

# A VECTOR QUANTIZER FOR IMAGE RESTORATION

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## ABSTRACT

An algorithm based on nonlinear interpolative vector quantization (NLIVQ) is presented which accomplishes image restoration concurrently with image compression. The algorithm is applied to the problem of deblurring noise-free diffraction-limited images by training with a large set of blurred and original image pairs. Simulation results demonstrate a quantitative improvement in images processed by the algorithm, as measured by image peak signal-to-noise ratio (PSNR), as well as a significant improvement in perceived image quality. A theoretical formulation of the algorithm is presented along with a discussion of implementation, training and simulation results.

## 1. INTRODUCTION

Vector quantization (VQ) is generally thought of as a tool for reducing the number of bits required to represent certain kinds of data, or in other words, for achieving data compression [1]. In recent years, however, researchers have presented VQ algorithms which perform other signal processing tasks concurrently with compression. These range from speech processing tasks such as speaker recognition and noise suppression, to image processing tasks like half-toning, edge detection, enhancement, classification, and reconstruction [2]. In this paper, we present VQ for image restoration.

Image restoration is concerned with correction of degradations found in images acquired by less than perfect imaging systems [3]. In this research, we use a nonlinear algorithm to remove blur caused by diffraction-

limited optics. Nonlinear algorithms have been employed successfully in many image restoration problems. Of particular note are the new generation of super-resolving algorithms. These algorithms are capable of restoring frequency information outside the passband of the optical system, and it has been shown that linear shift-invariant restoration methods cannot achieve super-resolution [4] [5]. We are also interested in the possibility of attaining super-resolution using this VQ approach.

This novel VQ application is based on the nonlinear interpolative VQ (NLIVQ) found in [6]. The NLIVQ described below uses a large training set consisting of blurred images and their corresponding original unblurred versions to accomplish the design. We assume that the training set accurately represents the statistics of the class of images to be restored.

## 2. ALGORITHM FORMULATION

Let  $\{F^i, G^i\}_{i=1}^n$  be a sequence of image pairs, where  $F^i$  and  $G^i$  are the original and diffraction-limited  $N \times N$  images, respectively. Decompose each image pair of the sequence into a series of  $M \times M$  blocks which will serve as the basic units for VQ processing. Let  $f^{ik}$  and  $g^{ik}$  be block  $k$  from  $F^i$  and  $G^i$ , respectively. As in [6], we assume that the encoder  $\mathbf{E}$ , decoder  $\mathbf{D}$ , and the associated codebook  $\mathbf{C}$ , are given for a VQ that minimizes the distortion

$$D = E \left[ \|g^{ik} - \tilde{g}^{ik}\|^2 \right]. \quad (1)$$

The process for choosing  $\tilde{g}^{ik}$  can be written as

$$\tilde{g}^{ik} = \mathbf{D}(\mathbf{E}(g^{ik})) = \arg \min_{c_l \in \mathbf{C}} \|g^{ik} - c_l\|^2, \quad (2)$$

where  $c_l$  refers to entry  $l$  of  $\mathbf{C}$ .

We define the nonlinear VQ restoration algorithm as a new decoder  $\mathbf{D}^*$ , and its associated codebook  $\mathbf{C}^*$ , which minimizes the conditional expectation

$$D = E \left[ \|f^{ik} - \tilde{f}^{ik}\|^2 \mid \mathbf{E}(g^{ik}) = l \right], \quad (3)$$

where  $\mathbf{E}$  returns the index of the matching codebook entry. For a given set of training data, let  $R_l = \{f^{ik} : \mathbf{E}(g^{ik}) = l\}$ . Define entry  $l$  of  $\mathbf{C}^*$  as the centroid of  $R_l$ , or

$$c_l^* = \left( \frac{1}{|R_l|} \right) \sum_{f^{ik} \in R_l} f^{ik}. \quad (4)$$

Finally, the nonlinear VQ restoration algorithm is given by

$$\tilde{f}^{ik} = \mathbf{D}^*(\mathbf{E}(g^{ik})) = c_{\mathbf{E}(g^{ik})}^*, \quad (5)$$

where  $\tilde{f}^{ik}$  is the restored image block.

### 3. SIMULATION RESULTS

We found it expedient to design the nonlinear VQ algorithm for mean-removed image blocks, leaving restoration of the mean as a separate task. The problem of estimating the mean of the restored block is not difficult. We have found that a simple inverse filter restoration can serve as a basis for this process.

The simulations here utilized a block size of 3x3 with 14 bits allocated for the mean removed image blocks. Assuming eight bits for the mean yields a bit rate of approximately 2.5 bits/pixel. The training set consisted of twenty  $512 \times 512$  images of varied subject matter. The originals were filtered with a diffraction-limited OTF (optical cutoff equal to half the folding frequency) to form the corresponding diffraction-limited training set. No noise was added to the filtered images.

Restoration results are shown in Fig. 1 where the original, diffraction-limited, and restored images are presented for an aerial image of an urban area. This image was not in the training set. These simulations use the correct value for the mean as computed from the original image in order to examine the performance of the VQ under ideal conditions. Other simulations were conducted in which the DC values were estimated from an image restored by a simple inverse filter. These produced nearly identical results. Peak signal to noise ratio (PSNR) values of images processed by our algorithm

improved by 1.5 to 2 dB, indicating a quantitative improvement in the images. Moreover, there is a substantial improvement in perceived image quality due to the human visual system (HVS) preference for sharp edges.

### 4. CONCLUSION

The application of NLIVQ to the image restoration problem is but one of many possible uses for this interesting technique. In this case, the NLIVQ training process determines the important statistical properties of the data and accomplishes the design of a nonlinear restoration algorithm. The results indicate that a reasonably good passband restoration was accomplished, and there is evidence to suggest modest super-resolution. Further research will involve attaining higher bit rates, using larger sets of training data, and experimenting with different encoder/decoder structures. It is also interesting to note that NLIVQ is similar in many respects to a multilayer neural network, which leads one to believe that similar results could be achieved with that approach.

### 5. REFERENCES

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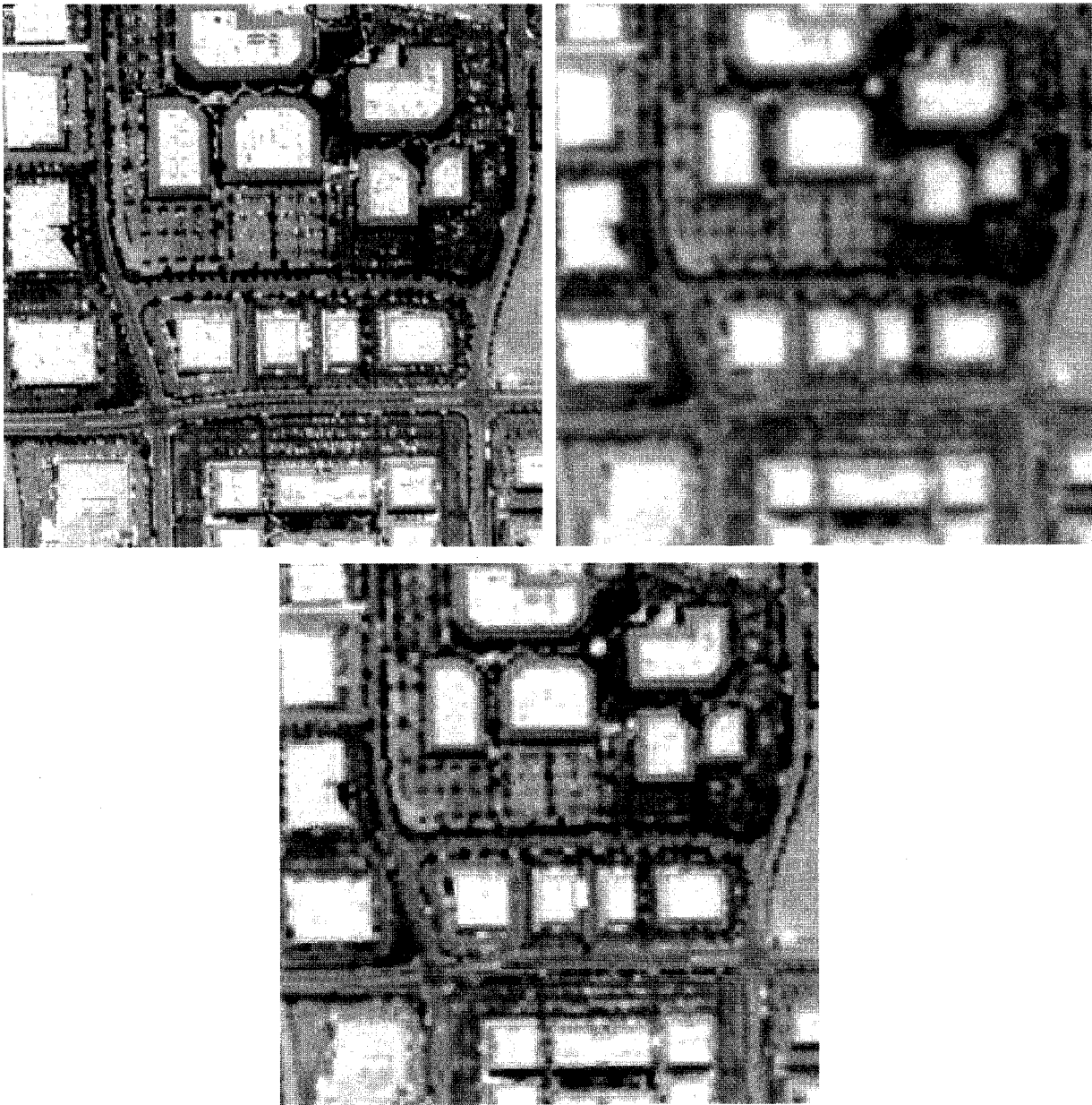


Figure 1: NLIVQ restoration of an urban image. Upper Left: chip from original test image; Upper Right: chip from diffraction limited image; Bottom: chip from NLIVQ restored image (2.5 bits/pixel).