SPArse Reconstruction using a ColLEction of bases (SPARCLE)

A. Bilgin[1,2], O. Guleryuz[3], T. P. Trouard[2,4], and M. I. Altbach[2]

[1]Electrical and Computer Engineering, University of Arizona, Tucson, AZ, United States, [2]Dept. of Radiology, University of Arizona, Tucson, AZ, United States, [3]Dept. of Electrical Engineering, Polytechnic Institute of NYU, Brooklyn, NY, United States, [4]Biomedical Engineering, University of Arizona, Tucson, AZ, United States

Introduction: The recently introduced Compressed Sensing (CS) theory has demonstrated that MR images can be reconstructed from a small number of k-space measurements [1-3]. The key assumption in CS MRI is that the image has a sparse representation in a predetermined basis. Selection of this sparsity basis is critically important in CS. In this work, we introduce a new sparse reconstruction framework (SPARCLE) where sparsity is enforced within a collection of bases rather than a single one. Reconstruction results indicate that this new framework can yield significantly improved image quality.

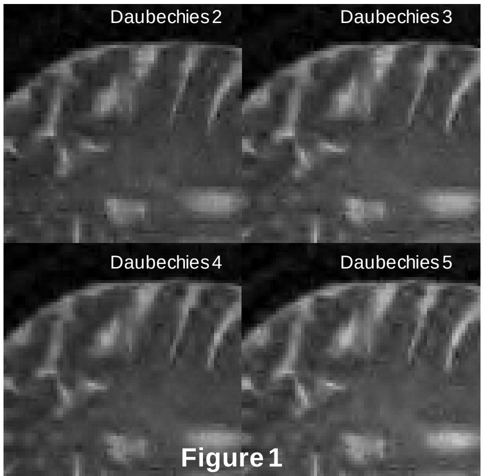

Figure 1

$\hat{\mathbf{x}}_S^{(0)} = \mathbf{F}_\Omega^H \mathbf{y}$
for $j = 1$ to J
  $\hat{\mathbf{x}}_0^{(j)} = \hat{\mathbf{x}}_S^{(j-1)}$
  for $i = 1$ to S
    $\hat{\alpha} = T_t(\mathbf{\Psi}_i \hat{\mathbf{x}}_{i-1}^{(j)})$
    $\hat{\mathbf{y}} = \mathbf{F}\Psi_i^H \hat{\alpha}$
    $\hat{\mathbf{y}}(\omega_{1:K}) = \mathbf{y}$
    $\hat{\mathbf{x}}_i^{(j)} = \mathbf{F}^H \hat{\mathbf{y}}$
  end
end
$\hat{\mathbf{x}} = \hat{\mathbf{x}}_S^{(J)}$

$\hat{\mathbf{x}}$: Reconstructed image ($1 \times N$).
$\mathbf{y}$: Fourier measurements.
J: Number of iterations.
$\mathbf{F}$: Fourier matrix (full $N \times N$).
$\mathbf{\Psi}_i$: Sparsity transforms $i = 1, \ldots, S$.
Ω: Set of measured frequencies ($|\Omega| = K << N$).
$\mathbf{F}_\Omega$: Undersampled Fourier measurement matrix ($K \times N$).
ω_k: Row indicies of the measured frequencies in $\mathbf{F}$, $k = 1, \ldots, K$.
$\hat{\mathbf{x}}_i^{(j)}$: Reconstruction at iteration j after using sparsity transform i.
$T_t(.)$: Thresholding function with threshold t.

Figure 2

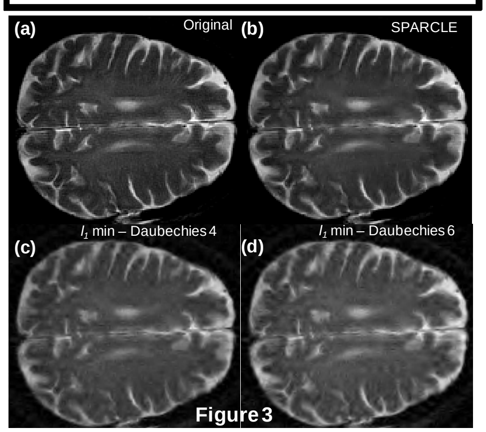

Figure 3

Theory: Let $\mathbf{\Psi}$ denote the sparsity transform, $\mathbf{F}_\Omega$ the undersampled Fourier measurement matrix, $\mathbf{y}$ the acquired k-space data, and $\hat{\mathbf{x}}$ the reconstructed image. The CS theory suggests that the image reconstruction can be performed by solving the constrained l_1 minimization problem: $\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{\Psi x}\|_1$ such that $\mathbf{y} = \mathbf{F}_\Omega \mathbf{x}$. An important consideration in l_1 minimization is the selection of the sparsity transform $\mathbf{\Psi}$. In practice, the sparsity transform is often selected from well-studied classes of transforms where MR images are known to be sparse. For example, orthonormal wavelet transforms are often used as the sparsity transform in CS MRI. Within this class of transforms, there is usually no strong preference to select a particular wavelet basis since many wavelets yield qualitatively and quantitatively similar reconstruction. This point is illustrated in Fig. 1. A T2-weighted axial MR image was retrospectively undersampled in k-space to retain coefficients along 64 radial lines and the above l_1 minimization framework was used to reconstruct images using orthonormal Daubechies wavelets with different number of vanishing moments as the sparsity transform. Sections of the resulting images are shown in Figure 1 to enable closer inspection of the reconstruction artifacts. In the figure, it is easy to see that while the overall reconstruction quality is very similar for the different bases, each image exhibits different reconstruction artifacts. Our proposed framework originates from this key observation.

Method: The requirement of incoherence between the sparsity and measurement bases in CS means that undersampling artifacts accumulate incoherently and manifest themselves as small coefficients in the sparsity basis. By eliminating the small coefficients that are most likely due to undersampling artifacts or noise, the desired signal is recovered. In our new approach, the sparsity constraints are enforced in a collection of bases. Each basis provides a sparse representation and captures unique characteristics of the signal. In addition, the undersampling artifacts are different in each basis. A large coefficient due to undersampling artifacts in one basis (which would normally be hard to distinguish from signal and would result in artifacts in the final image) is likely to result in small coefficients in the other basis. Thus, by requiring that the result be sparse in multiple bases, a significantly larger portion of the undersampling artifacts can be removed. The pseudo-code for the proposed algorithm is presented in Fig. 2. The signal is iteratively projected onto a collection of bases and the small coefficients in each basis are removed by thresholding. This step is followed by projection of the signal back onto the Fourier basis and assertion of consistency with measured data. These steps are repeated in an iterative fashion for a fixed number of iterations or until convergence is achieved. The algorithm was tested using a radial-FSE dataset which was acquired with TR=4.5s, FOV=26cm and ETL=4 with 8 coils using 256 radial views and 256 points along each radial view. The data was retrospectively subsampled along 64 radial lines in k-space and images were reconstructed using l_1 minimization as well as the proposed algorithm. In the proposed technique, Daubechies wavelets with vanishing moments of 1,3,5,7, and 9 were used.

Results: Fig. 3a shows the original image. The image obtained using the proposed technique is shown in Fig 3b. For reference, images obtained using l_1 minimization with the Daubechies wavelets 4 and 6 are illustrated in Figs. 3c and 3d, respectively. It can be seen that the images obtained using l_1 minimization contain excessive smoothing, ringing artifacts, and reduced contrast, while the image obtained using the proposed technique is much closer to the image obtained from the full dataset.

Conclusion: A novel sparse reconstruction technique is introduced that is based on simultaneous sparsity in a collection of bases. The proposed method is demonstrated in radial MRI and results in significantly improved reconstruction quality compared to conventional CS reconstruction. While results are presented employing a collection of wavelets bases, the proposed method is general and can be easily used with other bases as well.

Acknowledgements: AZ Alzheimer's Consortium, AZ Cancer Center, and NIH (HL085385). **References:** [1] Candes E et al. IEEE Transactions on Information Theory (2006). [2] Donoho D IEEE Transactions on Information Theory (2006). [3] Lustig M et al. MRM (2007).