Compressed Sensing using Dual-Tree Complex Wavelet Transform

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Introduction: Recently, a new theory called compressed sensing (CS) has been applied to MR image reconstruction with great success [1]. The CS theory states that a signal with a sparse representation can be reconstructed from much fewer measurements than previously suggested by the conventional Nyquist sampling theory [2-3]. Sparse representation is a key assumption of the CS theory, and reconstruction results greatly depend on the sparsifying transform. In this paper, we explore the use of dual-tree complex wavelet transform (DT-CWT) as a sparsifying transform in CS MRI.

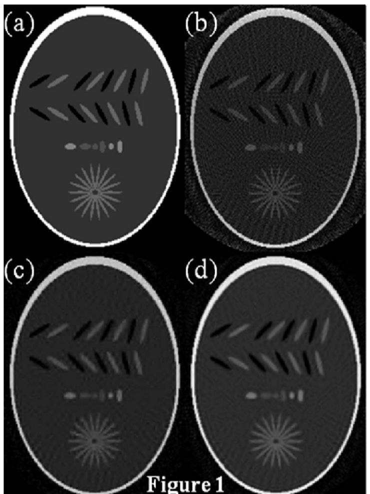

Figure 1

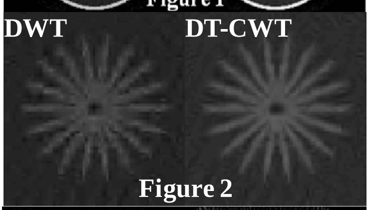

Figure 2

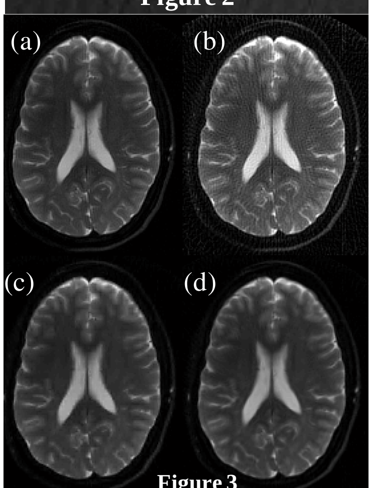

Figure 3

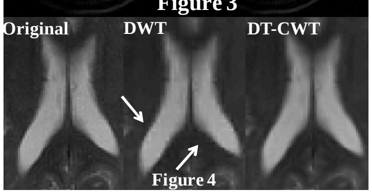

Figure 4

Theory: The CS MRI problem can be stated as follows: Let $\mathbf{f}$ denote the object being imaged, $\mathbf{M}$ the undersampled Fourier measurement matrix, and $\mathbf{g}$ the acquired k-space data such that $\mathbf{g} = \mathbf{Mf}$. If $\boldsymbol{\Psi}$ denotes a sparsifying transform such that $\boldsymbol{\Psi}\mathbf{f}$ is sparse, the CS theory states that the object can be recovered by solving the nonlinear convex optimization problem: $\min_{\mathbf{f}} \|\boldsymbol{\Psi}\mathbf{f}\|_1$ such that $\|\mathbf{Mf} - \mathbf{g}\|_2 < \varepsilon$, where ε accounts for noise. The sparsity transform plays a very critical role in CS. In general, the sparser the representation, the fewer the measurements that are needed for reconstruction. In addition, when there are insufficient and/or noisy measurements, the artifacts in the reconstructed image are strongly dependent on the sparsity transform. For example, while finite differences (i.e. total variation) is very effective to enforce sparsity in piecewise constant objects, it results in blotchy patterns and loss of detail in CS MRI when the number of measurements is low. Similarly, the (real) discrete wavelet transform (DWT), frequently used in CS MRI, also has some shortcomings: The DWT coefficients tend to oscillate significantly around singularities (i.e. edges) complicating extraction of such singularities. Thus, the inability to recover all the DWT coefficients around such singularities results in edge artifacts in CS MRI. In addition, the DWT is shift variant which means that a small shift in signal seriously perturbs wavelet coefficients. Thirdly, the non-ideal low-pass and high-pass filters in DWT result in considerable aliasing. While this aliasing is canceled during the inverse DWT under ideal conditions, any perturbation of the DWT coefficients (such as noise, thresholding, etc.) results in aliasing artifacts in the reconstructed image. Lastly, the DWT lacks directionality and cannot represent curves efficiently. The DT-CWT is a recent enhancement to DWT that alleviates these shortcomings by using complex valued wavelet and scaling functions [4].

Method: First, a computer generated phantom was used to compare different methods. This phantom was designed to contain directionally-oriented ellipses to highlight directional reconstruction artifacts. The *k*-space data were sampled along 64 radial views with 256 points along each view to yield roughly 6.3 times undersampling. Experiments were also carried out using a dataset acquired on a 1.5T clinical scanner (GE Healthcare, Waukesha, WI) with an eight-channel head coil. The T2-weighted dataset was acquired using a radial-FSE sequence with TR=4.5s, FOV=28cm, and ETL=4. The dataset contained 256 radial views and 256 points along each radial view. An image was reconstructed from this dataset using non-uniform FFT (NUFFT) followed by sum-of-squares combination of the coil images. This image was retrospectively subsampled in *k*-space using 100 radial views to simulate undersampled acquisition. For both the phantom and in vivo results, the orthogonal Daubechies wavelet with 4 vanishing moments was used as the DWT and DT-CWT utilized the biorthogonal Daubechies wavelet with the 9/7 filters at the first stage followed by the Kingsbury Q-filter for the later stages [4]. The regularization parameters for the CS experiments were determined using L-curve analysis.

Results and Discussion: Fig. 1a shows the original phantom image and the image obtained using NUFFT is shown in Fig. 1b. The CS reconstructions with the DWT and DT-CWT are displayed in Figs. 1c and 1d, respectively. Note that while both CS reconstructions are able to reduce the undersampling artifacts significantly, the image obtained using DWT contains significant ringing along directional edges. Such artifacts are reduced when DT-CWT is employed in CS reconstruction. To emphasize this point, expansions of the reconstructed phantom images are displayed in Fig. 2. The results for the in vivo dataset are displayed in Fig. 3. The original image and the image obtained using NUFFT are shown in Figs 3a and 3b, respectively. The images obtained using CS reconstruction with DWT and DT-CWT are shown in Figs. 3c and 3d, respectively. Similar to the phantom results, both CS reconstructions result in significant reduction of the undersampling artifacts. However, as illustrated in Fig. 4, the DWT images contain significant ringing along the ventricles and these artifacts are eliminated when DT-CWT is used.

Conclusion: The DT-CWT has been proposed as a sparsity transform in CS MRI. Experimental results indicate that DT-CWT can significantly reduce the artifacts associated with DWT, especially along directional structures.

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References: [1] Lustig M et al. MRM (2007). [2] Candes E et al. IEEE Transactions on Information Theory (2006). [3] Donoho D. IEEE Transactions on Information Theory (2006). [4] Selesnick I W et al. IEEE Signal Processing Magazine (2005).