# Rapid Imaging using Undersampled Radial Trajectories and L1 Reconstruction

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**Introduction:** Compressed Sensing (CS) is an emerging field that suggests that data compression can be implicitly incorporated into the data acquisition process [1,2,3]. CS is regarded by some as an alternative to Shannon's Nyquist sampling theory. The Nyquist sampling theory states that the number of samples required to perfectly reconstruct a signal is determined by its bandwidth. In contrast, the CS theory states that by using nonlinear algorithms based on convex optimization, certain class of signals can be reconstructed perfectly from what appears to be highly incomplete data. More specifically, the CS theory states that sparse or compressible signals can be recovered from a small number of random linear measurements. These results are of practical significance to MR imaging, since MR imaging is performed using linear measurements of the object (in k-space) and the MR images often have sparse (or compressible) representations (e.g. using wavelets or finite differences).

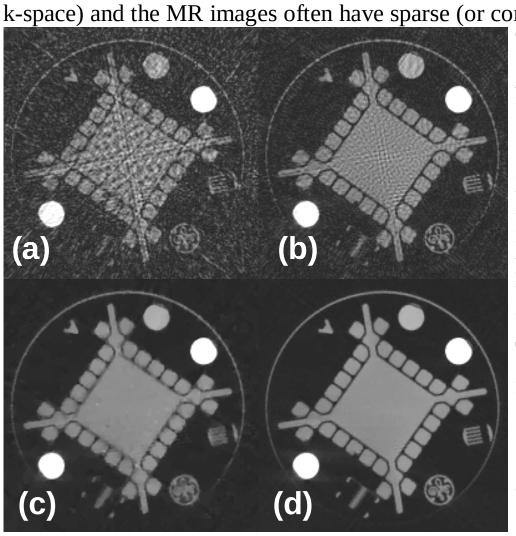


**Fig. 1**. Reconstruction from partial k-space data. Filtered Backprojection (a) 32 views (b) 64 views. TV minimization (c) 32 views (d) 64 views.

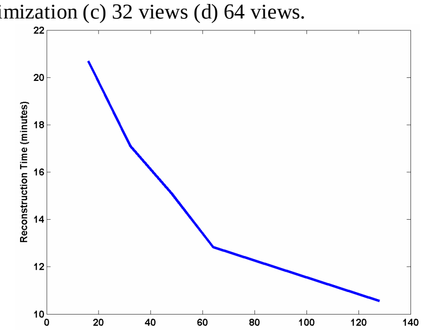


**Fig. 2**. Reconstruction time for different number of views for TV minimization.

The main difficulty in applying these initial theoretical results to MRI lay in performing random sampling in k-space. Since such random sampling is difficult for MRI hardware, recent reports have suggested some alternative strategies for randomizing the measurements. In [5], a rapid imaging method based on randomly perturbed and undersampled spirals was introduced. In [6], a randomly undersampled 3DFT trajectory was used for rapid imaging. Random ordering of the phase encodes in time was used in [7] to randomly undersample k-t space in dynamic cardiac imaging. However, recent developments in CS theory suggest that such random sampling may not always be necessary [4]. More specifically, these recent results indicate that CS methods can yield exact recovery if the sparsity basis and the measurement system obey the uniform uncertainty principle and are incoherent [4]. In this work, we illustrate that it is possible to use the CS theory principles with radial trajectories as suggested in [3].

**Theory:** Let $\mathbf{f}$ denote the object being imaged, $\mathbf{M}$ the measurement matrix, and $\mathbf{g}$ the measurements. In MRI, $\mathbf{M}$ is an undersampled Fourier matrix and $\mathbf{g}$ is the measured k-space data such that $\mathbf{g} = \mathbf{Mf}$. Furthermore, let $\Psi(\mathbf{f})$ denote the projection of the object $\mathbf{f}$ onto a sparsity basis $\Psi$ where $\Psi(\mathbf{f})$ is sparse. The main result of CS theory suggests that $\mathbf{f}$ is the unique solution to the convex optimization problem

$$\min_{\mathbf{f}} \|\Psi(\mathbf{f})\|_1 \text{ subject to } \mathbf{g} = \mathbf{Mf}$$

with very high probability [1,2,3]. Recent results also indicate that the theory can be further extended to cover compressible signals in addition to the signals that are truly sparse [4].

**Method:** We illustrate that the CS theory can be applied to reconstruct MR images from radially undersampled k-space data. We restate the reconstruction problem as a minimization of the total variation (TV) of the image $f$ with quadratic constraints

$$\min_{f} \|f\|_{TV} \text{ subject to } \|Mf - g\|_2 < \varepsilon$$

where $\varepsilon$ controls the fidelity between the measurements and the reconstruction and is used to account for noise in the measurements. TV of image $f$ is the sum of the magnitudes of the discretized spatial gradient of the image given by $\|f\|_{TV} = \sum_i \sum_j \sqrt{(f(i+1,j) - f(i,j))^2 + (f(i,j+1) - f(i,j))^2}$. In our experiments, we took an image which was originally acquired using 256 radial views and 256 points along each radial view using a SE pulse sequence. Using this image, we created undersampled datasets by only keeping partial data in k-space along a few radial lines. We then reconstructed images from these partial datasets using the TV minimization formulation as stated above. For comparison, we also reconstructed images using traditional filtered backprojection.

**Results and Discussion:** Figure 1 illustrates the images obtained using different reconstruction algorithms and different number of radial views. Figures 1a and 1b illustrate the images reconstructed using filtered backprojection from 32 and 64 radial views, respectively. Images reconstructed using the TV minimization method from 32 and 64 radial views are illustrated in Figs. 1c and 1d, respectively. Note that the TV minimization method yields high resolution images with significantly reduced aliasing artifacts. A plot of the TV minimization reconstruction times for different number of radial views using a Matlab™ implementation on a computer with a 3.4 Ghz Intel Xeon™ processor is given in Figure 2. It is interesting to note that the reconstruction time reduces with increased number of views since convergence can be achieved faster using more data. In comparison, the filtered backprojection reconstruction (implemented in C) requires only a few seconds (including I/O) on the same computer.

**Conclusion:** We illustrated that radial k-space trajectories can be used together with the emerging CS theory. While the reconstruction times of the proposed method is considerably longer than those of the traditional filtered backprojection, the method significantly reduces the undersampling artifacts and results in high resolution images.

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**References:** [1] Candes E et al. "Robust uncertainty principles: Exact recovery from highly incomplete Fourier information," IEEE Transactions on Information Theory. Feb. 2006. [2] Donoho DL, "Compressed Sensing," IEEE Transactions on Information Theory. Apr. 2006. [3] Candes E and Romberg J, "Practical Signal Recovery from Random Projections", preprint. [4] Candes E and Tao T, "Near optimal signal recovery from random projections: universal encoding strategies?" preprint. [5] Lustig M et al. Proc. ISMRM (2005). [6] Lustig M et al. Proc. ISMRM (2006). [7] Lustig M et al. Proc. ISMRM (2006).