6$ as N approaches infinity. Write a program to compute f(N) for N = 10000 by computing the sum from n = 1 to 10000. Then repeat the computation but in reverse order, that is from n = 10000 to 1 using increments of -1. Use single precision for all numbers.

Explain the difference between your results. Which number do you feel is more accurate?

Now, repeat the two directions again, but using double precision for all numbers. Explain what is different compared to what you had found using single precision numbers.

4) We began a Taylor series approximation of a two variable function in class. Complete your second order approximation to the function we used: $e^{2x}y^3$ for $x_{i+1} = 1$ and $y_{i+1} = 2.1$. Report your value after the first order corrections are added, after the second order corrections are added, and after your third order corrections are added. What is the absolute error after each order of correction?