

# Three-Dimensional Compressed Sensing for Dynamic MRI

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**Introduction:** Dynamic contrast enhanced (DCE) magnetic resonance imaging (MRI) is a valuable tool used in a number of clinical applications. However, imaging of time-varying objects is a challenging task when both high spatial resolution and high temporal resolution is desired. It has been demonstrated that radial imaging techniques can yield increased temporal resolution without sacrificing spatial resolution and are less susceptible to motion [1,2]. However, highly undersampled radial trajectories result in increased streaking artifacts and low SNR. The recently introduced Compressed Sensing (CS) theory illustrates that a small number of linear measurements can be sufficient to reconstruct sparse or compressible signals [3,4] and has the potential to significantly accelerate data acquisition in MRI [5,6,7]. In this work, we introduce a CS theory based method for reconstruction of time-varying radial k-space data by exploiting the spatio-temporal sparsity of DCE-MRI images.

**Theory:** Let us assume that we would like to reconstruct a dynamic object with  $N \times N$  pixels at  $T$  time instances and that we will acquire  $L_\theta$  radial views at each time instance with  $L_r$  points along each radial line. Let  $\mathbf{f}$  be the  $N^2 T$  dimensional vector representing the dynamic object being imaged,  $\mathbf{M}$  the  $L_\theta L_r T \times N^2 T$  measurement matrix, and  $\mathbf{g}$  a  $L_\theta L_r T$ -dimensional vector of k-space measurements. Note that since we are interested in reconstructing the time-varying object from severely undersampled data,  $L_\theta L_r T \ll N^2 T$ . Here,  $\mathbf{M}$  is an undersampled Fourier projection matrix that maps the dynamic object  $\mathbf{f}$  to the acquired k-space data:  $\mathbf{g} = \mathbf{M}\mathbf{f}$ . Dynamic MR images exhibit redundancy in both space and time. This redundancy can be exploited by using a linear transform such as a wavelet transform. Let  $\Psi$  denote the  $N^2 T \times N^2 T$  transform matrix such that  $\Psi\mathbf{f}$  is sparse. In other words,  $\Psi\mathbf{f}$  has only  $S$  non-zero values where  $S \ll N^2 T$ . CS theory suggests that  $\mathbf{f}$  can be recovered by solving the convex optimization problem “ $\min_{\mathbf{f}} \|\Psi\mathbf{f}\|_1$  subject to  $\|\mathbf{M}\mathbf{f} - \mathbf{g}\|_2 < \epsilon$ ” where  $\epsilon$  controls the fidelity between the measurements and the reconstruction and is used to account for noise in the measurements.

**Method:** Mice were prepared for DCE-MRI as described previously [8]. Radial-FSE data sets were acquired continuously with TR=100ms and TE=9ms for 64 time points using 256 radial views and 256 points along each radial view. Gd-DOTP (0.03 mmole/Kg) was injected at time point 20 during imaging. Co-registered pre-contrast images were also acquired at different TR times, for calculating a pre-contrast T1 map. The performances of three different reconstruction methods were compared on full datasets and subsampled datasets by reducing the number of radial views at each time point. The first method used non-uniform FFT (NUFFT) to reconstruct the radial views at each time point. In the second method (2D CS), the k-space data for each time point was processed independently using CS reconstruction. Note that this can be achieved by simply setting  $T=1$  in the above formulation. In the third method (3D CS), we processed all of the data jointly. When subsampling the original data set, we used bit-reversed view ordering such that the view angles of subsequent time points bisected those of the previous ones. Three-dimensional wavelet transforms were used to achieve sparsity. A Haar wavelet was used along the temporal dimension and Symlets were used along the spatial dimensions. The reconstructed dynamic images and pre-contrast T1 map were fitted to a 3-compartment pharmacokinetic model in order to quantitatively assess renal function in the mice.

**Results and Discussion:** Fig. 1 shows images obtained using different reconstruction algorithms and different undersampling factors at one time instance. Figs. 1a and 1b show NUFFT reconstructions with 16 and 64 radial views, respectively. Severe undersampling artifacts and low SNR can be observed in these images. 2D CS reconstructions of the same k-space data are shown in Figs. 1c and 1d. While the radial undersampling artifacts are reduced and the SNR is increased, the 2D CS images exhibit wavelet artifacts since only a limited number of wavelet coefficients could be recovered during the reconstruction. The corresponding 3D CS images are shown in Figs. 1e and 1f. In addition to the reduced undersampling artifacts and increased SNR, these images also demonstrate high spatial resolution. Our MATLAB implementation required ~25-50 seconds (depending on the number of radial views) for 2D CS reconstruction of each time frame on a computer with a 2.8GHz Intel Xeon processor. The 3D CS reconstruction required ~60-120 minutes for the  $256 \times 256 \times 64$  volume on the same computer. Fig. 2a shows a high resolution anatomical reference image, and Figs. 2b-2d show parameter maps of volumetric filtration function (mL/min per mL tissue) calculated by fitting the images from different reconstructions with 16 views to the 3-compartment model. Note that the separation between the renal cortex, medulla and pelvis is surprisingly visible for the CS datasets even when only 16 radial views are used. The parameter map obtained using 3D CS shows a better correspondence to the anatomical image (Fig. 2a) than the 2D CS map.

**Conclusion:** A CS theory based reconstruction method has been introduced for dynamic radial imaging. The proposed method significantly reduces undersampling artifacts and can provide high temporal and spatial resolution. The proposed method can also be combined with parallel imaging methods as well as other partial k-space acquisition methods such as homodyne reconstruction for further acceleration.

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**References:** [1] Peters, DC et al. MRM (2000). [2] Vigen, KK et al. MRM (2000). [3] Candes E et al. IEEE Transactions on Information Theory (2006). [4] Donoho DL, IEEE Transactions on Information Theory (2006). [5] Lustig M et al. Proc. ISMRM (2005). [6] Lustig M et al. Proc. ISMRM (2006). [7] Lustig M et al. Proc. ISMRM (2006). [8] Raghunand et al., MRM 55:1272-1280, 2006.

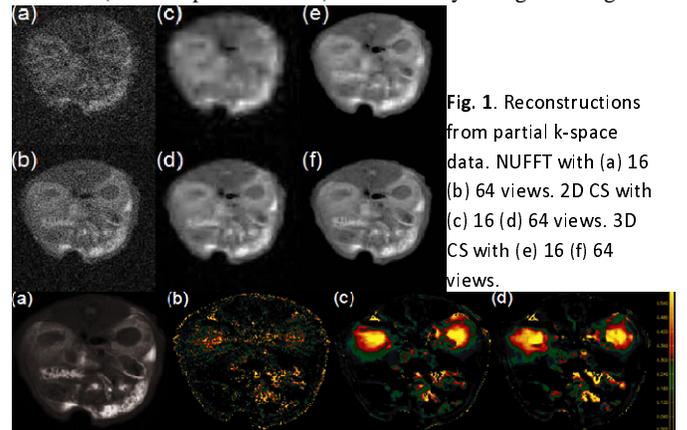


Fig. 1. Reconstructions from partial k-space data. NUFFT with (a) 16 (b) 64 views. 2D CS with (c) 16 (d) 64 views. 3D CS with (e) 16 (f) 64 views.

Fig. 2. (a) High resolution anatomical reference. Parameter maps of volumetric filtration function (b) NUFFT (c) 3D CS (d) 2D CS.